

MATHEMATICAL METHODS

Units 3 & 4 – Written examination 1



2009 Trial Examination

SOLUTIONS

Question 1

$$\begin{aligned}\text{a. } f'(x) &= \frac{-2(3\sin(2x))(\sin(2x)) - 3\cos(2x)(2\cos(2x))}{\sin^2(2x)} \\ &= \frac{-6(\sin^2(2x) + \cos^2(2x))}{\sin^2(2x)} \\ &= \frac{-6}{\sin^2(2x)}\end{aligned}$$

M1 + A1
2 marks

$$\begin{aligned}\text{b. } \therefore \int \frac{-6}{\sin^2(2x)} dx &= \frac{3\cos(2x)}{\sin(2x)} + c \\ -3 \int \frac{2}{\sin^2(2x)} dx &= \frac{3\cos(2x)}{\sin(2x)} + c \\ \int \frac{2}{\sin^2(2x)} dx &= \frac{-\cos(2x)}{\sin(2x)} + c \\ \therefore \int \left(\frac{2}{\sin^2(2x)} + 1 \right) dx &= -\frac{\cos(2x)}{\sin(2x)} + x + c\end{aligned}$$

M1 + A1
2 marks

Question 2**a.** Let $x = 0$

$$y = -3 \log_e(2) - 1$$

$$= \log_e(2^{-3}) - 1$$

$$= \log_e\left(\frac{1}{8}\right) - \log_e(e)$$

$$= \log_e\left(\frac{1}{8e}\right)$$

M1 + A1
2 marks

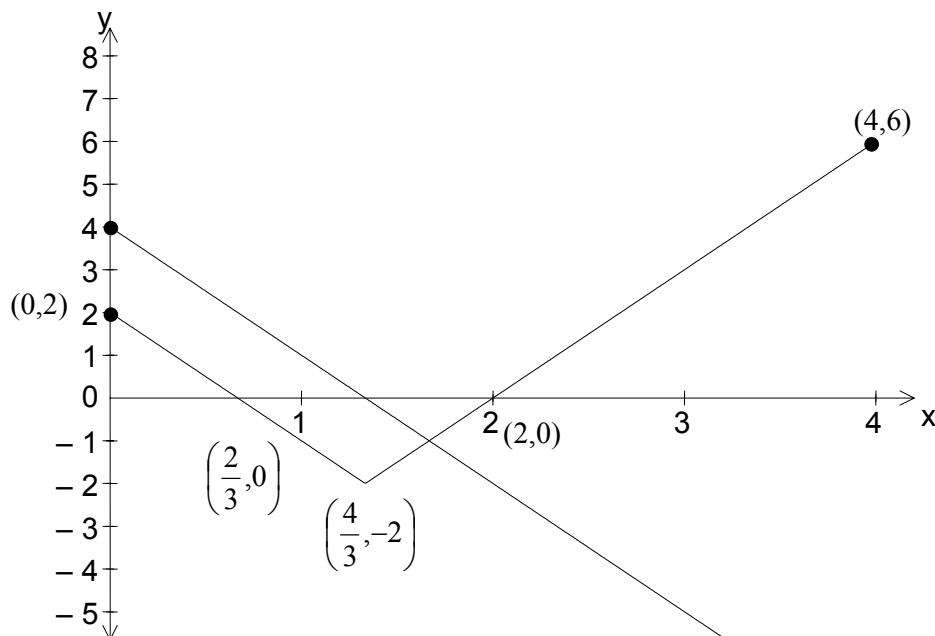
b. Let $x = -3 \log_e(y + 2) - 1$

$$x + 1 = -3 \log_e(y + 2)$$

$$y = e^{-\left(\frac{x+1}{3}\right)} - 2$$

$$\therefore f^{-1}(x) = e^{-\left(\frac{x+1}{3}\right)} - 2$$

M1 + A1
2 marks

Question 3**a.**

A1 + M1
2 marks

b. $h'(x)$ has domain $(0, 4) \setminus \left\{\frac{4}{3}\right\}$ or $\left(0, \frac{4}{3}\right) \cup \left(\frac{4}{3}, 4\right)$

A1
1 mark

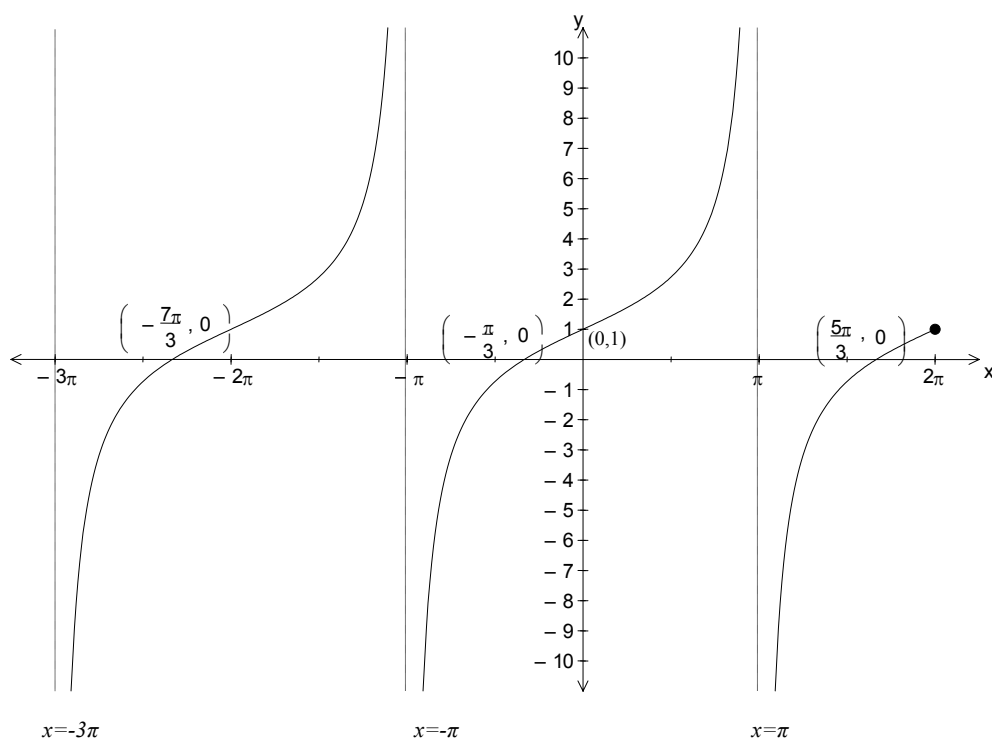
Question 4**a.**

$$\tan\left(\frac{x}{2}\right) = -\frac{1}{\sqrt{3}}, x \in \left[-\frac{3\pi}{2}, \pi\right]$$

$$\frac{x}{2} = -\frac{7\pi}{6}, -\frac{\pi}{6}, \frac{5\pi}{6}$$

$$x = -\frac{7\pi}{3}, -\frac{\pi}{3}, \frac{5\pi}{3}$$

M1 + A1
2 marks

b.

A3
3 marks

Question 5

$$\begin{aligned}
 \text{a. } \int_{-1}^1 (g(x) - f(x)) dx &= \\
 &= \int_{-1}^1 ((-x^3 - x^2 - x + 1) - (x^2 - 1)) dx \\
 &= \int_{-1}^1 (-x^3 - 2x^2 + x + 2) dx \\
 &= \left[-\frac{x^4}{4} - \frac{2x^3}{3} + \frac{x^2}{2} + 2x \right]_{-1}^1 \\
 &= \frac{-1}{4} - \frac{2}{3} + \frac{1}{2} + 2 - \left[\frac{-1}{4} + \frac{2}{3} + \frac{1}{2} - 2 \right] \\
 &= 4 - \frac{4}{3} \\
 &= 2\frac{2}{3} \text{ units}^2
 \end{aligned}$$

M2 + A1
3 marks

Question 6

$$\begin{aligned}
 f(x) &= x^{\frac{-1}{2}}, f'(x) = -\frac{1}{2}x^{\frac{-3}{2}} \\
 f(4.1) &\approx f(4) + 0.1 \times f'(4) \\
 &\approx \frac{1}{\sqrt{4}} + 0.1 \left(-\frac{1}{2} \right) (4)^{\frac{-3}{2}} \\
 &\approx \frac{1}{2} - \frac{1}{10} \times \frac{1}{2} \times \frac{1}{8} \\
 &\approx \frac{1}{2} - \frac{1}{160} \\
 &\approx \frac{79}{160}
 \end{aligned}$$

M2 + A1
3 marks

Question 7

- a. Median divides the data into halves. Therefore, the median is 2

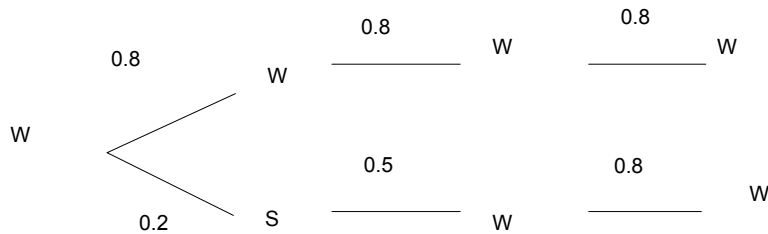
A1
1 mark

b.
$$\Pr(X = 5 | x > 2) = \frac{\Pr(X = 5)}{\Pr(X > 2)}$$
$$= \frac{0.15}{0.4}$$
$$= \frac{3}{8}$$

A1
1 mark

Question 8

- a.

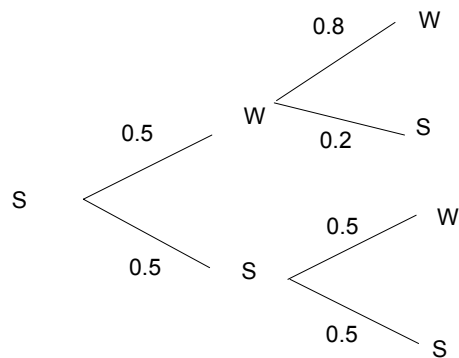


$$\Pr(\text{windy Sat and Sun} \setminus \text{windy Thurs}) = 0.512 + 0.080$$

$$= 0.592$$

M1 + A1
2 marks

- b.



$$\Pr(\text{windy on at least Sat or Sun} \setminus \text{still on Fri}) = 0.5 \times 0.8 + 0.5 \times 0.2 + 0.5 \times 0.5$$

$$= 0.750$$

M1 + A1
2 marks

Question 9

$$\begin{aligned}
 \text{a. } 1 &= \int_1^2 \left(\frac{x}{2} \right) dx + \int_2^4 (k) dx \\
 &= \left[\frac{x^2}{4} \right]_1^2 + [kx]_2^4 \\
 &= \frac{3}{4} + 2k \\
 \therefore k &= \frac{1}{8}
 \end{aligned}$$

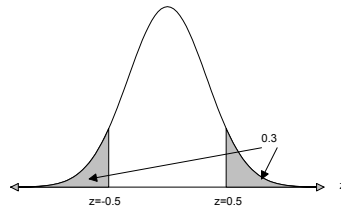
M1+A1
2 marks

$$\begin{aligned}
 \text{b. } \Pr(X \leq 3) &= \int_1^2 \left(\frac{x}{2} \right) dx + \int_2^3 \left(\frac{1}{8} \right) dx \\
 &= \frac{6+3-2}{8} = \frac{7}{8}
 \end{aligned}$$

A1
1 mark

Question 10

$$\text{a. } 0.5 = \frac{x-175}{10} \therefore x = 180 \text{ this is 5 above the mean so } \Pr(X > 170) = 0.7$$



M1+A1
2 marks

$$\text{b. } \Pr(X > 180 | X > 175) = \frac{\Pr(X > 180)}{0.5} = \frac{0.3}{0.5} = \frac{3}{5}$$

A1
1 mark

Question 11

$$\begin{aligned} \text{a. } SA &= \pi r^2 + 2\pi rh \\ 200\pi &= \pi(r^2 + 2rh) \\ \frac{200 - r^2}{2r} &= h \\ \therefore h &= \frac{100}{r} - \frac{r}{2} \end{aligned}$$

M1 + A1
2 marks

$$\begin{aligned} \text{b. } V &= \pi r^2 h \\ &= \pi r^2 \left(\frac{200 - r^2}{2r} \right) \\ &= 100\pi r - \frac{\pi r^3}{2} \end{aligned}$$

M1 + A1
2 marks

$$\begin{aligned} \text{c. } \frac{dV}{dr} &= 100\pi - \frac{3}{2}\pi r^2 \\ 0 &= 100\pi - \frac{3}{2}\pi r^2 \\ r^2 &= \frac{200}{3} \Rightarrow r = \sqrt{\frac{200}{3}}, \text{rationalise} \\ r &= \frac{10\sqrt{6}}{3} \text{ cm} \end{aligned}$$

M1 + A1
2 marks