

Student Name: \_\_\_\_\_

## MATHEMATICAL METHODS (CAS)

### Units 3 & 4 – Written examination 2



### 2009 Trial Examination

Reading time: 15 minutes

Writing time: 2 hours

### QUESTION & ANSWER BOOK

#### Structure of book

| <i>Section</i> | <i>Number of questions</i> | <i>Number of questions to be answered</i> | <i>Number of marks</i> | <i>Suggested times (minutes)</i> |
|----------------|----------------------------|-------------------------------------------|------------------------|----------------------------------|
| 1              | 22                         | 22                                        | 22                     | 33                               |
| 2              | 5                          | 5                                         | 58                     | 87                               |
| Total 80       |                            |                                           |                        |                                  |

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, one bound reference, one approved graphics calculator ( memory DOES NOT need to be cleared) and, if desired, one scientific calculator
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

#### Materials supplied

- Question and answer book of 24 pages including answer sheet for multiple-choice questions.

#### Instructions

- Print your name in the space provided on the top of this page and the multiple-choice answer sheet.
- All written responses must be in English.

**Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic communication devices into the examination room.**

**Instructions for Section 1**

Answer all questions on the answer sheet provided for multiple choice questions.

Choose the response that is **correct** for the question.

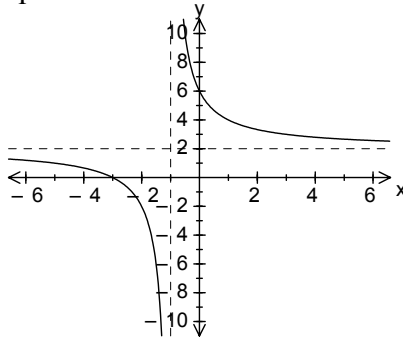
A correct answer scores 1, an incorrect answer scores 0.

Marks will **not** be deducted for incorrect answers.

No marks will be given if more than one answer is completed for any question.

**Question 1**

The graph shown is given by the equation:



A.  $y = \frac{1}{x-1} - 2$

B.  $y = \frac{2}{x+1} + 2$

C.  $y = \frac{2x+6}{x+1}$

D.  $y = \frac{1}{x+1} - 2$

E.  $y = \frac{x+1}{2x+2}$

**Question 2**

The derivative of  $x \log_e(x^2 - x)$  with respect to  $x$  is given by:

A.  $x(2x-1)\log_e(x^2 - x)$

B.  $\log_e(x^2 - x) + \frac{2x-1}{x-1}$

C.  $\frac{2x-1}{x^2 - x}$

D.  $\frac{2x-1}{x-x}$

E.  $x(x^2 - x) + 2x - 1$

**Question 3**

The **exact** range of the function  $f : (e, 5] \rightarrow R, f(x) = -2 \log_e x$ , is:

- A.  $[-2 \log_e(5), -2)$
- B.  $(-2, -2 \log_e(5)]$
- C.  $[-3.2, -2)$
- D.  $(-2, -3.2]$
- E.  $(-2 \log_e(5), -2]$

**Question 4**

The inverse of the function  $f : (-\infty, 2] \rightarrow R, f(x) = (x - 2)^2 + 3$  is:

- A.  $f^{-1} : [2, \infty) \rightarrow R, f^{-1}(x) = 2 - \sqrt{x - 3}$
- B.  $f^{-1} : [3, \infty) \rightarrow R, f^{-1}(x) = 2 + \sqrt{x - 3}$
- C.  $f^{-1} : [2, \infty) \rightarrow R, f^{-1}(x) = \sqrt{x} - 1$
- D.  $f^{-1} : [3, \infty) \rightarrow R, f^{-1}(x) = 2 - \sqrt{x - 3}$
- E.  $f^{-1} : [3, \infty) \rightarrow R, f^{-1}(x) = 2 + \sqrt{x - 3}$

**Question 5**

If the graph of  $y = g(x)$  is transformed by a reflection in the  $y$  axis, a dilation by a factor of 2 from the  $y$  axis and a translation 2 units horizontally (to the right), the resulting graph would have the equation:

- A.  $y = g(2 - 2x)$
- B.  $y = g\left(1 - \frac{x}{2}\right)$
- C.  $y = -g\left(\frac{x}{2} - 1\right)$
- D.  $y = g\left(2 - \frac{x}{2}\right)$
- E.  $y = -g\left(\frac{x}{2} - 2\right)$

**TURN OVER**

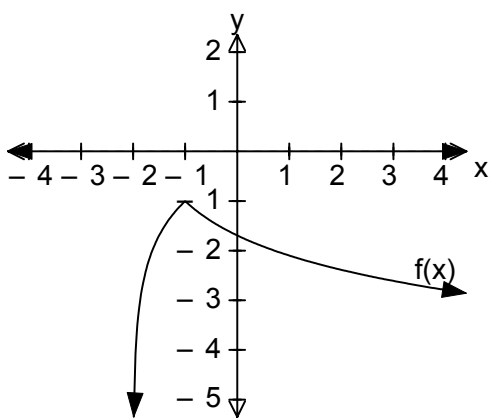
**Question 6**

The solution of the equation  $e^{2x} - e^x = 6$  is:

- A.  $\ln(6)$
- B.  $\ln(3), \ln(2)$
- C.  $\ln(3)$
- D. 1
- E.  $\ln(3), -\ln(2)$

**Question 7**

The graph of the function  $f$  is shown below:

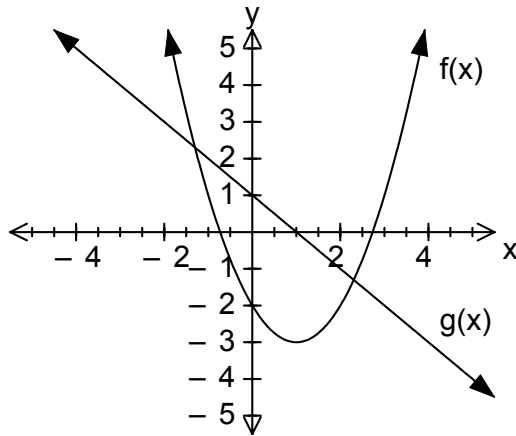


The equation of the function  $f$  is given by:

- A.  $f(x) = -|\log_e(x+1)| - 2$
- B.  $f(x) = |\log_e(x-2)| + 1$
- C.  $f(x) = -|\log_e(x-2)| + 1$
- D.  $f(x) = -|\log_e(x+2)| - 1$
- E.  $f(x) = |\log_e(x+2)| - 1$

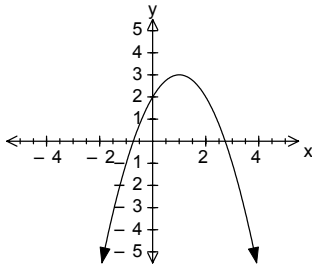
**Question 8**

The graphs of  $f(x) = (x-1)^2 - 3$ , and  $g(x) = 1 - x$  are shown below

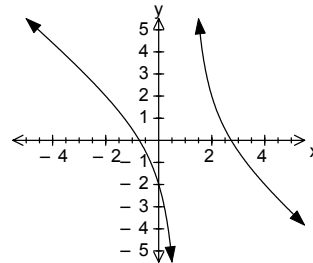


If  $h(x) = f(x).g(x)$ , then the graph of  $h$  is represented by:

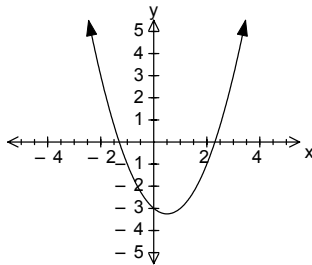
**A.**



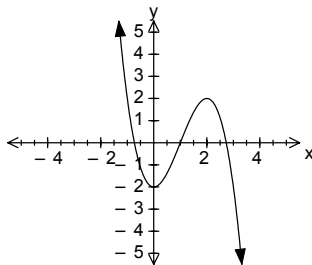
**D.**



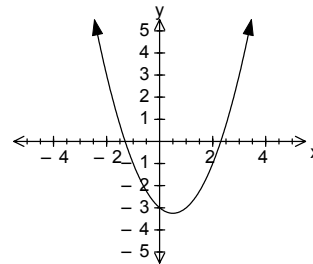
**B.**



**C.**



**E.**



**TURN OVER**

**Question 9**

A spherical ice-cube is slowly melting. The rate of change of the volume with respect to  $r$ , in  $\text{cm}^3 / \text{cm}$ , when the **diameter** is  $2 \text{ cm}$ , is:

A.  $16\pi$

B.  $8\pi$

C.  $4\pi$

D.  $\frac{64\pi}{3}$

E.  $\frac{4\pi}{3}$

**Question 10**

If  $\int_0^k (4x - 5)dx = 3$  then  $k$  is given by:

A.  $-\frac{1}{2}, 3$

B.  $-\frac{1}{2}$

C.  $3$

D.  $2$

E.  $2, 3$

**Question 11**

If  $\Pr(A|B) = 0.3$ ,  $\Pr(A \cup B) = 0.6$  and  $\Pr(B) = 0.4$ , then  $\Pr(A)$  is:

A.  $0.20$

B.  $0.32$

C.  $0.82$

D.  $0.10$

E.  $0.48$

**Question 12**

The value of  $E(X)$  for the following probability distribution is:

|              |      |      |      |      |
|--------------|------|------|------|------|
| $x$          | 0    | 1    | 2    | 3    |
| $\Pr(X = x)$ | 0.25 | $2k$ | 0.45 | $4k$ |

- A. 1.00
- B. 0.90
- C. 2.30
- D. 1.85
- E. 1.60

**Question 13**

The chance of Daisy scoring a goal in netball is 0.85. If she has 30 shots at goal during a game the probability (correct to 4 decimal places) that she scores at least 27 goals is:

- A. 0.8486
- B. 0.6783
- C. 0.1514
- D. 0.7972
- E. 0.3217

**Question 14**

$f(x)$  is a smooth, continuous curve, such that :

$f(2) = 2, f(5) = 7, f(0) = 5, f'(2) = 0, f'(5) = 0, f'(x) > 0$  for  $x \in (2,5) \cup (5,\infty)$  and

$f'(x) < 0$  for  $x \in (-\infty,2)$  then the general form of the equation of  $f$  is:

- A.  $f(x) = ax^4 + bx^3 + cx^2 + dx + e$
- B.  $f(x) = ae^{x-b} + c$
- C.  $f(x) = ax^3 + bx^2 + cx + d$
- D.  $f(x) = ax^2 + bx + c$
- E.  $f(x) = \frac{a}{x-b} + c$

**TURN OVER**

**Question 15**

If  $X \sim N(1.5, 0.0625)$ , then  $\Pr(x > 1.6 | X < 1.8)$ , correct to 4 decimal places is:

- A. 0.0548
- B. 0.2594
- C. 0.2295
- D. 1.0000
- E. 0.3894

**Question 16**

If  $X$  is a random variable with a probability density function defined by

$$f(x) = \begin{cases} \frac{1}{2} \sin\left(\frac{x}{2}\right) & \pi \leq x \leq 2\pi \\ 0 & \text{elsewhere} \end{cases}$$

The mode is given by:

- A.  $\frac{5\pi}{4}$
- B.  $2\pi$
- C.  $\pi$
- D.  $\frac{3\pi}{2}$
- E.  $\frac{7\pi}{4}$

**Question 17**

The  $x$  – intercepts for  $y = 4 \sin\left(\frac{x}{2}\right) + 2, x \in [-2\pi, 2\pi]$  are equal to:

- A.  $\frac{-2\pi}{3}, \frac{-4\pi}{3}$
- B.  $\frac{-5\pi}{6}, \frac{-\pi}{6}$
- C.  $\frac{-5\pi}{3}, \frac{-\pi}{3}, \frac{\pi}{3}, \frac{5\pi}{3}$
- D.  $\frac{-5\pi}{6}, \frac{-\pi}{6}, \frac{\pi}{6}, \frac{5\pi}{6}$
- E.  $\frac{-5\pi}{3}, \frac{-\pi}{3}$

**Question 18**

If  $\int_{-1}^3 (g(x))dx = -6$  then  $\int_{-1}^3 \left(5 - \frac{g(x)}{2}\right)dx$  is equal to:

- A. 23
- B. 7
- C. 13
- D. 8
- E. 2

**Question 19**

If  $f(x) = \frac{4}{3-2x}$ , then  $\int f(x+1)dx$  is equal to:

- A.  $-2 \log_e |1-2x| + c$
- B.  $8 \log_e |1-2x| + c$
- C.  $-4 + c$
- D.  $2 \log_e |1-2x| + c$
- E.  $-8 \log_e |1-2x| + c$

**Question 20**

The area of the region bounded by the curve  $f(x) = x^2 - 7x + 10, x \in R$ , the  $x$  - axis and  $y$  -axis in square units, is :

- A.  $13\frac{1}{6}$
- B.  $4\frac{1}{2}$
- C.  $4\frac{1}{6}$
- D.  $8\frac{2}{3}$
- E.  $6\frac{5}{6}$

**TURN OVER**

**Question 21**

If  $Z \sim N(0,1)$ , and  $\Pr(-z < Z < z) = 0.48$ , then the value of  $z$ , correct to 4 decimal places is:

- A. 0.6433
- B. 0.0502
- C. 0.3156
- D. 0.3874
- E. 0.7063

**Question 22**

A function has the rule  $p(t) = 26 \ln(3t + 1)$ . The average rate of change of  $p$  with respect to  $t$ , between  $t = 3$  and  $t = 5$  is **exactly** equal to:

- A.  $-13 \ln(1.6)$
- B. 6.11
- C. 3.00
- D.  $13 \ln\left(\frac{8}{5}\right)$
- E. 7.80

## SECTION 2

### Instructions for Section 2

Answer **all** questions in the spaces provided.

A decimal approximation will not be accepted if an **exact** answer is required to a question.

In questions where more than one mark is available, appropriate working **must** be shown.

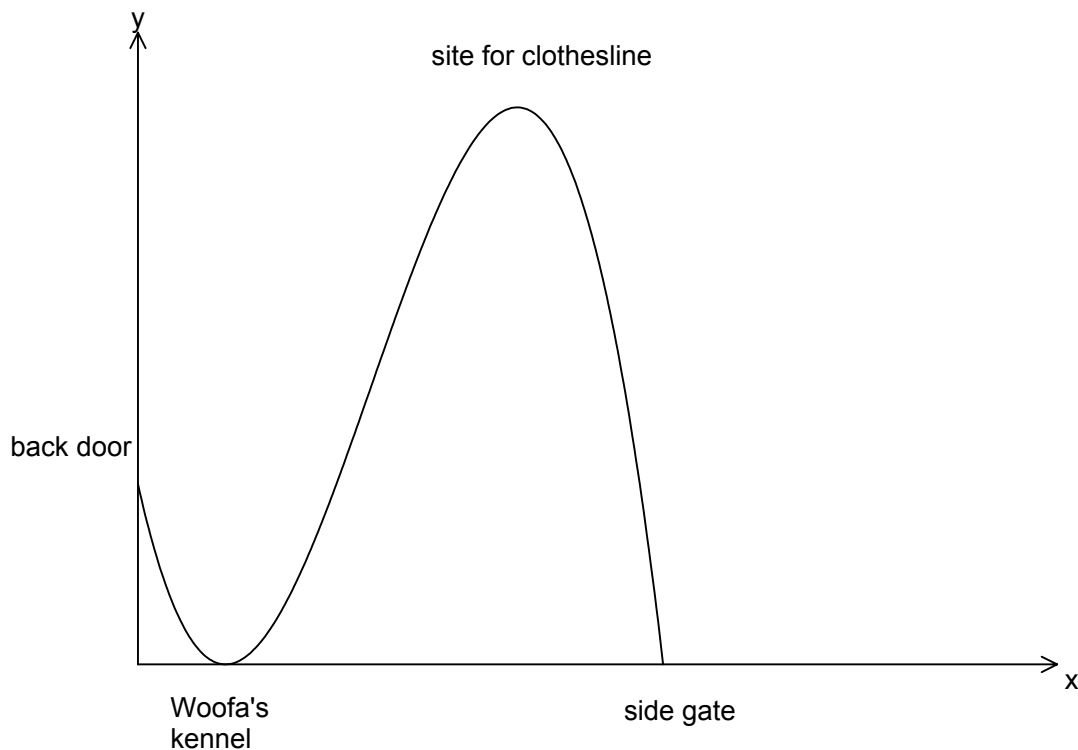
Where an instruction to **use calculus** is stated for a question, you must show an appropriate derivative or anti-derivative.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

### Question 1

Tom is landscaping his backyard. He decides to plot it on a grid, to help with the design. His back door is located at  $(0,3)$ , Woofa's kennel is located at  $(1,0)$  and the side gate is located at  $(6,0)$ .

He wants to install a path joining the points in the shape of a cubic curve, so that one of the turning points occurs at Woofa's kennel as shown in the diagram.



**TURN OVER**

- a. If the path has the general equation:  $y = a(x - b)^2(x - c)$ , show that  $a = \frac{-1}{2}$ ,  $b = 1$ ,  $c = 6$ .

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2 marks

- b. If the clothesline is to be located at the second turning point, find the exact co-ordinates of the clothesline.

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3 marks

- c. Tom marks out a straight line,  $f(x)$  between the back door and the point on the path (4,9). Find the equation of this line.

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2 marks

- d. The area between  $f(x)$  and the path will be covered with lawn. Use calculus to find the area of lawn that will be sown.

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2 marks

- e. Tom is thinking of putting in a fence for a vegetable patch, so that it is a **normal** to the curve at  $x = 2$ . Find the equation of  $g(x)$ , representing this fence.

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3 marks

- f. Show if this fence will fit in the yard before the side gate.

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1 mark

**TURN OVER**

**Question 2**

- a. If  $a$  and  $b$  are positive integers, such that  $a^x = b^{1-x}$ , show by using natural logarithms that

$$x = \frac{\ln(b)}{\ln(a) + \ln(b)}$$

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2 marks

- b. Hence, if  $9^x = 4^{1-x}$ , show that  $x = \frac{\ln(2)}{\ln(6)}$

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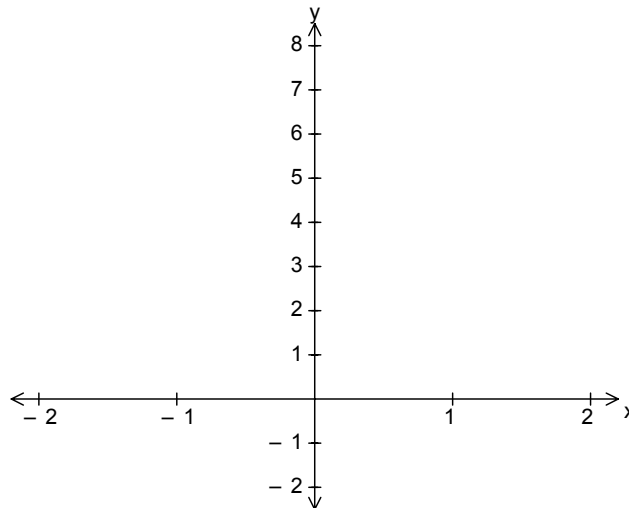
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2 marks

- c. Sketch  $f(x) = 5^x$ , and  $g(x) = e^{1-x}$ , on the same set of axes. Show asymptotes and axis intercepts.



2 marks

- d. State the exact value of  $x$  at the point where the curves intersect.

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2 marks

**Question 3**

The temperature in a butcher's cool room,  $t$  hours after 9 *am* is modelled by the equation

$$T(t) = -5 \cos\left(\frac{\pi}{12}(t + 6)\right) + 4$$

- a.** Find the temperature range in the cool room

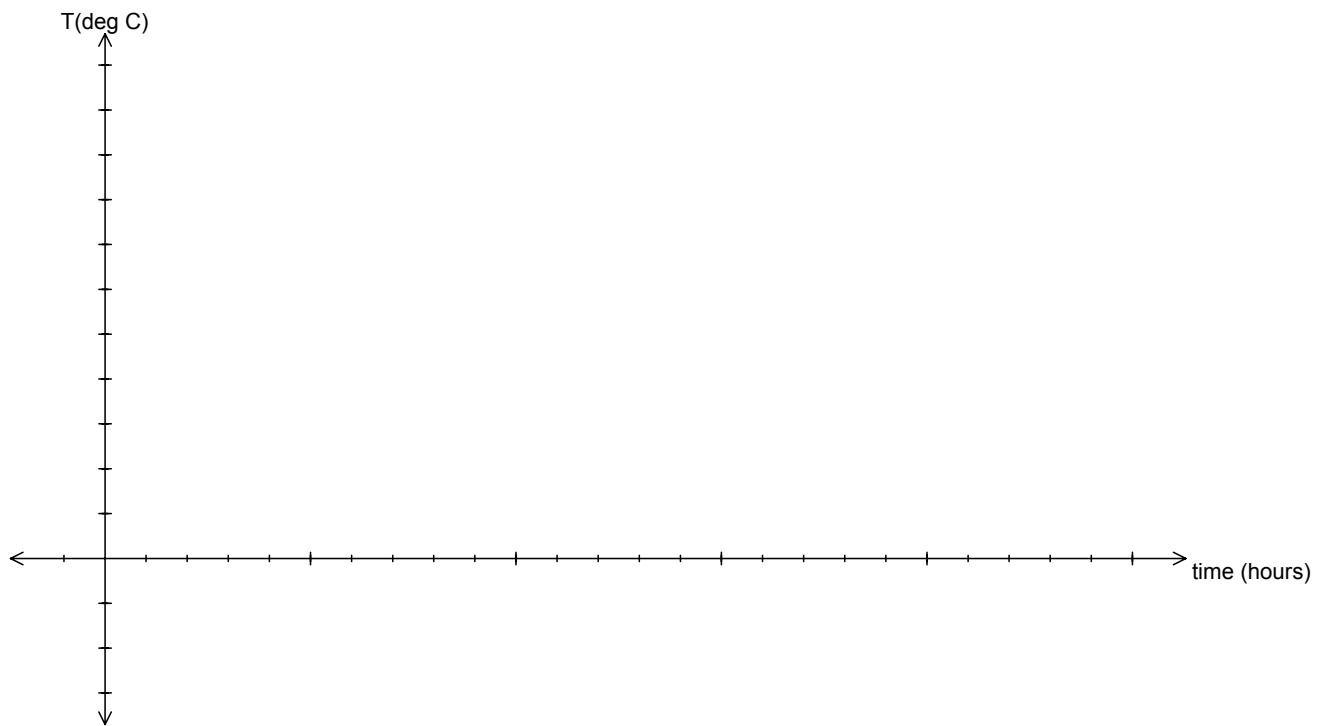
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1 mark

- b.** Sketch the graph of the temperature in the cool room, over a 24 hour period. Give the axis intercepts (correct to 1 decimal place) and turning points.



3 marks

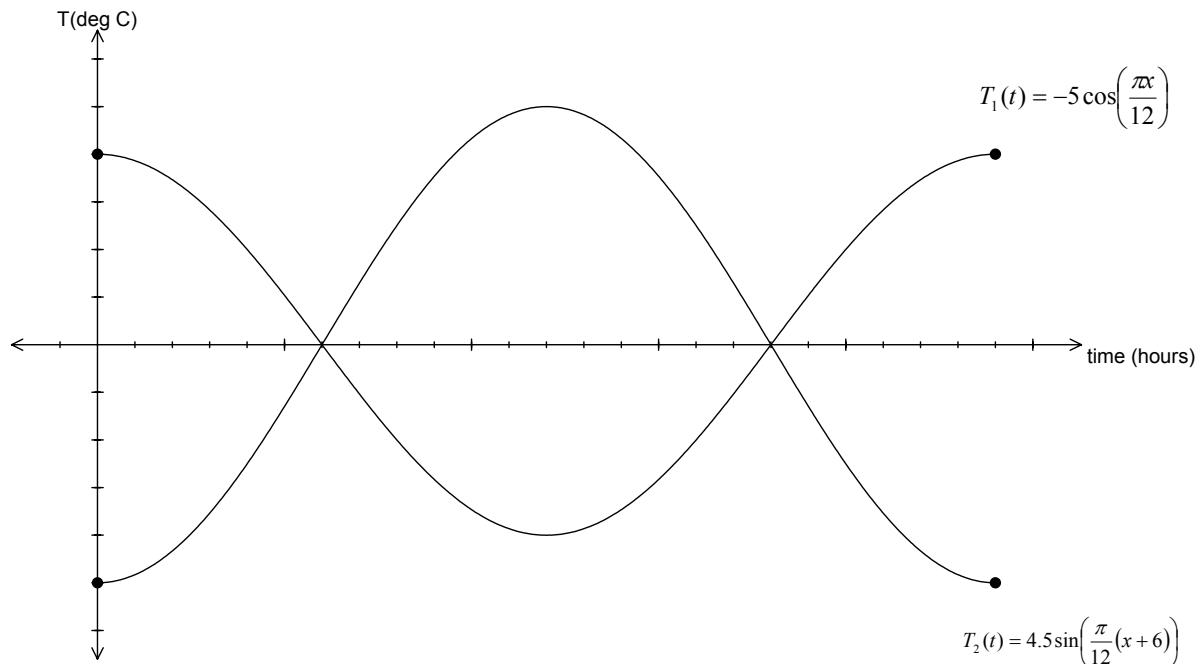
**TURN OVER**

This temperature range is too great, so the butcher investigates a new cool room that has two cooling systems in operation. The temperature produced individually by the two systems is given by the following equations:

$$T_1(t) = -5 \cos\left(\frac{\pi t}{12}\right) \text{ and } T_2(t) = 4 \sin\left(\frac{\pi}{12}(t + 6)\right)$$

The sum of these two functions,  $T_3 = T_1 + T_2$  gives the overall temperature in the cool room,  $t$  hours after 9 am. The graphs of  $T_1$  and  $T_2$  are shown below.

- c. Use addition of ordinates to sketch the new graph  $T_3$ , over a 24 hour period.



1 mark

- d. What is the temperature range of the new cool room?

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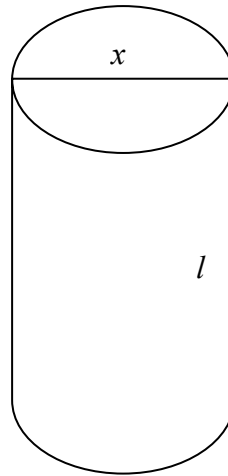
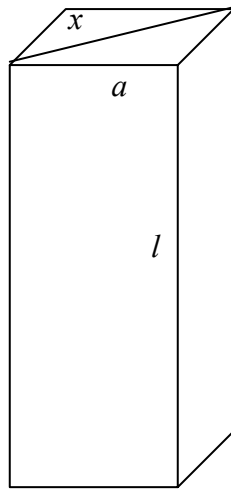


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1 mark

**Question 4**

A company makes small cylindrical rods and small square end rods that have the same maximum width  $x$  cm. The rods must be the same length, so they can be used for the same job.



- a.** If the cylindrical rods have a volume of  $100 \text{ cm}^3$ , show that the length of the rods is given by

$$l = \frac{400}{\pi x^2} \text{ cm}$$

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2 marks

- b.** Find the exact volume of the square end rods.

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2 marks

**TURN OVER**

## 2009 MATHMETHODS (CAS) EXAM 2

It is known that in the process of manufacturing, 3% of the rods are faulty. After manufacturing, the rods are boxed in cartons of 100.

- c. If a carton of rods is randomly selected, find the probability (to 4 decimal places) that no more than 2 rods are faulty in the carton.

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2 marks

- d. Find the probability (to 4 decimal places) that at least nine out of ten cartons have no more than 2 faulty rods per carton.

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2 marks

Their customers order their cartons of rods monthly. The company has found that 65% of their customers order round end rods, if their previous order was for round ends, whereas 75% of their customers order square end rods if their previous order was for square ends.

- e. If a customer orders round ended rods in January, find the probability (correct to 4 decimal places) that they will order round ended rods in April.

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3 marks

- f. If a customer orders round ended rods in January, find the probability (correct to 4 decimal places) that they will order square ended rods on at least 2 of their next 3 orders.

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2 marks

The cost of producing the rods  $\$C$  is a continuous random variable with a probability function

given by 
$$C(x) = \begin{cases} \frac{6}{5}(x^2 - x) & 1 \leq x \leq 2 \\ 0 & \text{elsewhere} \end{cases}$$

g.

- i. Find the median cost of production.

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3 marks

- ii. Find the probability (correct to 4 decimal places) that the cost of production of the rods will be less than \$1.60

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2 marks

**TURN OVER**

The weights  $W$  g of the rods are normally distributed with a mean of 108 g. It is known that 11% of the rods are under 107 g.

**h**

- i.** Find the standard deviation, correct to 2 decimal places.

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3 marks

- ii.** Find the interval between which approximately 95% of the weights of the rods would lie. Give values correct to 2 decimal places.

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1 mark

**Question 5**

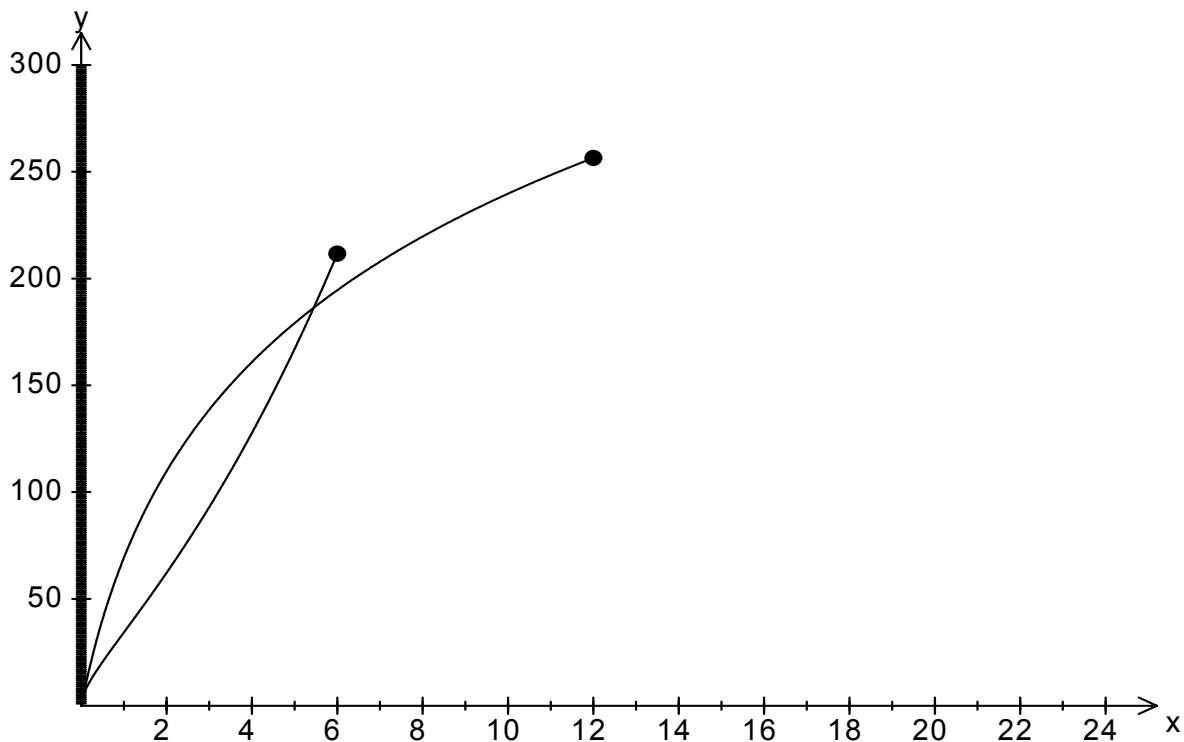
Two new drugs have been developed that help with pain relief for terminally ill patients. The tablets must be given together and they provide 24 hour relief to the patients. Tablet A is absorbed into the blood stream over twelve hours and it is cleared over the next twelve hours. Tablet B is absorbed into the blood stream over six hours and is cleared over the next six hours. This can be modelled by the following equations:

$$\text{Tablet A } \begin{cases} A_1 = 100 \ln(t+1) & 0 \leq t \leq 12 \\ A_2 = 100 \ln(25-t) & 12 < t \leq 24 \end{cases} \quad \text{and} \quad \text{Tablet B } \begin{cases} B_1 = 20(e^{\sqrt{x}} - 1) & 0 \leq t \leq 6 \\ B_2 = 20(e^{\sqrt{12-x}} - 1) & 6 < x \leq 24 \end{cases}$$

Where  $A_1$   $A_2$   $B_1$   $B_2$  represent the amount of active chemical in  $mg$  in the bloodstream after  $t$  hours

The graphs of  $A_1$  and  $B_1$  are shown below.

- a. On the same set of axes, sketch the graphs of  $A_2$  and  $B_2$ .



1 mark

**TURN OVER**

**b.**

- i.** Describe the transformations that have changed equation  $A_1$  to equation  $A_2$ .

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1 mark

- ii.** Describe the transformations that have changed equation  $B_1$  to equation  $B_2$ .

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1 mark

**c.**

- i.** What is the maximum amount of A absorbed into the bloodstream correct to 2 decimal places?

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1 mark

- ii.** What is the maximum amount of B absorbed into the bloodstream correct to 2 decimal places?

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1 mark

Maximum relief for the patients is reached when the amount of A and B are added together, giving the greatest concentration of drugs in the bloodstream.

- d.** What is the greatest concentration of the two drugs in the bloodstream?

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1 mark

- e. How long after initially taking the tablets will maximum relief be achieved (to the nearest minute)?

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1 mark

A period of very good relief is achieved when the level A is greater than  $125\text{ mg}$  **and** the level of B is greater than  $125\text{mg}$  at the same time.

- f. How long does this period of very good relief last for to the nearest minute?

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2 marks

**END OF QUESTION AND ANSWER BOOK**

**MULTIPLE CHOICE ANSWER SHEET**

Student Name: \_\_\_\_\_

Circle the letter that corresponds to each correct answer.

| Question |   |   |   |   |   |
|----------|---|---|---|---|---|
| 1        | A | B | C | D | E |
| 2        | A | B | C | D | E |
| 3        | A | B | C | D | E |
| 4        | A | B | C | D | E |
| 5        | A | B | C | D | E |
| 6        | A | B | C | D | E |
| 7        | A | B | C | D | E |
| 8        | A | B | C | D | E |
| 9        | A | B | C | D | E |
| 10       | A | B | C | D | E |
| 11       | A | B | C | D | E |
| 12       | A | B | C | D | E |
| 13       | A | B | C | D | E |
| 14       | A | B | C | D | E |
| 15       | A | B | C | D | E |
| 16       | A | B | C | D | E |
| 17       | A | B | C | D | E |
| 18       | A | B | C | D | E |
| 19       | A | B | C | D | E |
| 20       | A | B | C | D | E |
| 21       | A | B | C | D | E |
| 22       | A | B | C | D | E |

# MATHEMATICAL METHODS (CAS)

## Units 3 & 4 – Written examination 2



## 2009 Trial Examination

### SOLUTIONS

#### SECTION 1: Multiple-choice questions (1 mark each)

##### Question 1

*Answer:* C

*Explanation:*  $x=0$ ,  $y$  intercept is 6 and C is the only one that gives this result (or do long division and other features also become clear)

##### Question 2

*Answer:* B

*Explanation:* product rule:  $\ln(x^2 - x) + \frac{x(2x-1)}{x(x-1)}$   
 $\ln(x^2 - x) + \frac{(2x-1)}{(x-1)}$

##### Question 3

*Answer:* A

**B**  
*Explanation:* sub  $x=e$ ,  $y = -2$ , sub  $x=5$ ,  $y=-2\ln(5)$   
giving  $[-2\ln(5), -2]$

**Question 4***Answer:* D*Explanation:* swap  $x$  and  $y$  for inverse

$$x = (y - 2)^2 + 3$$

$$\pm \sqrt{x - 3} = y - 2, \text{ due to domain only negative}$$

$$y = -\sqrt{x - 3} + 2, \text{ domain of } f(x) \text{ is the range of the inverse function}$$

$$f^{-1} : [3, \infty) \rightarrow \mathbb{R}, f^{-1}(x) = -\sqrt{x - 3} + 2$$

**Question 5***Answer:* B

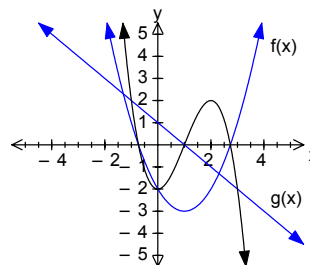
$$\begin{aligned} \text{Explanation: } y &= g\left(-\frac{1}{2}(x-2)\right) \\ &= g\left(-\frac{x}{2} + 1\right) \Rightarrow g\left(1 - \frac{x}{2}\right) \end{aligned}$$

**Question 6***Answer:* C

$$\text{Explanation: let } e^x = a, \Rightarrow a^2 - a - 6 = 0$$

$$(a - 3)(a + 2) = 0$$

$$e^x = 3, e^x = -2, \Rightarrow x = \ln(3), \text{ only}$$

**Question 7***Answer:* D*Explanation:* graph has been translated 2 units left, absolute valued, reflected in the  $x$ -axis then translated 1 unit down**Question 8***Answer:* C*Explanation:* product of functions key points are  $x=0, 1, -1$ 

**Question 9**

*Answer:* C

$$\begin{aligned} \text{Explanation: } V &= \frac{4}{3}\pi r^3 \\ \frac{dV}{dr} &= 4\pi r^2, r = 1 \\ \frac{dV}{dr} &= 4\pi \end{aligned}$$

**Question 10**

*Answer:* A

$$\begin{aligned} \text{Explanation: } [2x^2 - 5x]_0^k &= 3 \\ 2k^2 - 5k - 3 &= 0 \\ (2k + 1)(k - 3) &= 0 \\ k &= -\frac{1}{2}, 3 \end{aligned}$$

**Question 11**

*Answer:* B

$$\begin{aligned} \text{Explanation: } 0.3 &= \frac{\Pr(A \cap B)}{0.4} \\ \Pr(A \cap B) &= 0.12 \\ 0.6 &= \Pr(A) + 0.4 - 0.12 \\ \Pr(A) &= 0.32 \end{aligned}$$

**Question 12**

*Answer:* E

$$\begin{aligned} \text{Explanation: } 0.7 + 6k &= 1 \\ k &= 0.05 \\ E(X) &= 0 + 0.1 + 0.9 + 0.6 = 1.6 \end{aligned}$$

**Question 13**

Answer: E

Explanation:  $1 - \text{binomcdf}(30, 0.85, 26) = 0.3217$

**Question 14**

Answer: A

Explanation: turning point at (2,2), point of inflection at (5,7) makes it a quartic curve

**Question 15**

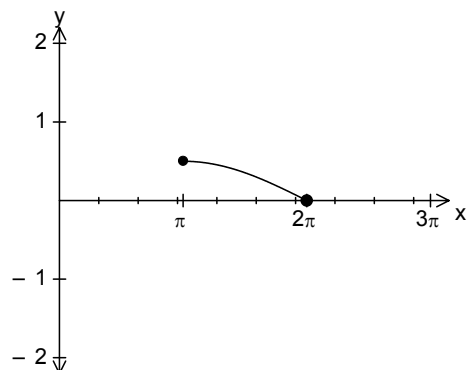
Answer: B

Explanation:  $\frac{\Pr(1.6 < X < 1.8)}{\Pr(X < 1.8)} = \frac{\text{normalcdf}(1.6, 1.8, 1.5, 0.25)}{\text{normalcdf}(-10^{99}, 1.8, 1.5, 0.25)} = 0.2594$

**Question 16**

Answer: C

Explanation: mode is the highest point,  $x = \pi$



**Question 17**

Answer: E

Explanation: let  $y = 0$

$$0 = 4 \sin\left(\frac{x}{2}\right) + 2, x \in [-\pi, \pi]$$

$$-\frac{1}{2} = \sin\left(\frac{x}{2}\right)$$

$$\frac{x}{2} = -\frac{\pi}{6}, -\frac{5\pi}{6} \Rightarrow x = -\frac{\pi}{3}, -\frac{5\pi}{3}$$

**Question 18***Answer:* A

*Explanation:*  $\int_{-1}^3 (5)dx - \frac{1}{2} \int_{-1}^3 (g(x))dx$

$$[5x]_{-1}^3 - \frac{1}{2}(-6)$$

$$15 + 5 + 3 = 23$$

**Question 19***Answer:* A

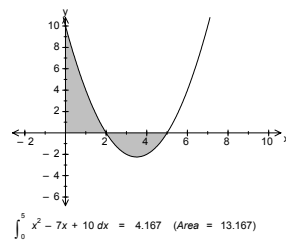
*Explanation:*  $\int \left( \frac{4}{3 - 2(x-1)} \right) dx$

$$\int \left( \frac{4}{1 - 2x} \right) dx$$

$$-\frac{4}{2} \ln|1 - 2x| + c \Rightarrow -2 \ln|1 - 2x| + c$$

**Question 20***Answer:* A*Explanation:*

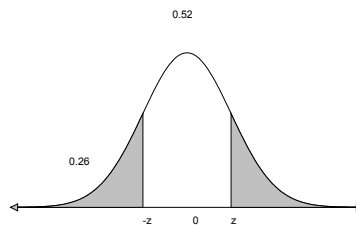
$$\int_0^2 (x^2 - 7x + 10)dx - \int_2^5 (x^2 - 7x + 10)dx = 13\frac{1}{6} \text{ sq. units}$$



**Question 21**

*Answer:* A

*Explanation:*  $\text{invNorm}(0.26, 0, 1) = 0.6433$



**Question 22**

*Answer:* D

*Explanation:*  $x = 5, y = 26 \ln(16), \text{ and } x = 3, y = 26 \ln(10)$

$$\text{avrate} = \frac{26 \ln(16) - 26 \ln(10)}{5 - 3}$$

$$= \frac{26 \left( \ln \left( \frac{16}{10} \right) \right)}{2}$$

$$= 13 \ln \left( \frac{8}{5} \right)$$

**SECTION 2: Analysis Questions****Question 1**

**a.** Turning point at  $x = 1 \therefore b = 1$ ,  $x$  intercept at  $x = 6 \therefore c = 3$ ,

At back door  $(0,3)$  sub into equation  $3 = a(-1)^2(-6)$

$$3 = -6a$$

$$a = -\frac{1}{2}, b = 1, c = 6$$

M1+A1  
2 marks

**b.** expand (or use product rule) or use CAS calculator.

$$y = -\frac{1}{2}x^3 + 4x^2 - \frac{13}{2}x + 3$$

$$\frac{dy}{dx} = -\frac{3}{2}x^2 + 8x - \frac{13}{2}, \quad \frac{dy}{dx} = 0$$

$$0 = -3x^2 + 16x - 13$$

$$\therefore TP.is\left(4\frac{1}{3}, 9\frac{7}{27}\right)$$

M2+A1  
3 marks

**c.**  $(0,3)(4,9) \Rightarrow m = \frac{9-3}{4-0} = \frac{3}{2}$

$$y - 3 = \frac{3}{2}(x - 0)$$

$$y = \frac{3}{2}x + 3$$

M1+A1  
2 marks

**d.** Find point on curve  $Area = \int_0^4 \frac{3x + 6 + x^3 - 8x^2 + 13x - 6}{2} dx$

$$\int_0^4 8x + \frac{x^3}{2} - 4x^2 dx$$

$$\left[ 4x^2 + \frac{1}{8}x^4 - \frac{4}{3}x^3 \right]_0^4$$

$$10\frac{2}{3} sq units$$

M1+A1  
2 marks

e. Let  $x=2, y=2$  ( $f(2)=2$ )

$$\frac{dy}{dx} = -\frac{3}{2}x^2 + 8x - \frac{13}{2}, x=2$$

$$m_t = 3.5, \Rightarrow m_n = -\frac{2}{7}$$

$$y-2 = -\frac{2}{7}(x-2)$$

$$\therefore g(x) = -\frac{2x}{7} + 2\frac{4}{7}$$

M2+A1  
3 marks

f. Sub  $y=0, x=9$  so no, it won't fit before the side gate

A1  
1 mark

## Question 2

a.  $a^x = \frac{b}{b^x}$

$$a^x b^x = b$$

$$(ab)^x = b$$

$$\log_{ab} b = x$$

$$x = \frac{\ln(b)}{\ln(ab)}$$

$$\frac{\ln(b)}{\ln(a) + \ln(b)}$$

M1+A1  
2 marks

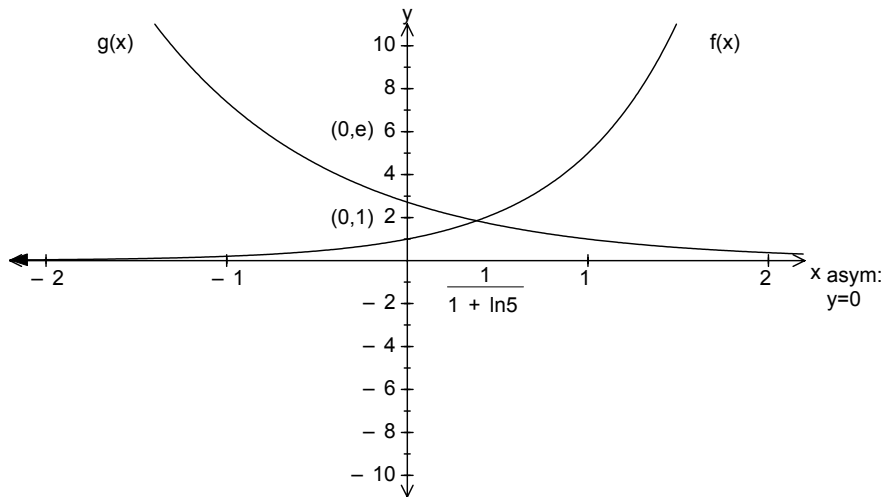
b. 
$$\frac{\ln(4)}{\ln(9) + \ln(4)}$$
  

$$\frac{2\ln(2)}{2\ln(3) + 2\ln(2)}$$
  

$$\frac{\ln(2)}{\ln(6)}$$

M1+A1  
2 marks

- c. Correct shape, show important points,  $(0,e)$ ,  $(0,1)$



A2  
2 marks

- d. point of intersection at  $x = \frac{1}{1 + \ln(5)}$

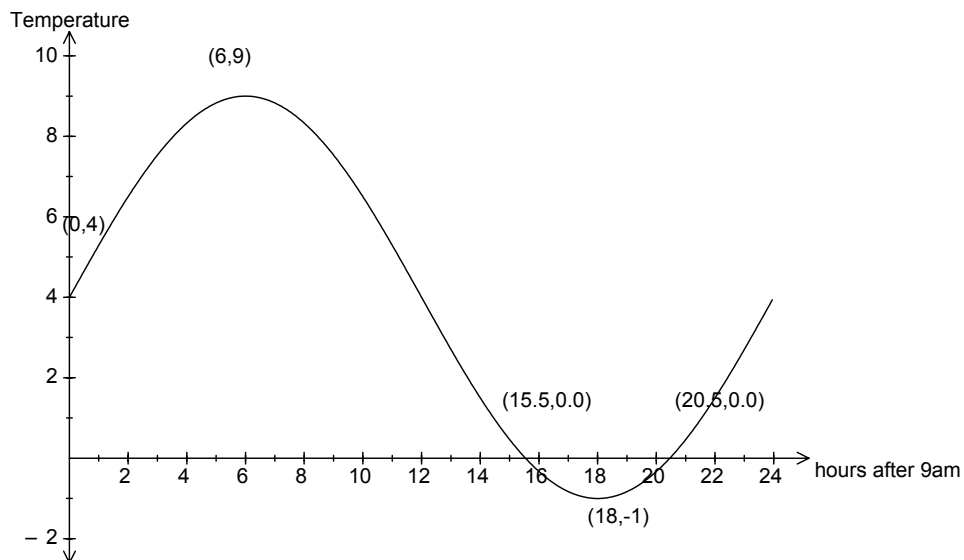
M1+A1  
2 marks

### Question 3

- a. Temperature range is  $[-1,9]$

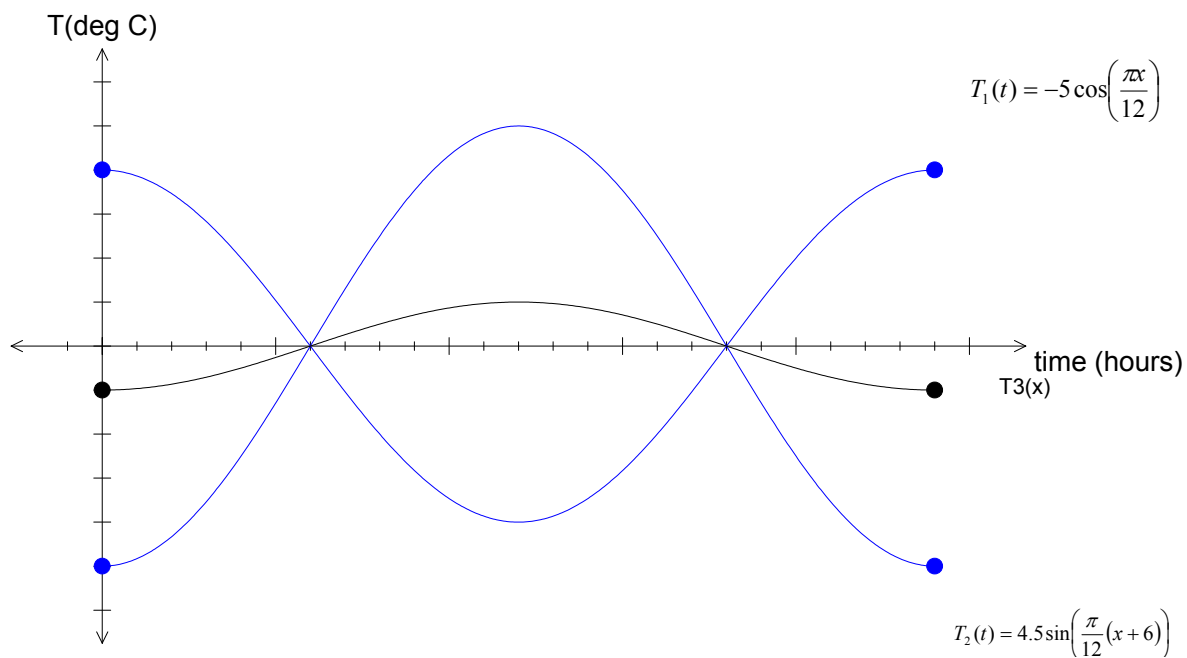
A1  
1 mark

- b.



A3  
3 marks

c.



A1  
1 mark

d.  $[-1,1]$

A1  
1 mark

#### Question 4

a.  $V = \pi \left(\frac{x}{2}\right)^2 l$

$$100 = \pi \left(\frac{x}{2}\right)^2 l$$

$$l = \frac{400}{\pi x^2}$$

M1+A1  
2 marks

b.  $V = a^2 l$

$$= \frac{x^2}{2} \times \frac{400}{\pi x^2}$$

$$= \frac{200}{\pi} \text{ cm}^3$$

M1+A1  
2 marks

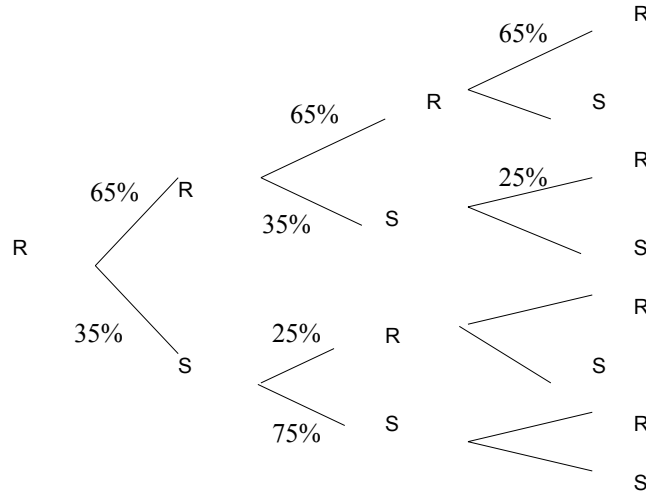
c.  $\Pr(X \leq 2) = \text{binomcdf}(100, .03, 2) = 0.4198$

M1+A1  
2 marks

d.  $\Pr(X \geq 9) = 1 - \text{binomcdf}(10, 0.4198, 1) = 0.0025$

M1+A1  
2 marks

e.



Pr (round in April|round in Jan)

$$= (0.65)^3 + 0.65 \times 0.35 \times 0.25 + 0.35 \times 0.25 \times 0.65 + 0.35 \times 0.75 \times 0.25$$

$$= 0.4540$$

M2+A1  
3 marks

f. Pr (square on at least 2 of next 3 | round in Jan)

$$= 0.65 \times 0.35 \times 0.75 + 0.35 \times 0.25 \times 0.35 + 0.35 \times 0.75 \times 0.25 + 0.35 \times (0.75)^2$$

$$= 0.4638$$

M1+A1  
2 marks

g.

i.  $\frac{6}{5} \int_1^m (x^2 - x) dx = 0.5$

$$\frac{6}{5} \left[ \frac{x^3}{3} - \frac{x^2}{2} \right]_1^m = 0.5$$

$$\frac{6}{5} \left[ \frac{m^3}{3} - \frac{m^2}{2} - \frac{1}{3} + \frac{1}{2} \right] = 0.5$$

Intersect on calculator  $m = 1.75$  therefore median cost is \$1.75

M2+A1  
3 marks

ii.  $\frac{6}{5} \int_1^{1.6} (x^2 - x) dx = 0.3024$

M1+A1  
2 marks

h.

i.  $\Pr(Z < z) = 0.11 \Rightarrow z = \text{invNorm}(0.11, 0, 1) = -1.2265$

$$\sigma = \frac{107 - 108}{-1.2265} = 0.82$$

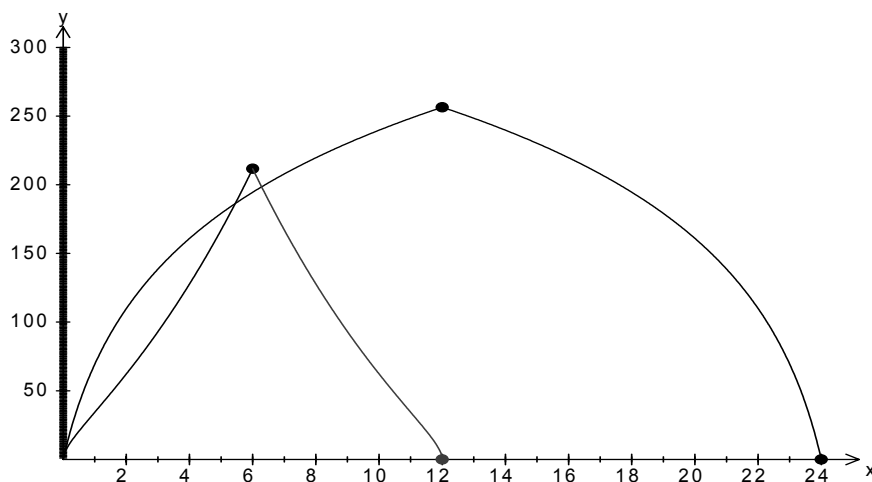
M2+A1  
3 marks

ii.  $\mu \pm 2\sigma \Rightarrow [106.36, 109.64]g$

A1  
1 mark

### Question 5

a.



A1  
1 mark

- b. i. reflected in y axis, then translated 24 units right  
ii. reflected in y axis then translated 12 units right

A2  
2 marks

- c. i. On calculator  $A = 256.49 g$   
ii.  $B = 211.56 g$

A2  
2 marks

- d.  $198.54025 \times 2 = 397.08g$  (2 x y part of intersection of  $A_2$  and  $B_1$ )

A1  
1 mark

- e. 6.28 hours = 6 hours 17min ( $x$  part of intersection of  $A_2$  and  $B_1$ )

A1  
1 mark

- f.  $X$  parts of intersection of  $A_1$  and  $y=125$  and  $A_2$  and  $y=125$  times are 3.9244 and 8.0756 hours  
Giving 4.1512 hours or 4 hours 9 min

M1+A1  
2 marks