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VCE

Mathematical Methods

Solutions

Trial Examination 2



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SECTION 1

ANSWERS

1	A	B	C	D	E
2	A	B	C	D	E
3	A	B	C	D	E
4	A	B	C	D	E
5	A	B	C	D	E
6	A	B	C	D	E
7	A	B	C	D	E
8	A	B	C	D	E
9	A	B	C	D	E
10	A	B	C	D	E
11	A	B	C	D	E
12	A	B	C	D	E
13	A	B	C	D	E
14	A	B	C	D	E
15	A	B	C	D	E
16	A	B	C	D	E
17	A	B	C	D	E
18	A	B	C	D	E
19	A	B	C	D	E
20	A	B	C	D	E
21	A	B	C	D	E
22	A	B	C	D	E

SECTION 1

Question 1

Answer D

 $f(x) = g(x) \log_e(2x)$ differentiating using the product rule

$$f'(x) = g'(x) \log_e(2x) + \frac{g(x)}{x}$$

$$f'\left(\frac{e}{2}\right) = g'\left(\frac{e}{2}\right) \log_e(e) + \frac{2}{e} g\left(\frac{e}{2}\right)$$

$$f'\left(\frac{e}{2}\right) = 1 \times 1 + \frac{e}{2} \times \frac{2}{e} = 2$$

Question 2

Answer A

$$f(x) = \sin\left(\frac{1}{x}\right)$$

Let $y = \sin(u)$ $u = \frac{1}{x} = x^{-1}$ chain rule

$$\frac{dy}{du} = \cos(u) \quad \frac{du}{dx} = -x^{-2} = -\frac{1}{x^2}$$

$$\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx} = -\frac{1}{x^2} \cos\left(\frac{1}{x}\right)$$

$$f'(a) = -\frac{1}{a^2} \cos\left(\frac{1}{a}\right)$$

Question 3

Answer B

$$f(x) = x^3 + e^{2x} \quad \text{now} \quad f(2) = 8 + e^4 \quad f(0) = 1$$

$$\text{average rate is } \frac{f(2) - f(0)}{2 - 0} = \frac{8 + e^4 - 1}{2} = \frac{7 + e^4}{2}$$

Question 4

Answer E

All of **A. B. C.** and **D.** are true, **E.** is false although $\int_0^{\frac{2\pi}{n}} (a + b \sin(nx)) dx = \frac{2\pi a}{n}$

This will only be the area if $a > |b|$ and $a > 0$.

Question 5**Answer C**

Let $y_1 = mx + c$ and $y_2 = -x^2 + 3x - 3$, the tangent to the graph at

the point P , where $x = 3$. $\frac{dy_2}{dx} = -2x + 3$ $\left. \frac{dy_2}{dx} \right|_{x=3} = -3 = m$ so **B.** is true.

At $x = 3$ $y_2 = -9 + 9 - 3 = -3$ $P(3, -3)$ is on the tangent,

$y_1 = mx + c \Rightarrow -3 = -9 + c \Rightarrow c = 6$ so **A.** is true, also **D.** is true.

The area $A = \int_a^b (y_1 - y_2) dx$ $a = 0$ $b = 3$, so that $A = \int_0^3 (x^2 + (m-3)x + (c+3)) dx$

E. is true, **C.** is false.

Question 6**Answer A**

$f(x+h) \approx f(x) + hf'(x)$ with $f(x) = \frac{1}{x^2}$ $x = 4$ $h = -0.01$

so that $\frac{1}{3.99} \approx f(4) - 0.01f'(4)$

Question 7**Answer B**

The required area is below the x -axis, so taking the absolute value, makes the area

positive. **A.** is true $\left| \int_{-a}^a (x^2 - a^2) dx \right|$ this is also equal to **D.** which is true $\int_{-a}^a (a^2 - x^2) dx$,

by symmetry **C.** is true $2 \int_0^a (a^2 - x^2) dx$. The graph of $y = x^2 - a^2$, this crosses the y -axis

at $-a^2$, now the inverse function is $x = y^2 - a^2 \Rightarrow y^2 = x + a^2 \Rightarrow y = \sqrt{x + a^2}$, the

area bounded by the curve and the y -axis is $2 \left| \int_{-a^2}^0 \sqrt{x + a^2} dx \right|$, so that **E.** is true, **B.** is false.

Question 8**Answer C**

$f: y = x^2 + a$ $\text{dom } f = \mathbb{R}^-$ $\text{ran } f = (a, \infty)$

f^{-1} $x = y^2 + a$ transposing

$y^2 = x - a$ $y = \pm \sqrt{x - a}$ but $\text{ran } f^{-1} = \mathbb{R}^-$ $\text{dom } f^{-1} = (a, \infty)$ so we must take the

negative, $f^{-1}: (a, \infty) \rightarrow \mathbb{R}$, $f^{-1}(x) = -\sqrt{x - a}$

Question 9**Answer A**

$f(x) = f(-x)$, $f(x)$ is an even function, and $\int_{-6}^6 f(x) dx = 10$, then $\int_0^6 f(x) dx = 5$

$$\int_0^6 (2f(x) - 1) dx = 2 \int_0^6 f(x) dx - [x]_0^6 = 2 \times 5 - (6 - 0) = 4$$

Question 10**Answer B**

The function is not defined when $x = 0$,

all of **A**, **C**, **D**. and **E**. are false,

The function is an even function,
symmetrical about the y-axis.

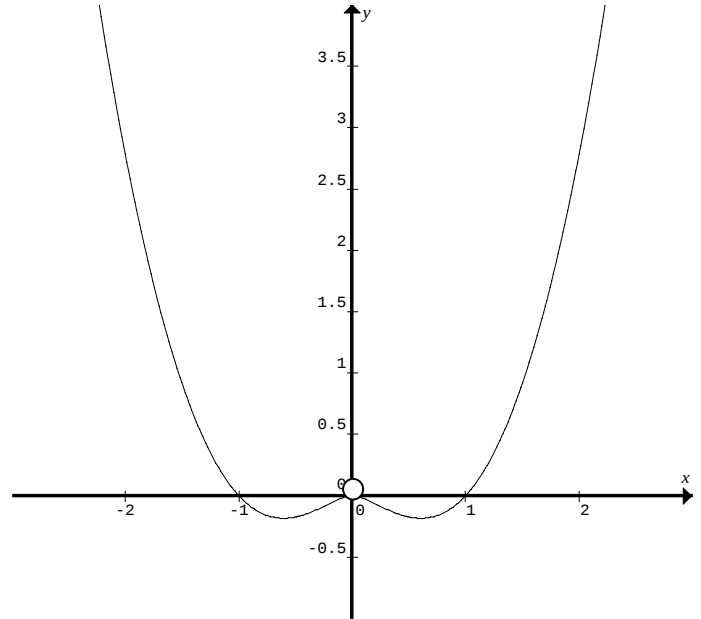
$$y = x^2 \log_e(x)$$

$$\frac{dy}{dx} = 2x \log_e(x) + x = x(2 \log_e(x) + 1)$$

for turning points, $\frac{dy}{dx} = 0$, since $x \neq 0$

$$\log_e(x) = -\frac{1}{2} \quad x = e^{-\frac{1}{2}} = \frac{1}{\sqrt{e}},$$

the graph has minimums at $x = \pm \frac{1}{\sqrt{e}}$

**Question 11****Answer E**

$$f: y = b + \frac{a}{x-b}$$

$$f^{-1} \quad x = b + \frac{a}{y-b} \quad \Rightarrow x-b = \frac{a}{y-b} \quad \Rightarrow y-b = \frac{a}{x-b}$$

$$f^{-1}(x) = y = b + \frac{a}{x-b} \quad \text{so} \quad f^{-1} = f$$

The domain and range of both f and f^{-1} are $R \setminus \{b\}$.

Since $a \neq 0$ and $b \neq 0$, the graph of $y = f(x)$ passes through $\left(0, b - \frac{a}{b}\right)$

and the graph of $y = f^{-1}(x)$ passes through $\left(b - \frac{a}{b}, 0\right)$.

All of **A**, **B**, **C**, **D**. are true, however **E**. is false

The graph of $y = f(x)$ and $y = f^{-1}(x)$ always intersects on the line $y = x$ at the points $(b \pm \sqrt{a}, b \pm \sqrt{a})$ only if $a > 0$.

Question 12**Answer C**

$$\frac{dy}{dx} = 2 \cos\left(\frac{x}{2}\right) \Rightarrow y = \int 2 \cos\left(\frac{x}{2}\right) dx = 4 \sin\left(\frac{x}{2}\right) + c \quad \text{to find } c, \text{ use } y\left(\frac{5\pi}{3}\right) = 0$$

$$0 = 4 \sin\left(\frac{5\pi}{6}\right) + c = 2 + c = 0 \Rightarrow c = -2$$

$$y = 4 \sin\left(\frac{x}{2}\right) - 2 \quad \text{now when } x = 0 \quad y = 4 \sin(0) - 2 = -2$$

Question 13**Answer D**

$y = \frac{bx}{x-a} = \frac{bx-ab+ab}{x-a} = b + \frac{ab}{x-a}$ has $y = b$ as a horizontal asymptote and $x = a$ as a vertical asymptote.

Question 14**Answer D**

Let $f: [0, \pi] \rightarrow R$, $f(x) = 2 \cos\left(\frac{x}{2}\right) - 2$. The period is $T = \frac{2\pi}{\frac{1}{2}} = 4\pi$

The graph of f is transformed by a reflection in the x -axis, the rule is

$g(x) = 2 - 2 \cos\left(\frac{x}{2}\right)$, we only have one-quarter of a cycle

now a dilation of factor 2 from the y -axis, replace x with $\frac{x}{2}$

$g: [0, 2\pi] \rightarrow R$, $g(x) = 2 - 2 \cos\left(\frac{x}{4}\right)$ since we must have one-quarter of a cycle,

the new domain is $[0, 2\pi]$

then a dilation by a factor of 3 from the x -axis, multiply y by 3

the equation becomes $g: [0, 2\pi] \rightarrow R$, $g(x) = 6 - 6 \cos\left(\frac{x}{4}\right)$

Question 15**Answer E**

$$\Pr(A' \cap B') + b - p = 1 - a \quad \text{or}$$

$$\Pr(A' \cap B') + a - p = 1 - b$$

$$\Pr(A' \cap B') = 1 + p - (a + b)$$

	A	A'	
B	p	$b - p$	b
B'	$a - p$	$?$	$1 - b$
	a	$1 - a$	

Question 16**Answer D**

$$f(x) = x^4 - 4x^2$$

$$f'(x) = 4x^3 - 8x$$

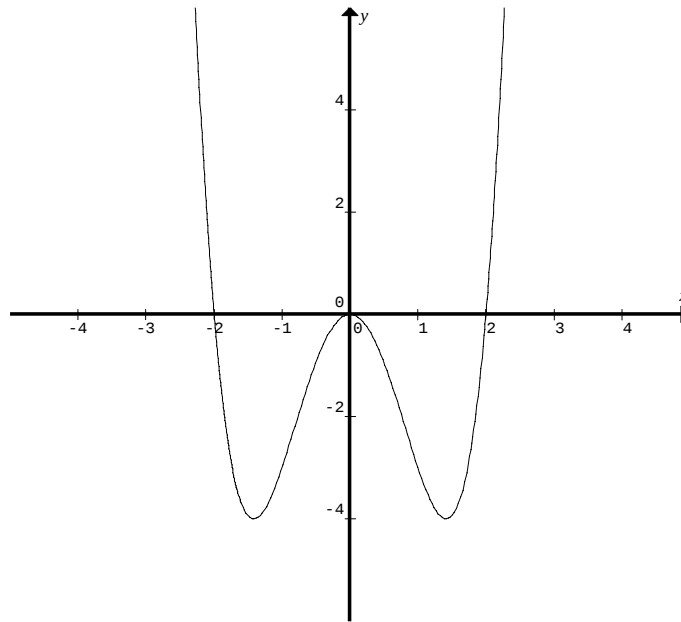
$$f'(x) = 4x(x^2 - 2)$$

turning points at

$$x = 0, x = \pm\sqrt{2}$$

for the function to be one-one, we require

$$a < -\sqrt{2}$$

**Question 17****Answer B**
 $X \stackrel{d}{=} Bi(n=?, p=0.6)$ Betty winning a game.

$$\Pr(X=0) = 0.4^n \leq 0.01$$

$$n \log_e(0.4) \leq \log_e(0.01)$$

$$n \geq \frac{\log_e(0.01)}{\log_e(0.4)} = 5.02 \quad \text{so } n = 6$$

Question 18**Answer B**

$$X \stackrel{d}{=} N\left(\mu_X = \mu, \sigma_X^2 = \frac{9\mu^2}{4}\right)$$

$$\Pr(X > 2\mu) = \Pr\left(Z > \frac{2\mu - \mu}{\frac{3}{2}\mu}\right) = \Pr\left(Z > \frac{2}{3}\right) = 0.252$$

Question 19**Answer C**

$$\text{Let } g(x) = \int_0^x f(t) dt \text{ then } g'(x) = f(x)$$

$$g(0) = 0 \quad g'(0) = f(0) = 9 \quad g'(3) = f(3) = 0$$

Question 20**Answer D**

Option **D.** has $f(x) = e^x$ $g(x) = -x^2$ and $f(g(x)) = e^{-x^2}$

Which is the graph required.

Question 21**Answer A**

$$A = \frac{3\sqrt{3}}{2} L^2 \quad \frac{dA}{dL} = 3\sqrt{3}L \quad \text{given} \quad \frac{dL}{dt} = \sqrt{3} \text{ cm/s}$$

$$\frac{dA}{dt} = \frac{dA}{dL} \cdot \frac{dL}{dt} = 3\sqrt{3}L \times \sqrt{3} = 9L$$

$$\left. \frac{dA}{dt} \right|_{L=\sqrt{3}} = 9\sqrt{3} \text{ cm}^2/\text{s}$$

Question 22**Answer E**

$$\text{Since } \sum \Pr(X = x) = 1 \Rightarrow \frac{a}{2} + a + b + \frac{b}{2} = \frac{3a}{2} + \frac{3b}{2} = 1 \Rightarrow 3(a+b) = 2 \quad \mathbf{A.} \text{ is true}$$

$$E(X) = \sum x \Pr(X = x) = -2 \times \frac{a}{2} - a + b + 2 \times \frac{b}{2} = -2a + 2b = 2(b-a) \quad \mathbf{B.} \text{ is true}$$

$$E(X^2) = \sum x^2 \Pr(X = x) = (-2)^2 \times \frac{a}{2} + (-1)^2 a + (1)^2 b + (2)^2 \times \frac{b}{2} = 2a + a + b + 2b = 3(a+b) = 2$$

C. is true, since **A.** is true.

$$\text{var}(X) = E(X^2) - (E(X))^2 = 2 - 4(b-a)^2 = 2 - 4b^2 + 8ab - 4a^2 \quad \mathbf{D.} \text{ is true}$$

$$\mathbf{E.} \quad \text{is false, } E\left(\frac{1}{X}\right) = \sum \frac{1}{x} \Pr(X = x) = -\frac{a}{4} - a + b + \frac{b}{4} = \frac{5}{4}(b-a)$$

END OF SECTION 1 SUGGESTED ANSWERS

SECTION 2

Question 1

a.i $f(x) = x^3 - 3x^2 + cx + d$

$$f'(x) = 3x^2 - 6x + c$$

but $x = -1$ is a turning point so $f'(x) = (x+1)(3x-9) = 3(x+1)(x-3)$

Expanding gives $c = -9$,

also $u = 3$,

$$f(-1) = 5 = -1 - 3 - c + d = -4 + 9 + d \quad \text{so that}$$

$$d = 0$$

$$f(3) = v = 27 - 27 - 27 = -27$$

$$v = -27$$

A1

A1

A1

- ii.** The graph of $y = x^3 - 3x^2 - 9x$ has a maximum value of 5, and a minimum value of -27 , and crosses the x -axis at three distinct points. The graph of $y = x^3 - 3x^2 - 9x + d$ will therefore cross the x -axis at three distinct points, provided that $d \in (-5, 27)$ or $-5 < d < 27$

A2

b. $f(x) = x^3 - 3x^2 + cx + d$

$f'(x) = 3x^2 - 6x + c$, for two distinct turning points, we require

$$\Delta = 36 - 12c > 0$$

$$c < 3 \quad \text{and} \quad d \in \mathbb{R}$$

M1

A1

c. $f(x+p) = (x+p)^3 - 3(x+p)^2 + c(x+p) + d$

$$f(x+p) = x^3 + 3x^2p + 3xp^2 + p^3 - 3(x^2 + 2xp + p^2) + cx + cp + d$$

$$f(x+p) = x^3 + x^2(3p-3) + x(3p^2-6p+c) + p^3-3p^2+cp+d = x^3$$

$$\text{therefore } 3p-3=0 \Rightarrow p=1$$

A1

$$\text{and } 3p^2-6p+c=0 \quad \text{since } p=1 \quad c=6-3=3$$

$$c=3$$

A1

$$\text{and } p^3-3p^2+cp+d=0$$

$$\text{so } d=-1$$

A1

alternative method, if $y = (x-1)^3 = x^3 - 3x^2 + 3x - 1 \rightarrow y = x^3$

so that $p=1 \quad c=3 \quad \text{and} \quad d=-1$

d. $A = \int_a^b f(x) dx$ $a = 0$ $b = 2$ $h = \frac{1}{2}$ $n = 4$ $f(x) = x^3 - 3x^2 + cx + d$

(1) $L = 10 = h \left[f(0) + f\left(\frac{1}{2}\right) + f(1) + f\left(\frac{3}{2}\right) \right]$ M1

(2) $R = 6 = h \left[f\left(\frac{1}{2}\right) + f(1) + f\left(\frac{3}{2}\right) + f(2) \right]$

(1) $\Rightarrow 20 = f(0) + f\left(\frac{1}{2}\right) + f(1) + f\left(\frac{3}{2}\right)$

(2) $\Rightarrow 12 = f\left(\frac{1}{2}\right) + f(1) + f\left(\frac{3}{2}\right) + f(2)$ subtracting gives M1

$8 = f(0) - f(2) = d - (8 - 12 + 2c + d) = 4 - 2c$

$2c = -4$

$c = -2$ A1

Question 2

a. the amplitude is 1.5, so that $a = 1.5$

one-half cycle is 8, so that $T = \frac{2\pi}{n} = 16 \Rightarrow n = \frac{\pi}{8}$ A1

b. $y = 16(1 - e^{-kx})$ passes through the origin $O(0,0)$ and $B(4,8)$

$8 = 16(1 - e^{-4k}) \Rightarrow \frac{1}{2} = 1 - e^{-4k}$

$e^{-4k} = \frac{1}{2} \quad e^{4k} = 2$ M1

$4k = \log_e(2)$

$k = \frac{1}{4} \log_e(2)$

c.i reflect in the y-axis

translate 8 units, to the right, away from the y-axis A1

or translate 8 units, to the right parallel to the x-axis.

- ii. $f : [4, 8] \rightarrow R$, $f(x) = 16(1 - e^{k(x-8)})$ A2
must give domain.

d.i $A = 2 \int_0^4 \left(16(1 - e^{-kx}) - \frac{3}{2} \sin\left(\frac{\pi x}{8}\right) \right) dx$ A1

ii. $A = 2 \left[16x + \frac{16}{k} e^{-kx} + \frac{12}{\pi} \cos\left(\frac{\pi x}{8}\right) \right]_0^4$ each term A1

$$A = 2 \left[\left(64 + \frac{16}{k} e^{-4k} + \frac{12}{\pi} \cos\left(\frac{\pi}{2}\right) \right) - \left(0 + \frac{16}{k} + \frac{12}{\pi} \cos(0) \right) \right] \text{ but } e^{-4k} = \frac{1}{2}$$
 M1

$$A = 2 \left[64 + \frac{8}{k} - \frac{16}{k} - \frac{12}{\pi} \right]$$

$$A = 128 - \frac{16}{k} - \frac{24}{\pi}$$
 A2

$$p = 128 \quad q = -16 \quad \text{and} \quad r = -24$$

Question 3

- a. the function is continuous $f(4) = 16b = 4c \Rightarrow c = 4b$ M1
the total area under the curve is one.

$$b \int_0^4 t^2 dt + c \int_4^8 (8-t) dt = 1$$

$$b \left[\frac{1}{3} t^3 \right]_0^4 + c \left[8t - \frac{1}{2} t^2 \right]_4^8 = 1$$
 A1

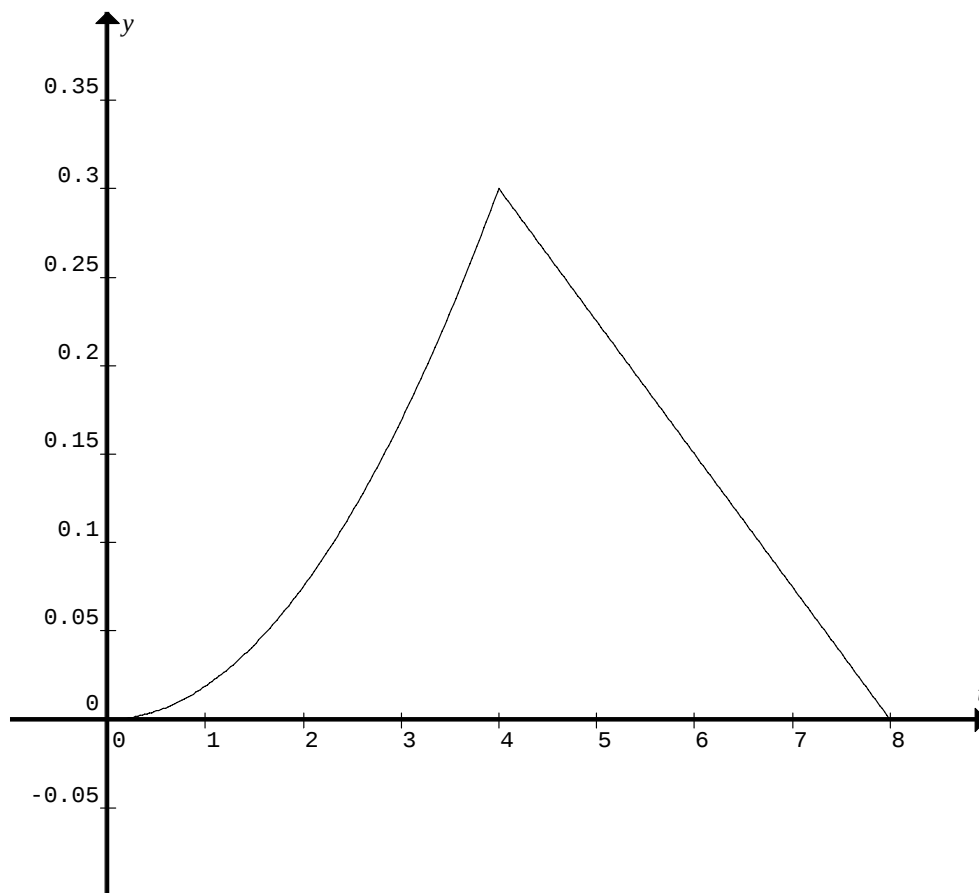
$$b \left(\frac{1}{3} (64 - 0) \right) + c ((64 - 32) - (32 - 8)) = 1$$

$$\frac{64b}{3} + 8c = 1 \quad \text{but} \quad c = 4b$$

$$b \left(\frac{64}{3} + 32 \right) = 1 \Rightarrow b = \frac{3}{160} \quad \text{and} \quad c = \frac{3}{40}$$
 A1

- b. must show point at $(4, 0.3)$ and zero for $t \geq 8$ and $t \leq 0$

G1



c. $\Pr(T > 6) = \frac{3}{40} \int_6^8 (8-t) dt$ or the area of a triangle as

M1

$$\Pr(T > 6) = \frac{1}{2} \times 2 \times 2c = \frac{3}{20}$$

$$\Pr(T > 6) = \frac{3}{20}$$

A1

d. $E(T) = \frac{3}{160} \int_0^4 t^3 dt + \frac{3}{40} \int_4^8 t(8-t) dt$

M1

$$E(T) = 1.2 + 3.2 = 4.4 \text{ minutes}$$

A1

- e. Since $\frac{3}{160} \int_0^4 t^2 dt = 0.4$ the median time m is given by

$$\frac{3}{40} \int_4^m (8-t) dt = 0.1 \quad \text{A1}$$

$$\left[8t - \frac{1}{2}t^2 \right]_4^m = \frac{4}{3}$$

$$\left(8m - \frac{m^2}{2} \right) - (32 - 8) = \frac{4}{3} \quad \text{M1}$$

$$m^2 - 16m + \frac{152}{3} = 0 \quad \text{solving for } m, \text{ and } 4 < m < 8$$

$$m = 4.35 \text{ minutes} \quad \text{A1}$$

- f. X is the running time of the movie in minutes

$$X \stackrel{d}{=} N(\mu = 94, \sigma^2 = 10^2)$$

$$\Pr(X > 109) = \Pr\left(Z > \frac{109 - 94}{10}\right) = \Pr(Z > 1.5)$$

$$= 0.0668 \quad \text{A1}$$

- g. $Y \stackrel{d}{=} Bi(n = 4, p = 0.0668)$

$$\Pr(Y \geq 2) = 1 - [\Pr(Y = 0) + \Pr(Y = 1)] \quad \text{M1}$$

$$\Pr(Y \geq 2) = 1 - [0.9332^4 + {}^4C_1 0.0668 \times 0.9332^3]$$

$$\Pr(Y \geq 2) = 0.0244 \quad \text{A1}$$

- h. $\Pr(2 \text{ comedies}) = ACC + CAC + CCA \quad \text{M1}$

$$= 0.45 \times 0.55 \times 0.65 + 0.55 \times 0.35 \times 0.55 + 0.55 \times 0.65 \times 0.35$$

$$= 0.392 \quad \text{A1}$$

Question 4

a. $P\left(a, \frac{4}{a^2}\right) \quad O(0,0)$

$$s = d(OP) = \sqrt{(a-0)^2 + \left(\frac{4}{a^2} - 0\right)^2} \quad \text{M1}$$

$$s = \sqrt{a^2 + \frac{16}{a^4}} = \sqrt{\frac{16 + a^6}{a^4}} \quad \text{since } a > 0$$

$$s = \frac{1}{a^2} \sqrt{16 + a^6} \quad \text{A1}$$

b.i. $\frac{ds}{da} = \frac{\frac{1}{2} \times 6a^5 \times \frac{1}{\sqrt{16+a^6}} \times a^2 - 2a\sqrt{16+a^6}}{a^4}$ differentiating using the quotient rule

$$\frac{ds}{da} = \frac{1}{a^4} \left[\frac{3a^7 - 2a(16 + a^6)}{\sqrt{16 + a^6}} \right] \quad \text{M1}$$

$$\frac{ds}{da} = 0 \quad \text{for minimum distance}$$

$$\frac{ds}{da} = \frac{a^6 - 32}{a^3 \sqrt{16 + a^6}} = 0 \quad \text{A1}$$

$$a = \sqrt[6]{32} = 2^{\frac{5}{6}} \quad \text{A1}$$

ii. $S_{\min} = \sqrt[3]{2} \cdot \sqrt{3} \approx 2.182 \quad \text{A1}$

c.i at the point $P\left(a, \frac{4}{a^2}\right) \quad f'(x) = -8x^{-3} \quad m_T = -\frac{8}{a^3} \quad \text{A1}$

$$m_N = \frac{a^3}{8}$$

normal $y - \frac{4}{a^2} = \frac{a^3}{8}(x - a) \quad \text{or} \quad y = \frac{a^3 x}{8} - \frac{a^4}{8} + \frac{4}{a^2} \quad \text{A1}$

ii. normal passes through origin (0,0) then

$$-\frac{a^4}{8} + \frac{4}{a^2} = 0$$

$$a^6 = 32$$

$$a = \sqrt[6]{32} = 2^{\frac{5}{6}} \quad \text{A1}$$

Question 5

$$f : [0, 2\pi] \rightarrow \mathbb{R}, \quad f(x) = \sqrt{3}\sin(2x) + \cos(2x)$$

a. $f(x) = 0$

$$\sqrt{3}\sin(2x) + \cos(2x) = 0$$

$$\sqrt{3}\sin(2x) = -\cos(2x)$$

$$\tan(2x) = -\frac{1}{\sqrt{3}}$$

M1

$$2x = -\frac{\pi}{6}, -\frac{\pi}{6} + \pi, -\frac{\pi}{6} + 2\pi, -\frac{\pi}{6} + 3\pi, -\frac{\pi}{6} + 4\pi$$

$$2x = \frac{5\pi}{6}, \frac{11\pi}{6}, \frac{17\pi}{6}, \frac{23\pi}{6}$$

$$x = \frac{5\pi}{12}, \frac{11\pi}{12}, \frac{17\pi}{12}, \frac{23\pi}{12}$$

A1

b.i. $f'(x) = 2\sqrt{3}\cos(2x) - 2\sin(2x) = 0$

A1

$$\sqrt{3}\cos(2x) = \sin(2x)$$

$$\tan(2x) = \sqrt{3}$$

$$2x = \frac{\pi}{3}, \frac{\pi}{3} + \pi, \frac{\pi}{3} + 2\pi, \frac{\pi}{3} + 3\pi$$

M1

$$2x = \frac{\pi}{3}, \frac{4\pi}{3}, \frac{7\pi}{3}, \frac{10\pi}{3}$$

$$x = \frac{\pi}{6}, \frac{2\pi}{3}, \frac{7\pi}{6}, \frac{5\pi}{3}$$

A1

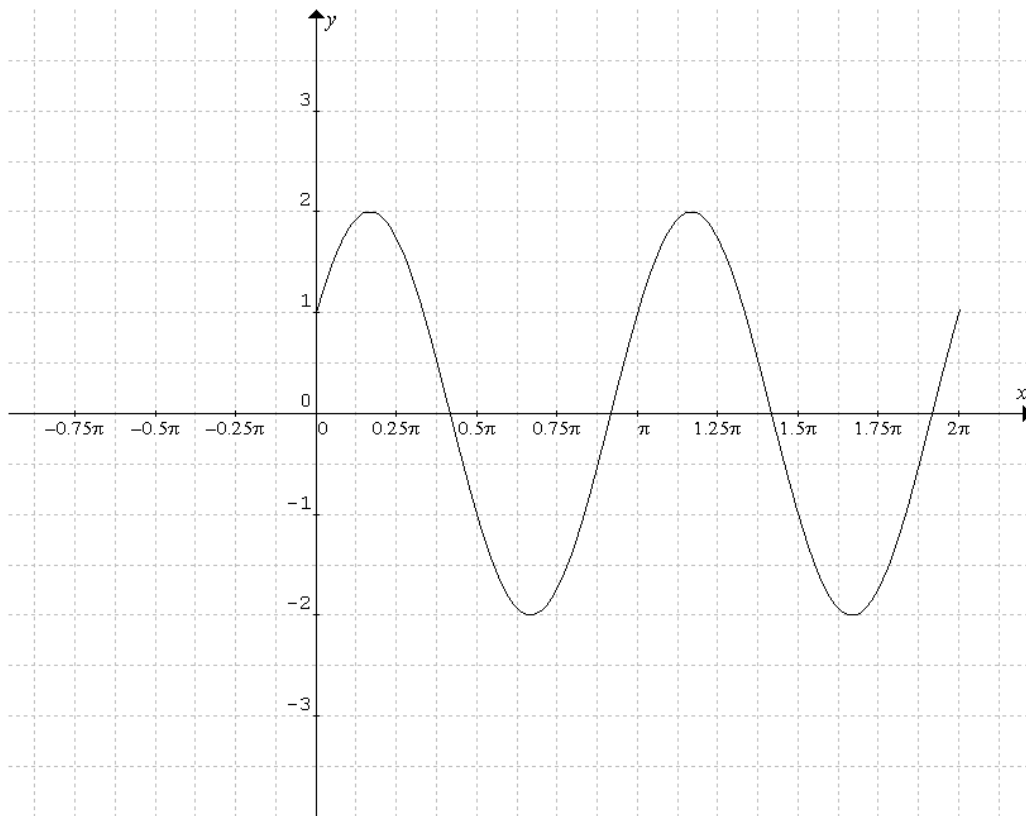
ii. $\max \left(\frac{\pi}{6}, 2 \right) \text{ and } \left(\frac{7\pi}{6}, 2 \right), \min \left(\frac{2\pi}{3}, -2 \right) \text{ and } \left(\frac{5\pi}{3}, -2 \right)$

A1

c. graph on correct domain, correct x-intercepts
and correct max and min.

G1

G1



d. $f(x) = \sqrt{3} \sin(2x) + \cos(2x) = 2 \sin\left(2x + \frac{\pi}{6}\right) = 2 \sin\left(2\left(x + \frac{\pi}{12}\right)\right)$

translate $2 \sin(2x)$, $\frac{\pi}{12}$ to the left parallel to the x-axis

$$A = 2$$

A1

$$\alpha = \frac{\pi}{12}$$

A1

END OF SECTION 2 SUGGESTED ANSWERS