

Year 2009

VCE

Mathematical Methods
and
Mathematical Methods
(CAS)
Solutions
Trial Examination 1



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Question 1

$$y = \frac{\log_e(2x)}{2x^2} \text{ differentiating using the quotient rule}$$

$$\text{let } u = \log_e(2x) \quad v = 2x^2$$

$$\frac{du}{dx} = \frac{1}{x} \quad \frac{dv}{dx} = 4x \quad \text{M1}$$

$$\frac{dy}{dx} = \frac{2x^2 \times \frac{1}{x} - 4x \log_e(2x)}{4x^4} = \frac{2x - 4x \log_e(2x)}{4x^4}$$

$$\frac{dy}{dx} = \frac{1}{2x^3} (1 - 2 \log_e(2x)) \quad \text{A1}$$

Question 2

$$\log_6(x+2) + \log_6(2x-2) = 2$$

$$\log_6((x+2)(2x-2)) = 2$$

$$2(x+2)(x-1) = 6^2 = 36$$

$$x^2 + x - 2 = 18$$

$$x^2 + x - 20 = 0$$

$$(x+5)(x-4) = 0$$

$$x = -5 \quad x = 4$$

$$\text{but } x > -2 \text{ so there is only one answer } x = 4 \quad \text{A1}$$

Question 3

For the function to be differentiable it must be continuous and the gradients must match.

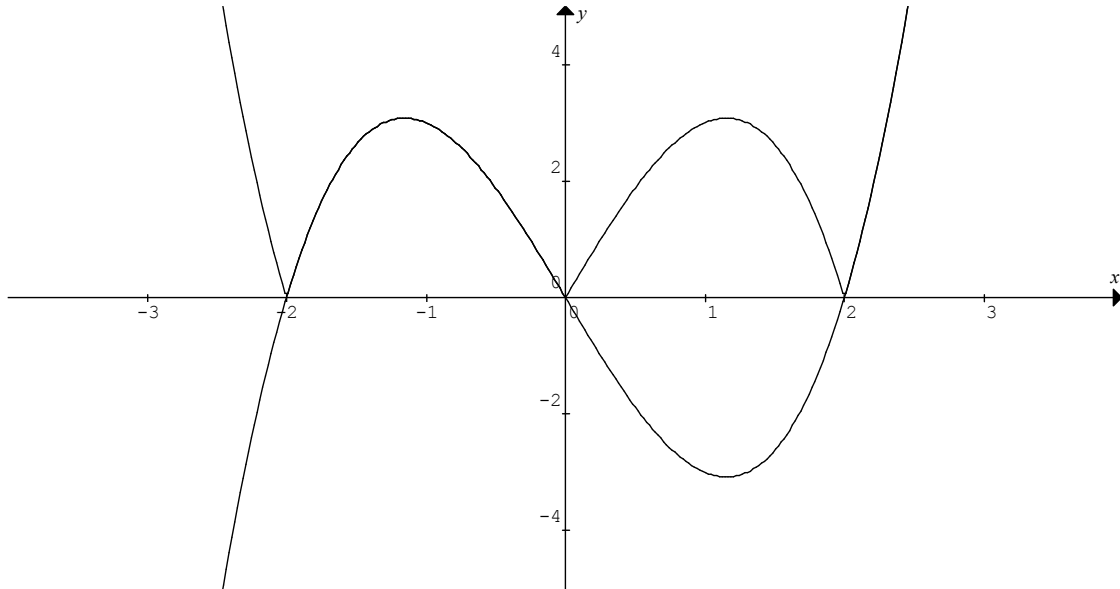
$$f(2) = a\sqrt{4} = 2a = 2m + 3 \quad \text{M1}$$

$$f'(x) = \begin{cases} \frac{a}{2\sqrt{x+2}} & x > 2 \\ m & x < 2 \end{cases} \Rightarrow f'(2) = \frac{a}{4} = m \quad \text{A1}$$

$$a = 4m \Rightarrow 8m = 2m + 3 \quad 6m = 3$$

$$m = \frac{1}{2} \quad \text{A1}$$

$$a = 2 \quad \text{A1}$$

Question 4**a.****G1**

b. $A = 2 \int_0^2 (4x - x^3) dx$ by symmetry

$$A = 2 \left[2x^2 - \frac{x^4}{4} \right]_0^2$$

M1

$$A = 2[(8 - 4) - (0)]$$

$$= 8 \text{ units}^2$$

A1**Question 5**

f $y = \frac{1}{3}(1 - e^{-2x})$ interchanging y and x

f^{-1} $x = \frac{1}{3}(1 - e^{-2y})$ transposing to make y the subject

M1

f^{-1} $3x = 1 - e^{-2y} \Rightarrow e^{-2y} = 1 - 3x \Rightarrow -2y = \log_e(1 - 3x)$

$$y = f^{-1}(x) = -\frac{1}{2} \log_e(1 - 3x) = \frac{1}{2} \log_e \left(\frac{1}{1 - 3x} \right)$$

A1

the domain of f^{-1} , needs to be stated as we are asked for a function

since $1 - 3x > 0 \Rightarrow x < \frac{1}{3}$ $\text{dom } f^{-1} = \left(-\infty, \frac{1}{3} \right)$

$f^{-1} : \{x : x < \frac{1}{3}\} \rightarrow \mathbb{R}$, $f^{-1}(x) = -\frac{1}{2} \log_e(1 - 3x)$

A1

Question 6

a. $V = \frac{\pi h}{45}(h^2 + 75h) = \frac{\pi}{45}(h^3 + 75h^2)$

when $h = 1$ $\Delta h = 0.01$ find ΔV

$$\frac{dV}{dh} = \frac{\pi}{45}(3h^2 + 150h) = \frac{\pi}{15}(h^2 + 50h) \approx \frac{\Delta V}{\Delta h} \quad \text{M1}$$

$$\Delta V \approx \frac{\pi}{15}(1 + 50) \times 0.01 = \frac{51\pi}{1500} \text{ cm}^3$$

$$\text{the volume increases by } \frac{17\pi}{500} \text{ cm}^3 \quad \text{A1}$$

b. Now given $\frac{dh}{dt} = 10 \text{ cm/min}$

$$\text{By the chain rule } \frac{dV}{dt} = \frac{dV}{dh} \frac{dh}{dt} = \frac{10\pi}{15}(h^2 + 50h) \quad \text{M1}$$

$$\text{when } h = 1 \quad \left. \frac{dV}{dt} \right|_{h=1} = \frac{2\pi(1+50)}{3} = 34\pi \text{ cm}^3/\text{min} \quad \text{A1}$$

Question 7

a. $\int_0^{\frac{\pi}{3}} k \sin(3x) dx = 1$

$$-\frac{k}{3} [\cos(3x)]_0^{\frac{\pi}{3}} = 1$$

$$-\frac{k}{3} (\cos(\pi) - \cos(0)) = \frac{2k}{3} = 1 \quad \text{M1}$$

$$k = \frac{3}{2}$$

b. $\frac{d}{dx}(x \cos(3x)) = \cos(3x) - 3x \sin(3x) \quad \text{A1}$

$$3 \int x \sin(3x) dx = \int \cos(3x) dx - x \cos(3x) = \frac{1}{3} \sin(3x) - x \cos(3x) \quad \text{M1}$$

$$\text{Now } E(X) = \frac{3}{2} \int_0^{\frac{\pi}{3}} x \sin(3x) dx \quad \text{A1}$$

$$E(X) = \frac{3}{2} \times \frac{1}{3} \left[\frac{1}{3} \sin(3x) - x \cos(3x) \right]_0^{\frac{\pi}{3}} \quad \text{M1}$$

$$E(X) = \frac{1}{2} \left[\left(\frac{1}{3} \sin(\pi) - \frac{\pi}{3} \cos(\pi) \right) - \left(\frac{1}{3} \sin(0) - 0 \right) \right]$$

$$E(X) = \frac{\pi}{6} \quad \text{A1}$$

Question 8

X	1	2
$\Pr(X = x)$	$\sin(k)$	$2\sin^2(k)$

a. $\sum \Pr(X = x) = \sin(k) + 2\sin^2(k) = 1$
 $2\sin^2(k) + \sin(k) - 1 = 0$ M1
 $(2\sin(k) - 1)(\sin(k) + 1) = 0$
 $\sin(k) = -1$ not possible, since, probabilities must be positive A1
and $\sin(k) = \frac{1}{2}$
 $k = \frac{\pi}{6}, \frac{5\pi}{6}$ A1

b. $E(X) = \sum x \Pr(X = x) = \sin(k) + 4\sin^2(k)$
 $E(X) = \frac{1}{2} + 4 \times \frac{1}{4}$
 $E(X) = 1.5$ A1

Question 9

a. $g(x) = \tan(x)$ and $h(x) = \frac{\sqrt{x}}{2}$ A1

b. let $f(x) = y = \tan(u)$ $u = \frac{\sqrt{x}}{2} = \frac{1}{2}x^{\frac{1}{2}}$ chain rule M1
 $\frac{dy}{du} = \frac{1}{\cos^2(u)}$ $\frac{du}{dx} = \frac{1}{4}x^{-\frac{1}{2}} = \frac{1}{4\sqrt{x}}$

$$f'(x) = \frac{dy}{dx} = \frac{1}{4\sqrt{x} \cos^2\left(\frac{\sqrt{x}}{2}\right)}$$
 A1

$$\left. \frac{dy}{dx} \right|_{x=\frac{\pi}{6}} = f'\left(\frac{\pi^2}{9}\right) = \frac{1}{4 \times \frac{\pi}{3} \cos^2\left(\frac{\pi}{6}\right)} = \frac{3}{4\pi \times \left(\frac{\sqrt{3}}{2}\right)^2} = \frac{3}{4\pi} \times \frac{4}{3}$$

$$f'\left(\frac{\pi^2}{9}\right) = \frac{1}{\pi}$$
 A1

Question 10

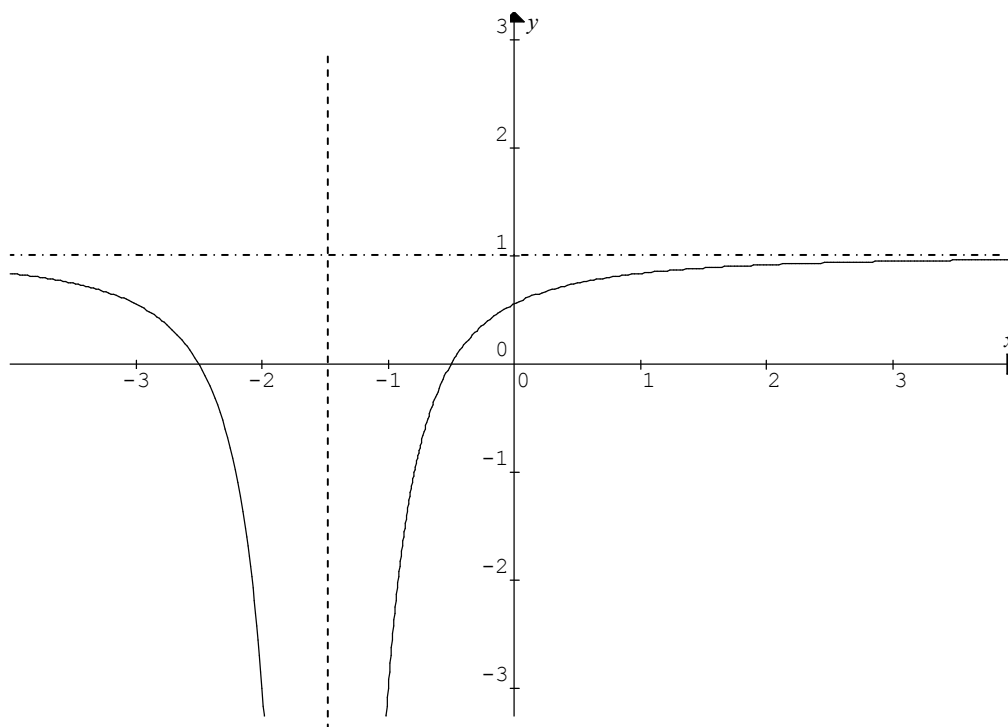
- a. $x = -\frac{3}{2}$ is a vertical asymptote, $y = 1$ is a horizontal asymptote

the y-intercept is $y = 1 - \frac{4}{9} = \frac{5}{9}$ $\left(0, \frac{5}{9}\right)$ A1

crosses the x-axis, when $y = 0$, $(2x+3)^2 = 4$ $2x+3 = \pm 2$

$2x = -1, -5$ $x = -\frac{1}{2}, -\frac{5}{2}$ $\left(-\frac{1}{2}, 0\right) \left(-\frac{5}{2}, 0\right)$

correct graph, correct axial intercepts and correct asymptotes G1



- b. The area is $A = \int_0^1 \left(1 - \frac{4}{(2x+3)^2}\right) dx$ A1

$A = \left[x + \frac{2}{2x+3} \right]_0^1 = \left(\left(1 + \frac{2}{5}\right) - \left(0 + \frac{2}{3}\right) \right)$ M1

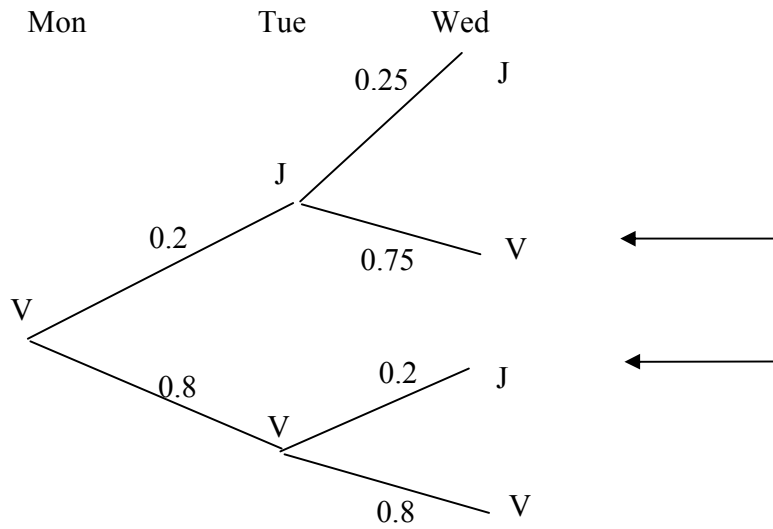
$A = \frac{11}{15} \text{ units}^2$ A1

Question 11

Pr(Vegimite on two days)

M1

$= V V J$ or $V J V$



$$= 0.2 \times 0.75 + 0.8 \times 0.2$$

$$= \frac{1}{5} \times \frac{3}{4} + \frac{4}{5} \times \frac{1}{5} = \frac{3}{20} + \frac{4}{25} = \frac{15+16}{100}$$

A1

$$= 0.31$$

A1

END OF SUGGESTED SOLUTIONS