

MATHEMATICAL METHODS (CAS)

Unit 1 – Written examination 2



2009 Trial Examination

SOLUTIONS

SECTION 1: Multiple-choice questions (1 mark each)

Question 1

Answer: E

Explanation:

$$\begin{aligned} & -\frac{15000}{3} \\ & = -5000L/h \end{aligned}$$

Question 2

Answer: C

Explanation:

$$\begin{aligned} m_n &= -\frac{1}{3} \\ y - 5 &= -\frac{1}{3}(x + 6) \\ y &= -\frac{1}{3}x + 3 \end{aligned}$$

Question 3

Answer: A

Explanation:

$\mathbb{R} \setminus \text{the interval } (-3, 7]$.

Question 4

Answer: B

Explanation:

Local maxima occurs on the x -axis when $y = 0, x = -\frac{3}{2}$ and $x = 1$

Question 5

Answer: E

Explanation:

Point of inflection at $(3,1)$ and y intercept at $(0, 2)$

Question 6

Answer: D

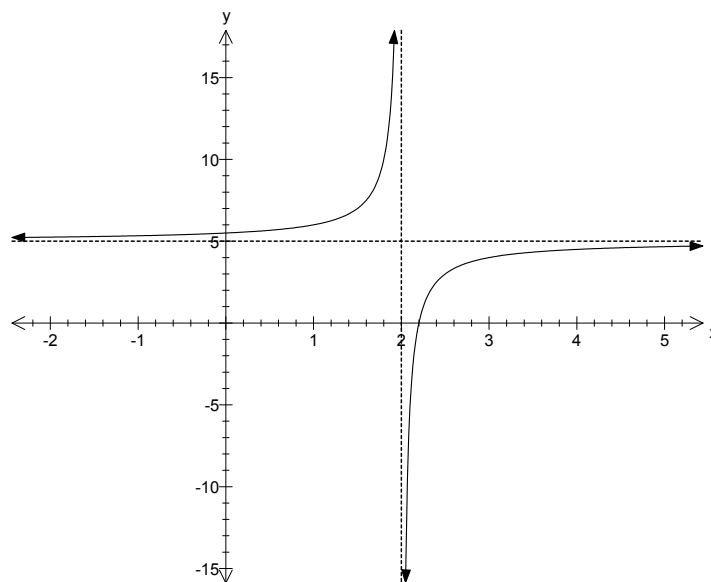
Explanation:

horizontal line cuts once, vertical line crosses twice or more.

Question 7

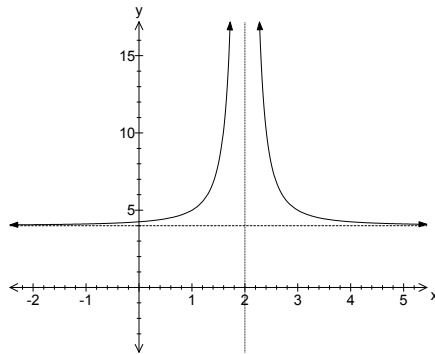
Answer: B

Explanation: from graph



Question 8

Answer: C
 $y > 4$



Question 9

Answer: A

Explanation:

$$\Delta = 25 - 4 \times 2 \times 1 = 17 > 0$$

Therefore 2 real solutions exist.

Question 10

Answer: D

Explanation:

$$\text{for } y, x = 0 \therefore y = 2$$

$$\text{for } x, y = 0 \therefore x = 4, -7$$

Question 11

Answer: E

Explanation:

if $P(a) = 0$ then $(x - a)$ is a factor therefore, $(x + 3)$ is the factor of $P(x)$

Question 12

Answer: E

Explanation:

gradient is positive **above** the x axis $x < -10$ or $x > -1$

Question 13*Answer:* B*Explanation:*

$$\begin{aligned}
 & \frac{3(a^{-2})^{-1}b}{(5ab^2)^3} \div \frac{9(a^2b^{-1})^3}{25b^2} \\
 &= \frac{3a^2b}{125a^3b^6} \times \frac{25b^2}{9a^6b^{-3}} \\
 &= \frac{a^2b^3}{15a^9b^3} \\
 &= \frac{1}{15a^7}
 \end{aligned}$$

Question 14*Answer:* C*Explanation:*

$$\begin{aligned}
 5^{-(2x-1)} &= 5^3 \\
 -(2x-1) &= 3 \\
 -2x+1 &= 3 \\
 x &= -1
 \end{aligned}$$

Question 15*Answer:* D*Explanation:*

$$\begin{aligned}
 & \frac{\log_2 7^2}{\log_2 7} \\
 &= \frac{2\log_2 7}{\log_2 7} \\
 &= 2
 \end{aligned}$$

Question 16

Answer: B

Explanation:

$$2 \times 2^{2x} - 9 \times 2^x + 4 = 0$$

$$\text{Let } y = 2^x$$

$$\Rightarrow 2y^2 - 9y + 4 = 0$$

$$\Rightarrow (2y - 1)(y - 4) = 0$$

$$\Rightarrow y = \frac{1}{2} \text{ or } y = 4$$

$$2^x = 2^{-1}, \quad 2^x = 2^2$$

$$x = -1 \quad x = 2$$

Question 17

Answer: D

Explanation:

$$(x - k)^2 + (y - h)^2 = r^2$$

$$k = -2, h = 1, r = \sqrt{16} = 4$$

$$(x + 2)^2 + (y - 1)^2 = 16$$

Question 18

Answer: A

Explanation:

Start: $x = 0, y = 0$ and after 4 hours: $x = 4, y = 128$

$$m = \frac{128 - 0}{4 - 0} \text{ therefore average rate} = 32 \text{ km/h}$$

Question 19

Answer: D

Explanation:

$$\begin{aligned} & (x^2 - 4)(9x^2 - 24x + 16) \\ &= 9x^4 - 24x^3 + 16x^2 - 36x^2 + 96x - 64 \\ &= 9x^4 - 24x^3 - 20x^2 + 96x - 64 \end{aligned}$$

Question 20

Answer: E

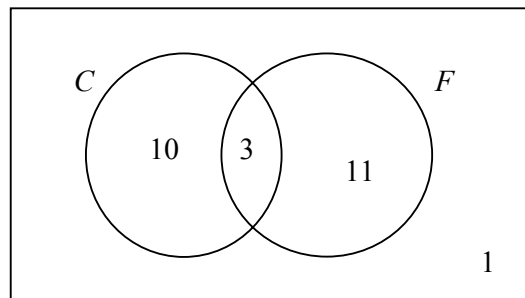
Explanation:

Find $\Pr(A \cap B)$ first: $\Pr(A \cap B) = \Pr(A) \times \Pr(B \mid A) = 0.2 \times 0.35 = 0.07$
 $\Pr(B) = \Pr(A \cup B) - \Pr(A) + \Pr(A \cap B) = 0.56 - 0.2 + 0.07 = 0.43$

Question 21

Answer: A

Explanation:



Question 22

Answer: A

Explanation:

	R	R'	
P	21	27	48
P'	18	34	52
	39	61	100

SECTION 2 - Analysis Questions

Question 1

a. $f(x) = x^2 + 6x + 9 - 9 - 7 = (x + 3)^2 - 16$

M1 + A1

2 marks

b. let $x = 0$: $y = -7$

let $y = 0$: $x = -7, x = 1$

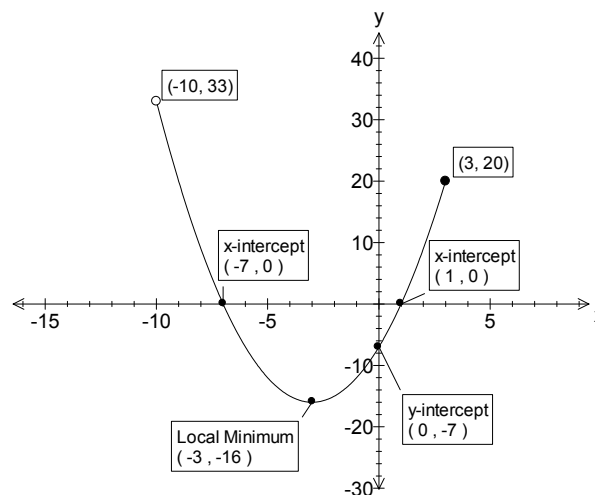
A2

2 marks

c. Show turning point, intercepts and endpoints

Endpoints are (3,20) and (-10, 33), x – intercepts (-7, 0), (1, 0) and y –intercept (0, -7)

A3



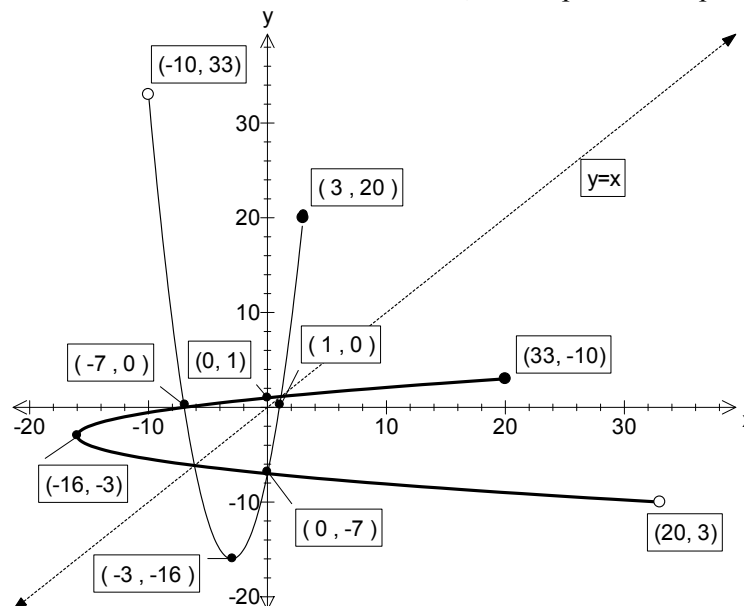
3 marks

d. range $[-16, 33]$

A1

1 mark

e. Sketch of inverse should show: Reflected TP, intercepts and endpoints



A2

2 marks

- f. the largest domain $(-10, -3]$ makes it a one to one function

A1
1 mark

g. $f(x) = (x+3-3)^2 - 16 - 2 = x^2 - 18$

M1 + A1
2 marks
Total 13 marks

Question 2

a.

$$s(1) = -3 + 18 - 36 + 21 = 0$$

$$\begin{array}{r} -3x^2 + 15x - 21 \\ (x-1) \overline{) -3x^3 + 18x^2 - 36x + 21} \end{array}$$

$$\dots\dots\dots -3x^3 + 3x^2$$

$$\dots\dots\dots 15x^2 - 36x$$

$$\dots\dots\dots 15x^2 - 15x$$

$$\dots\dots\dots -21x + 21$$

$$\dots\dots\dots -21x + 21$$

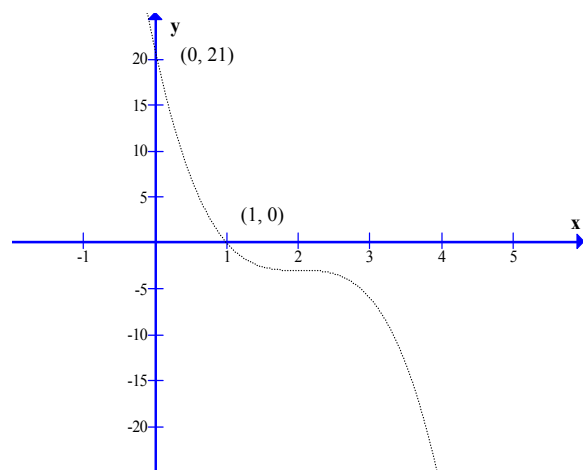
$$\dots\dots\dots 0$$

$$-3x^2 + 15x - 21 = -3(x^2 - 5x + 7)$$

$$\therefore s(x) = -3(x-1)(x^2 - 5x + 7)$$

M3 + A1
4 marks

- b. Show correct intercepts and shape
y-intercept $(0, 21)$
x-intercept $(1, 0)$



A2
2 marks

c.

$$d = \sqrt{(-2-0)^2 + (0-21)^2}$$

$$d = \sqrt{4+441} = \sqrt{445} \quad \text{must include correct rounding}$$

$$d = 21.10m$$

M1 + A1
2 marks

d.

$$\text{midpoint} = \left(\frac{0-2}{2}, \frac{21-0}{2} \right) = \left(-1, 10\frac{1}{2} \right)$$

M1 + A1
2 marks

e.

$$m = 0$$

$$y = 10\frac{1}{2}$$

M1 + A1
2 marks

f.

Centre of the circle = $(1+7+5, 0) = (13, 0)$

Eqn of circle:

$$(x-k)^2 + (y-h)^2 = r^2$$

$$k = 13, h = 0, r = \sqrt{25} = 5$$

$$\therefore (x-13)^2 + y^2 = 25$$

M2 + A1
3 marks
Total 15 marks

Question 3

a.

- i. $0 \text{ s} < t < 20 \text{ s}$
- ii. $40 \text{ s} < t < 50 \text{ s}$
- iii. $60 \text{ s} < t < 70 \text{ s}$

A3
3 marks

b.

- i. Bjorn is stationary 50 m from the starting point about to go back to the initial position.
- ii. He is moving away from the starting point moving with a constant velocity.

A2 + A1
3 marks

c.

i. $\frac{55-40}{30-20} = 1.5m/sec$

ii. $\frac{0-55}{70-52} = -3.06m/sec$

A2

2 marks

d. $\frac{0-65}{70-60} = -6.5m/sec$

A1

1 mark

Total 9 marks

Question 4

a.

$$5^{2x} - 20(5^x) - 125 = 0 \text{ Let } a = 5^x \Rightarrow a^2 - 20a - 125 = 0$$

$$(a+5)(a-25) = 0$$

$$a = -5, a = 25$$

$$5^x = -5 \text{ (impossible) or } 5^x = 5^2$$

$$\therefore x = 2$$

M3 + A1

4 marks

b. Use CAS or:

$$\log_{10} 3^{2x+1} = \log_{10} 12$$

$$(2x+1)\log_{10} 3 = \log_{10} 12$$

$$2x-1 = \frac{\log_{10} 12}{\log_{10} 3}$$

$$2x-1 = 2.2619$$

$$\therefore x = 1.6309$$

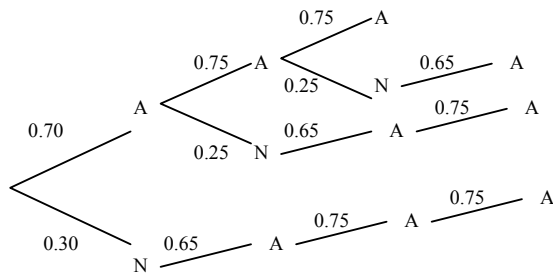
M3 + A1

4 marks

Total 8 marks

Question 5

- a. must realise that some of the branches are not needed



4 correct branches with correct values

A4
4 marks

b. $\Pr(AA) = 0.7 \times 0.75$
 $= 0.525$

M1 + A1
2 marks

c. $\Pr(AAN) + \Pr(ANA) + \Pr(NAA) = 0.7 \times 0.75 \times 0.25 + 0.7 \times 0.25 \times 0.65 + 0.30 \times 0.65 \times$
 $0.75 = 0.39$

M1+A1
2 marks

d.

	P	P'	
GM	0.55	0.25	0.8
GM'	0.1	0.1	0.2
	0.65	0.35	1

A2
2 marks

e. $\Pr(P \cap GM') = 0.1$

A1
1 mark

f. $\Pr(\text{no As} \mid \text{no As in GM}) = \frac{\Pr(GM' \cap P')}{\Pr(GM')}$
 $= \frac{0.10}{0.20}$
 $= 0.5$

M1 + A1
2 marks
Total 13 marks