

**Mathematical Association of Victoria
Trial Exam 2009**

**MATHEMATICAL METHODS / MATHEMATICAL METHODS
(CAS)**

STUDENT NAME _____

Written Examination 1

Reading time: 15 minutes

Writing time: 1 hour

QUESTION AND ANSWER BOOK

Structure of book

<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
9	9	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers.
- Students are NOT permitted to bring into the examination room: notes of any kind, blank sheets of paper, white out liquid/tape or a calculator of any type.

Materials supplied

- Question and answer book of 10 pages, with a detachable sheet of miscellaneous formulas at the back
- Working space is provided throughout the book.

Instructions

- Detach the formula sheet from the back of this book during reading time.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

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Instructions

Answer **all** questions in the spaces provided.

A decimal approximation will not be accepted if an **exact** answer is required to a question.

In questions where more than one mark is available, appropriate working must be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1

Let $f(x) = (e^{2x} - 3)^4$. Evaluate $f'(0)$.

3 marks

Question 2

Solve $2\log_e(x-2) - \log_e(x) = 0$ for x , where $x > 2$.

3 marks

Question 3

Solve $2\sin\left(2x - \frac{\pi}{6}\right) = 1$ for $-\pi < x < \pi$.

3 marks

Question 4

Consider the function $g: [-3, \infty) \rightarrow \mathbb{R}$, with rule $g(x) = 2(x+3)^2 - 1$.

- a. Find the **rule** of the inverse function, g^{-1} .

- b. Find the **domain** of the inverse function, g^{-1} .

2 + 1 = 3 marks

Question 5

Let $h(x) = \frac{1}{\tan(x)} + x$.

- a. Show that $h'(x) = -\frac{1}{\sin^2(x)} + 1$.

- b. Hence find $\int \left(\frac{1}{\sin^2(x)} dx \right)$.

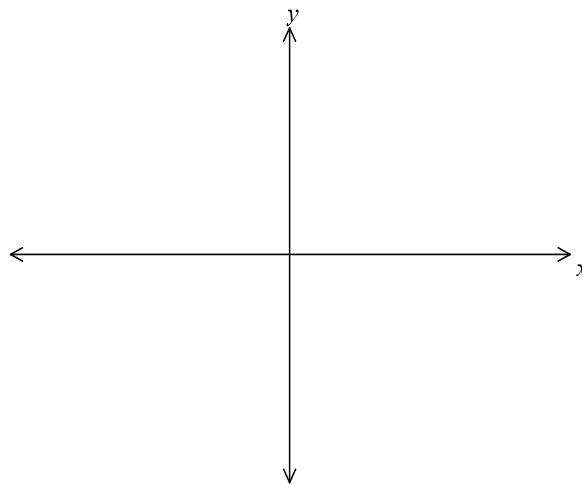
2 + 3 = 5 marks

Question 6

Consider the function $f : \mathbb{R} \setminus \{1\} \rightarrow \mathbb{R}$, $f(x) = 2 - \frac{2}{(x-1)^2}$.

- a. Find the **coordinates** of the x -axis intercepts of the graph of f .

- b. On the axes below, sketch the graph of $y = |f(x)|$. Label all axes intercepts with their coordinates. Label each asymptote with its equation.

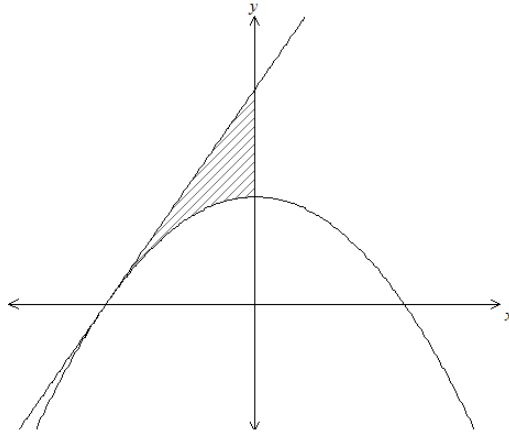


- c. Write down the domain of the derivative function, $\frac{d}{dx}(|f(x)|)$.

1 + 3 + 1 = 5 marks

Question 7

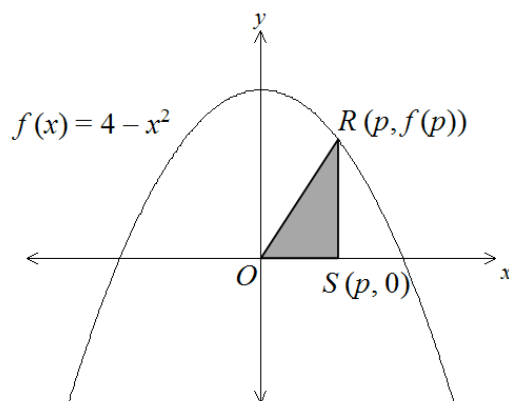
The graph of $f : \mathbb{R} \rightarrow \mathbb{R}$, $f(x) = 4 - x^2$ and the tangent to the graph of f , where it crosses the negative x -axis, are shown.



- a.** Find the equation of the tangent to the graph of f where it crosses the negative x -axis.

- b.** Find the area of the shaded region.

- c. Consider the triangle ORS , where vertex R is on the graph of f , with coordinates $(p, f(p))$, and vertices O and S are on the x -axis, with coordinates $(0, 0)$ and $(p, 0)$, respectively.



If $p \in [-2, 2]$, find the value(s) of p for which the area of triangle ORS is a maximum.

2 + 3 + 3 = 8 marks

Question 8

A spherical balloon is being inflated at a rate of $10 \text{ cm}^3/\text{s}$. The balloon will burst when the surface area reaches $3600\pi \text{ cm}^2$.

- a. The surface area of a sphere of radius r is given by $4\pi r^2$. Show that the radius of the balloon is 30 cm at the instant when the balloon bursts.

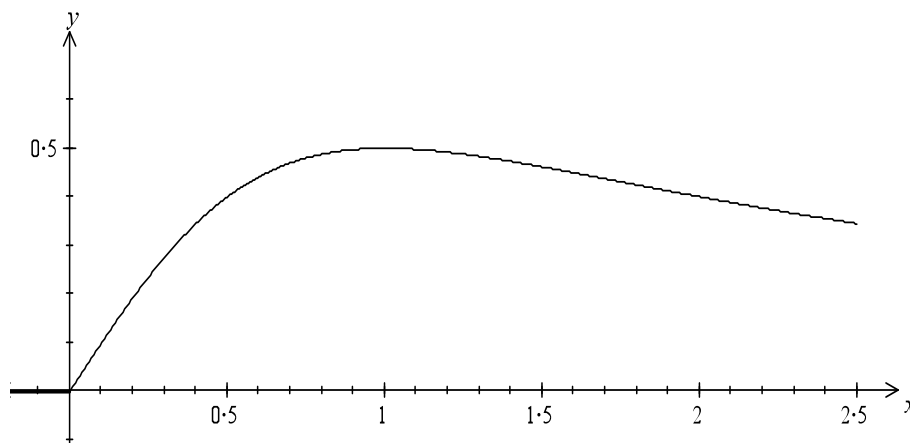
- b. Find the rate at which the radius of the balloon is changing, in cm per second, at the instant when the balloon bursts.

1 + 3 = 4 marks

Question 9

A continuous random variable X has a probability density function $f(x) = \begin{cases} \frac{x}{x^2+1} & \text{for } 0 \leq x \leq a \\ 0 & \text{elsewhere} \end{cases}$, where a

is a real constant. Part of the graph of f , where $0 \leq x \leq a$, is shown below.



- a. Given that $\int \frac{x}{x^2+1} dx = \frac{1}{2} \log_e (x^2+1) + c$, where c is a real constant, find the value of a .

2 marks

- b. Find the mode of X .

- c. Find $\Pr(0.25 \leq X \leq 0.75)$. Put your answer in the form $\log_e\left(\frac{a}{\sqrt{b}}\right)$ where a and b are positive integers.

$2 + 2 + 2 = 6$ marks

END OF QUESTION AND ANSWER BOOK

Mathematical Methods and Mathematical Methods (CAS)

Formulas

Mensuration

area of a trapezium:	$\frac{1}{2}(a+b)h$	volume of a pyramid:	$\frac{1}{3}Ah$
curved surface area of a cylinder:	$2\pi rh$	volume of a sphere:	$\frac{4}{3}\pi r^3$
volume of a cylinder:	$\pi r^2 h$	area of a triangle:	$\frac{1}{2}bc \sin A$
volume of a cone:	$\frac{1}{3}\pi r^2 h$		

Calculus

$\frac{d}{dx}(x^n) = nx^{n-1}$	$\int x^n dx = \frac{1}{n+1} x^{n+1} + c, n \neq -1$
$\frac{d}{dx}(e^{ax}) = ae^{ax}$	$\int e^{ax} dx = \frac{1}{a} e^{ax} + c$
$\frac{d}{dx}(\log_e(x)) = \frac{1}{x}$	$\int \frac{1}{x} dx = \log_e x + c$
$\frac{d}{dx}(\sin(ax)) = a \cos(ax)$	$\int \sin(ax) dx = -\frac{1}{a} \cos(ax) + c$
$\frac{d}{dx}(\cos(ax)) = -a \sin(ax)$	$\int \cos(ax) dx = \frac{1}{a} \sin(ax) + c$
$\frac{d}{dx}(\tan(ax)) = \frac{a}{\cos^2(ax)} = a \sec^2(ax)$	

product rule: $\frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}$ quotient	rule: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$
chain rule: $\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}$ approximation:	$f(x+h) \approx f(x) + hf'(x)$

Probability

$\Pr(A) = 1 - \Pr(A')$	$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$
$\Pr(A B) = \frac{\Pr(A \cap B)}{\Pr(B)}$	
mean: $\mu = E(X)$	variance: $\text{var}(X) = \sigma^2 = E((X - \mu)^2) = E(X^2) - \mu^2$

probability distribution		mean	variance
discrete	$\Pr(X = x) = p(x)$	$\mu = \sum x p(x)$	$\sigma^2 = \sum (x - \mu)^2 p(x)$
continuous	$\Pr(a < X < b) = \int_a^b f(x) dx$	$\mu = \int_{-\infty}^{\infty} x f(x) dx$	$\sigma^2 = \int_{-\infty}^{\infty} (x - \mu)^2 f(x) dx$