



THE SCHOOL FOR EXCELLENCE
UNIT 3 & 4 MATHEMATICAL METHODS 2009
COMPLIMENTARY WRITTEN EXAMINATION 1 - SOLUTIONS

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MARKING SCHEME

- $(A4 \times \frac{1}{2} \downarrow)$ means four answer half-marks rounded **down** to the next integer.
Rounding occurs at the end of a part of a question.
- **M1** = 1 **M**ethod mark.
- **A1** = 1 **A**nswer mark (it **must** be this or its equivalent).
- **H1** = 1 consequential mark (**H**is/**H**er mark...correct answer from incorrect statement or slip).

QUESTION 1

(a) $y = x^2 e^{-3x}$

Using the Product Rule: $\frac{dy}{dx} = x^2 \times \frac{d}{dx}(e^{-3x}) + e^{-3x} \times \frac{d}{dx}(x^2)$ **M1**

$$= (x^2 \times -3e^{-3x}) + (e^{-3x} \times 2x)$$
A1

$$= e^{-3x}(-3x^2 + 2x)$$

$$= \frac{-3x^2 + 2x}{e^{3x}} \text{ and so } a = -3 \text{ and } b = 2$$
A($2 \times \frac{1}{2} \downarrow$)

(b) $f(x) = \frac{x^2 + 1}{x^2 - 1}$

Using the Quotient Rule: $f'(x) = \frac{(x^2 - 1)2x - (x^2 + 1)2x}{(x^2 - 1)^2}$ **M1**

$$= \frac{2x(x^2 - 1 - x^2 - 1)}{(x^2 - 1)^2}$$

$$= \frac{-4x}{(x^2 - 1)^2}$$
A1

Therefore, $f'(2) = -\frac{8}{9}$ **H1**

Total = 6 marks

QUESTION 2

(a) $P(x) = x^4 + x^3 + ax^2 - x + b$

$P(1) = 0$ and so $1 + 1 + a - 1 + b = 0$.

Therefore $a + b = -1$ (i) **A1**

$P(-2) = 12$ and so $16 - 8 + 4a + 2 + b = 12$.

Therefore $4a + b = 2$ (ii) **A1**

Subtracting (i) from (ii) gives $3a = 3$ and so $a = 1$ as required.

Substituting $a = 1$ into (i) gives $1 + b = -1$ and so $b = -2$ as required.

(b) Knowing that $(x-1)(x+1)$ are factors of $P(x)$ then dividing $P(x)$ by x^2-1 gives

$$\begin{array}{r}
 x^2-1 \overline{) x^4 + x^3 + x^2 - x - 2} \\
 \underline{x^4 + 0x^3 - x^2} \\
 x^3 + 2x^2 - x \\
 \underline{x^3 + 0x^2 - x} \\
 2x^2 - 2 \\
 \underline{2x^2 - 2} \\
 0
 \end{array}$$

M1

The factors of $P(x)$ are $(x-1)(x+1)(x^2+x+2)$ **A1**

Total = 4 marks

QUESTION 3

$$\log_4(x) - \log_4(x-4) = 1$$

$$\log_4\left(\frac{x}{x-4}\right) = \log_4 4$$

M1

$$\frac{x}{x-4} = 4$$

A1

$$4x - 16 = x$$

$$x = \frac{16}{3}$$

$$a = 16 \text{ and } b = 3$$

A1

Total = 3 marks

QUESTION 4

(a) $2\pi \div \left(\frac{\pi}{8}\right) = 16 \text{ hours}$ **A1**

(b) 25° C **A1**

(c) $35^\circ \text{ C when } t = 8 \text{ (8.00 am)}$ **A1**

Total = 3 marks

QUESTION 5

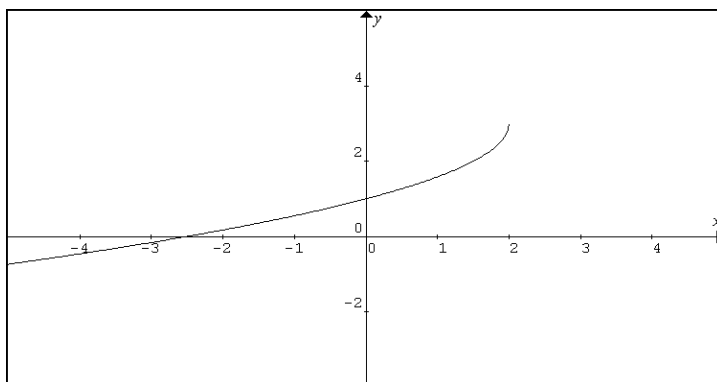
- (a) Dilation by a factor of either $\sqrt{2}$ from the x -axis **or** by a factor 0.5 from the y -axis;
Reflection in the x -axis; Translations 2 to the left and 3 up

(Order is important with the translations coming last).

A2 ($4 \times \frac{1}{2} \downarrow$)

-1 if incorrect order

(b)



Vertex clearly at (2, 3); y -intercept (0, 1); x -intercept (-2.5, 0); Shape.

A2 ($4 \times \frac{1}{2} \downarrow$)

(c) $f(x) = 3 - \sqrt{4 - 2x}$

M1

Interchanging x for y and vice-versa gives $x = 3 - \sqrt{4 - 2y}$

$$x - 3 = -\sqrt{4 - 2y}$$

Squaring both sides: $4 - 2y = (x - 3)^2$ and so $2y = 4 - (x - 3)^2$

$$f^{-1}(x) = 2 - \frac{1}{2}(x - 3)^2 \text{ for } x \in (-\infty, 3]$$

A1 + A1

Total = 7 marks

QUESTION 6

$$V = \frac{4}{3}\pi r^3$$

$$\frac{dV}{dt} = 16\pi \text{ and } \frac{dV}{dr} = 4\pi r^2 \quad \text{M1}$$

$$\frac{dr}{dt} = \frac{dV}{dt} \times \frac{dr}{dV}$$
$$= 16\pi \div 4\pi r^2 \quad \text{H1}$$

$$= \frac{16\pi}{4\pi r^2}$$

$$= \frac{4}{400} \text{ at } r = 20$$

Answer: 0.01 cm/minute A1

Total = 3 marks

QUESTION 7

$$E(aX + b) = aE(X) + b \quad \text{M1}$$

$$E(Y) = E(4X - 5) = 4E(X) - 5$$
$$= 4 \times 10 - 5$$
$$= 35 \quad \text{A1}$$

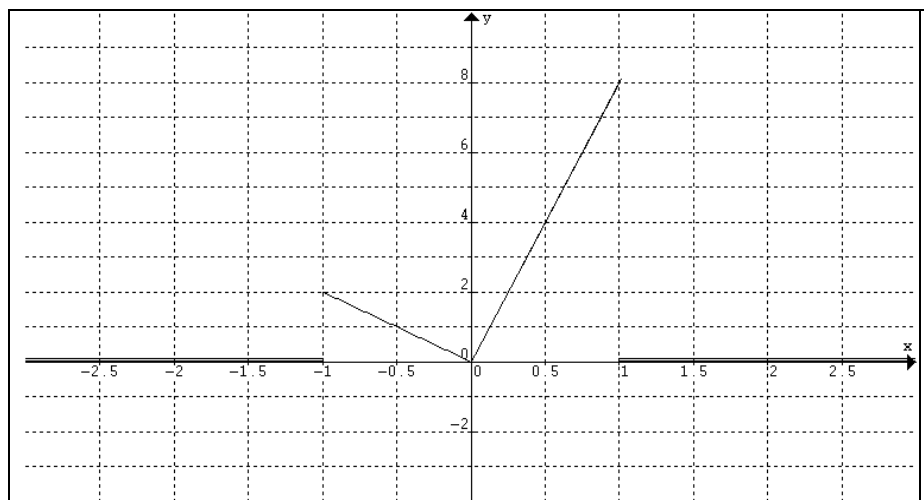
$$\text{Variance}(aX + b) = a^2 \text{Variance}(X)$$

$$\text{Variance}(Y) = \text{Variance}(4X - 5)$$
$$= 4^2 \text{Variance}(X)$$
$$= 16 \times 2$$
$$= 32 \quad \text{A1}$$

Total = 3 marks

QUESTION 8

(a)



A1 The points $(-1, 2)$ and $(1, 8)$ should be filled in and the points $(-1, 0)$ and $(1, 0)$ empty.

A1 Horizontal lines.

A1 Oblique lines.

(b) The area under the curve must be 1 for a probability density function. There are two triangles whose area needs to be found: $\left(\frac{1}{2} \times 1 \times 2k\right) + \left(\frac{1}{2} \times 1 \times 8k\right) = k + 4k = 5k$

Hence $5k = 1$ and so $k = 0.2$

A1

(c) The equation of the straight line for the interval $[-1, 0)$ is $y = -0.4x$ and the equation of the straight line for the interval $[0, 1]$ is $y = 1.6x$. The mean is given by $\int x \times g(x) dx$.

$$\begin{aligned} \text{Mean} &= \int_{-1}^0 -0.4x^2 dx + \int_0^1 1.6x^2 dx \\ &= -\left[\frac{0.4}{3}x^3\right]_{-1}^0 + \left[\frac{1.6}{3}x^3\right]_0^1 \\ &= \left(0 + \frac{0.4}{3}\right) + \left(\frac{1.6}{3} - 0\right) \\ &= \frac{2}{3} \end{aligned}$$

M1 for two "bits"

A1

Total = 6 marks

QUESTION 9

(a) If $f(x) = \sqrt{(x+2)}$ then $f'(x) = \frac{1}{2\sqrt{(x+2)}}$ **A1**

At $x = -1$, $y = 1$ and $f'(-1) = \frac{1}{2\sqrt{(-1+2)}} = \frac{1}{2}$ **H1**

Equation of tangent: $y - 1 = \frac{1}{2}(x + 1)$ and so $2y = x + 3$. **A1**

(b) Area required $= \int_{-3}^{-1} \left(\frac{1}{2}x + \frac{3}{2} \right) dx - \int_{-2}^{-1} \sqrt{(x+2)} dx$ **A1**

$$= \left[\frac{x^2}{4} + \frac{3x}{2} \right]_{-3}^{-1} - \int_{-2}^{-1} (x+2)^{0.5} dx$$

$$= \left(\frac{1}{4} - \frac{9}{4} \right) - \left(\frac{9}{4} - \frac{9}{4} \right) - \left[\frac{1}{1.5} (x+2)^{1.5} \right]_{-2}^{-1}$$

$$= 1 - \frac{2}{3}(1-0)$$

$$= \frac{1}{3}$$
 A1

Total = 5 marks

END OF SOLUTIONS TO EXAMINATION 1