Year 2009

VCE

Mathematical Methods
and
Mathematical Methods
(CAS)
Solutions
Trial Examination 1

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Question 1

$y = \dfrac{\log_e(2x)}{2x^2}$ differentiating using the quotient rule

let $u = \log_e(2x)$ $\qquad v = 2x^2$

$\dfrac{du}{dx} = \dfrac{1}{x} \qquad \dfrac{dv}{dx} = 4x$ M1

$$\frac{dy}{dx} = \frac{2x^2 \times \frac{1}{x} - 4x\log_e(2x)}{4x^4} = \frac{2x - 4x\log_e(2x)}{4x^4}$$

$\dfrac{dy}{dx} = \dfrac{1}{2x^3}\left(1 - 2\log_e(2x)\right)$ A1

Question 2

$\log_6(x+2) + \log_6(2x-2) = 2$

$\log_6\left((x+2)(2x-2)\right) = 2$

$2(x+2)(x-1) = 6^2 = 36$

$x^2 + x - 2 = 18$

$x^2 + x - 20 = 0$

$(x+5)(x-4) = 0$ M1

$x = -5 \quad x = 4$

but $x > -2$ so there is only one answer $x = 4$ A1

Question 3

For the function to be differentiable it must be continuous and the gradients must match.

$f(2) = a\sqrt{4} = 2a = 2m + 3$ M1

$f'(x) = \begin{cases} \dfrac{a}{2\sqrt{x+2}} & x > 2 \\ m & x < 2 \end{cases} \quad \Rightarrow \quad f'(2) = \dfrac{a}{4} = m$ A1

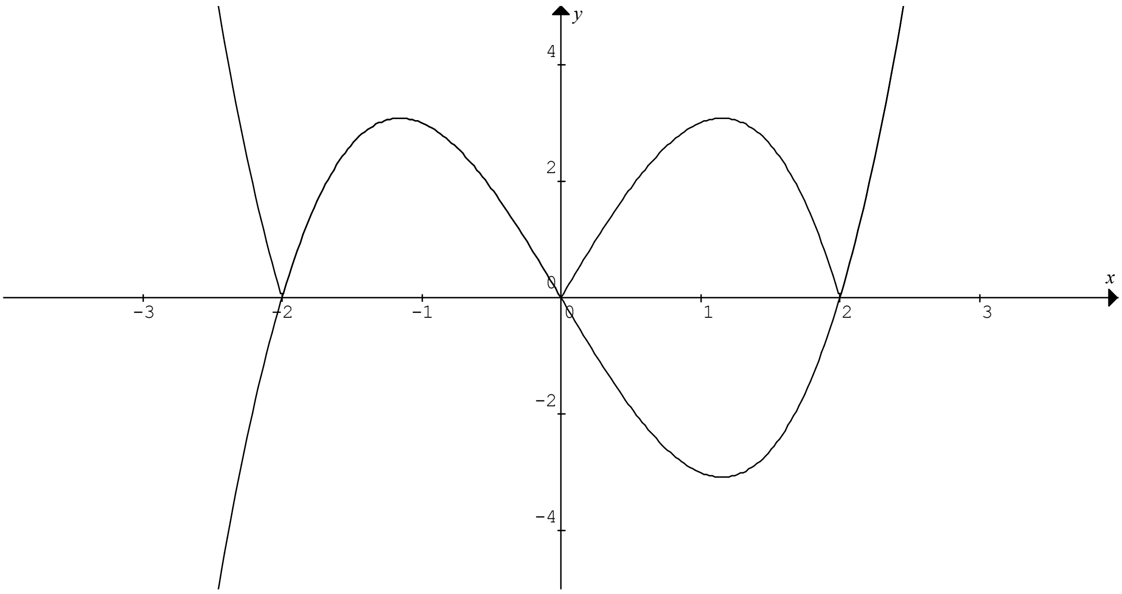
$a = 4m \qquad \Rightarrow 8m = 2m + 3 \qquad 6m = 3$

$m = \dfrac{1}{2}$ A1

$a = 2$ A1

Question 4

a. G1



b. $A = 2\int_0^2 \left(4x - x^3\right) dx$ by symmetry

$A = 2\left[2x^2 - \dfrac{x^4}{4}\right]_0^2$ M1

$A = 2\left[(8-4)-(0)\right]$

$= 8 \text{ units}^2$ A1

Question 5

f $\quad y = \dfrac{1}{3}\left(1 - e^{-2x}\right)$ interchanging y and x

f^{-1} $\quad x = \dfrac{1}{3}\left(1 - e^{-2y}\right)$ transposing to make y the subject M1

f^{-1} $\quad 3x = 1 - e^{-2y} \quad \Rightarrow \quad e^{-2y} = 1 - 3x \quad \Rightarrow \quad -2y = \log_e(1-3x)$

$y = f^{-1}(x) = -\dfrac{1}{2}\log_e(1-3x) = \dfrac{1}{2}\log_e\left(\dfrac{1}{1-3x}\right)$ A1

the domain of f^{-1}, needs to be stated as we are asked for a function

since $1 - 3x > 0 \quad \Rightarrow x < \dfrac{1}{3}$ $\quad$ dom $f^{-1} = \left(-\infty, \dfrac{1}{3}\right)$

$f^{-1}: \{x : x < \dfrac{1}{3}\} \to R$, $f^{-1}(x) = -\dfrac{1}{2}\log_e(1-3x)$ A1

Question 6

a. $V = \frac{\pi h}{45}\left(h^2 + 75h\right) = \frac{\pi}{45}\left(h^3 + 75h^2\right)$

when $h = 1$ $\Delta h = 0.01$ find ΔV

$\frac{dV}{dh} = \frac{\pi}{45}\left(3h^2 + 150h\right) = \frac{\pi}{15}\left(h^2 + 50h\right) \approx \frac{\Delta V}{\Delta h}$ M1

$\Delta V \approx \frac{\pi}{15}(1+50) \times 0.01 = \frac{51\pi}{1500}$ cm^3

the volume increases by $\frac{17\pi}{500}$ cm^3 A1

b. Now given $\frac{dh}{dt} = 10$ cm/min

By the chain rule $\frac{dV}{dt} = \frac{dV}{dh}\frac{dh}{dt} = \frac{10\pi}{15}\left(h^2 + 50h\right)$ M1

when $h = 1$ $\left.\frac{dV}{dt}\right|_{h=1} = \frac{2\pi(1+50)}{3} = 34\pi$ cm^3/min A1

Question 7

a. $\int_0^{\frac{\pi}{3}} k\sin(3x)\,dx = 1$

$-\frac{k}{3}\left[\cos(3x)\right]_0^{\frac{\pi}{3}} = 1$

$-\frac{k}{3}\left(\cos(\pi) - \cos(0)\right) = \frac{2k}{3} = 1$ M1

$k = \frac{3}{2}$

b. $\frac{d}{dx}\left(x\cos(3x)\right) = \cos(3x) - 3x\sin(3x)$ A1

$3\int x\sin(3x)\,dx = \int \cos(3x)\,dx - x\cos(3x) = \frac{1}{3}\sin(3x) - x\cos(3x)$ M1

Now $E(X) = \frac{3}{2}\int_0^{\frac{\pi}{3}} x\sin(3x)\,dx$ A1

$E(X) = \frac{3}{2} \times \frac{1}{3}\left[\frac{1}{3}\sin(3x) - x\cos(3x)\right]_0^{\frac{\pi}{3}}$ M1

$E(X) = \frac{1}{2}\left[\left(\frac{1}{3}\sin(\pi) - \frac{\pi}{3}\cos(\pi)\right) - \left(\frac{1}{3}\sin(0) - 0\right)\right]$

$E(X) = \frac{\pi}{6}$ A1

Question 8

X	1	2
$\Pr(X = x)$	$\sin(k)$	$2\sin^2(k)$

a. $\sum \Pr(X = x) = \sin(k) + 2\sin^2(k) = 1$

$2\sin^2(k) + \sin(k) - 1 = 0$

$(2\sin(k) - 1)(\sin(k) + 1) = 0$ M1

$\sin(k) = -1$ not possible, since, probabilities must be positive A1

and $\sin(k) = \frac{1}{2}$

$k = \frac{\pi}{6}$, $\frac{5\pi}{6}$ A1

b. $E(X) = \sum x \Pr(X = x) = \sin(k) + 4\sin^2(k)$

$E(X) = \frac{1}{2} + 4 \text{ x } \frac{1}{4}$

$E(X) = 1.5$ A1

Question 9

a. $g(x) = \tan(x)$ and $h(x) = \frac{\sqrt{x}}{2}$ A1

b. let $f(x) = y = \tan(u)$ $u = \frac{\sqrt{x}}{2} = \frac{1}{2}x^{\frac{1}{2}}$ chain rule M1

$\frac{dy}{du} = \frac{1}{\cos^2(u)}$ $\frac{du}{dx} = \frac{1}{4}x^{-\frac{1}{2}} = \frac{1}{4\sqrt{x}}$

$f'(x) = \frac{dy}{dx} = \frac{1}{4\sqrt{x}\cos^2\left(\frac{\sqrt{x}}{2}\right)}$ A1

$\left.\frac{dy}{dx}\right|_{x=\frac{\pi}{6}} = f'\left(\frac{\pi^2}{9}\right) = \frac{1}{4 \text{ x } \frac{\pi}{3}\cos^2\left(\frac{\pi}{6}\right)} = \frac{3}{4\pi \text{ x } \left(\frac{\sqrt{3}}{2}\right)^2} = \frac{3}{4\pi} \text{ x } \frac{4}{3}$

$f'\left(\frac{\pi^2}{9}\right) = \frac{1}{\pi}$ A1

Question 10

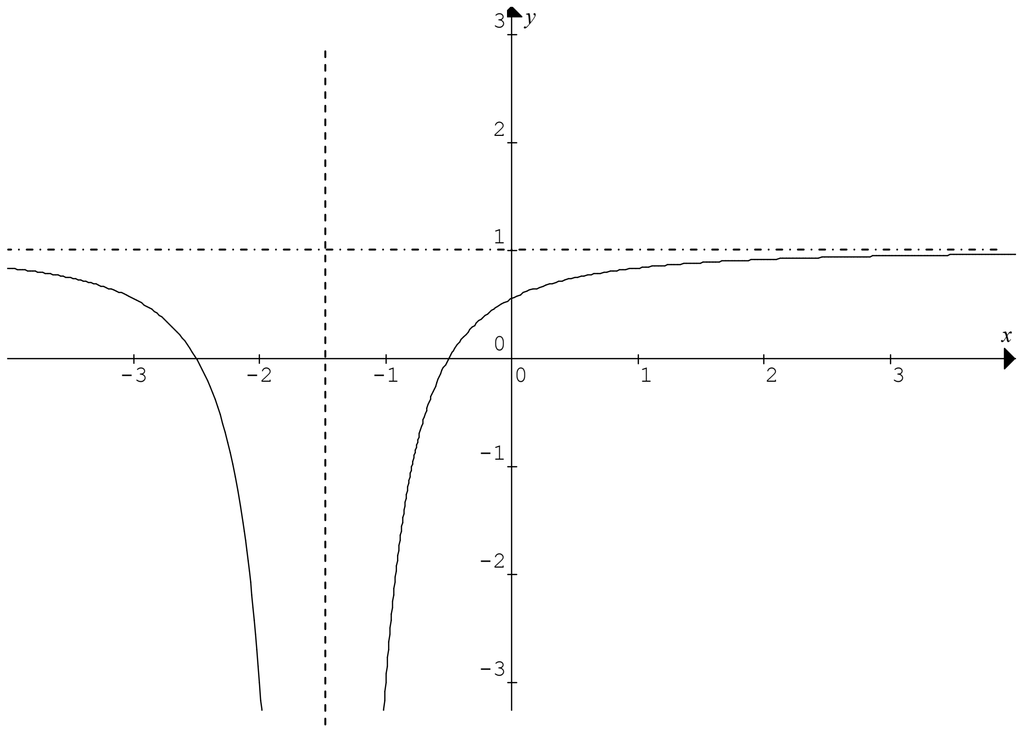
a. $x = -\frac{3}{2}$ is a vertical asymptote, $y = 1$ is a horizontal asymptote

the y-intercept is $y = 1 - \frac{4}{9} = \frac{5}{9}$ $\left(0, \frac{5}{9}\right)$ A1

crosses the x-axis, when $y = 0$, $(2x+3)^2 = 4$ $2x + 3 = \pm 2$

$2x = -1, -5$ $x = -\frac{1}{2}, -\frac{5}{2}$ $\left(-\frac{1}{2}, 0\right)$ $\left(-\frac{5}{2}, 0\right)$

correct graph, correct axial intercepts and correct asymptotes G1



b. The area is $A = \int_0^1 \left(1 - \frac{4}{(2x+3)^2}\right) dx$ A1

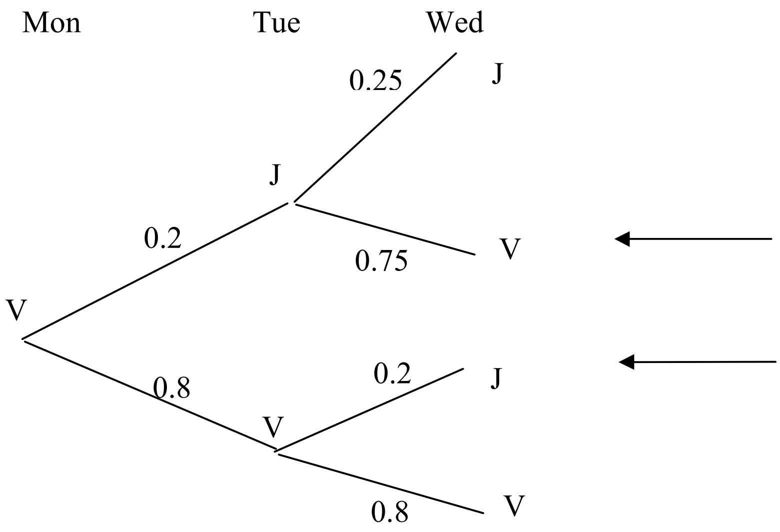
$A = \left[x + \frac{2}{2x+3}\right]_0^1 = \left(\left(1 + \frac{2}{5}\right) - \left(0 + \frac{2}{3}\right)\right)$ M1

$A = \frac{11}{15}$ units^2 A1

Question 11

$\Pr(\text{Vegimite on two days})$ M1

$= V\ V\ J \text{ or } V\ J\ V$



$= 0.2 \times 0.75 + 0.8 \times 0.2$

$= \frac{1}{5} \times \frac{3}{4} + \frac{4}{5} \times \frac{1}{5} = \frac{3}{20} + \frac{4}{25} = \frac{15+16}{100}$ A1

$= 0.31$ A1

END OF SUGGESTED SOLUTIONS