

Q1a The product rule: $\frac{dy}{dx} = 1 + \log_e x$

Q1b The quotient rule: $f'(x) = \frac{-(2x+2)\sin x - 2\cos x}{(2x+2)^2}$
 $= \frac{-(x+1)\sin x - \cos x}{2(x+1)^2}$
 $f'(\pi) = \frac{-(\pi+1)\sin \pi - \cos \pi}{2(\pi+1)^2} = \frac{1}{2(\pi+1)^2}$

Q2a $\int \frac{1}{1-2x} dx = -\frac{1}{2} \log_e |1-2x|$

Q2b $\int_1^4 (\sqrt{x} + 1) dx = \left[\frac{2x^{\frac{3}{2}}}{3} + x \right]_1^4 = \frac{23}{3}$

Q3 Equation of $f(x)$: $y = \frac{3}{x} - 4$

Equation of inverse: $x = \frac{3}{y} - 4$, i.e. $y = \frac{3}{x+4}$

$f^{-1}: R \setminus \{-4\} \rightarrow R$ where $f^{-1}(x) = \frac{3}{x+4}$

Q4 $\tan(2x) = \sqrt{3}$, $x \in \left(-\frac{\pi}{4}, \frac{\pi}{4}\right) \cup \left(\frac{\pi}{4}, \frac{3\pi}{4}\right)$,

$2x = \frac{\pi}{3}, \frac{4\pi}{3}$

$x = \frac{\pi}{6}, \frac{2\pi}{3}$

Q5a $\frac{1}{4} \times \frac{1}{3} = \frac{1}{12}$

Q5b $\Pr(1,4) + \Pr(2,3) + \Pr(3,2) + \Pr(4,1) = \frac{1}{12} + \frac{1}{12} + \frac{1}{12} + \frac{1}{12} = \frac{1}{3}$

Q5c Conditional probability $= \frac{\frac{1}{12}}{\frac{1}{3}} = \frac{1}{4}$

Q6 $\frac{dV}{dt} = 10$, $V = 2\pi r^2$, $\frac{dV}{dr} = 4\pi r$

$\frac{dV}{dt} = \frac{dV}{dr} \times \frac{dr}{dt}$

$\frac{dr}{dt} = \frac{\frac{dV}{dt}}{\frac{dV}{dr}} = \frac{10}{4\pi r}$

When $r = 30$, $\frac{dr}{dt} = \frac{1}{12\pi}$ mm per min

Q7a $\Pr(X > 1 | X \leq 3) = \frac{\Pr(X > 1 \cap X \leq 3)}{\Pr(X \leq 3)}$
 $= \frac{0.4 + 0.2}{0.1 + 0.2 + 0.4 + 0.2} = \frac{2}{3}$

Q7b $\bar{X} = 0 \times 0.1 + 1 \times 0.2 + 2 \times 0.4 + 3 \times 0.2 + 4 \times 0.1 = 2$
 $Var(x) = 0^2 \times 0.1 + 1^2 \times 0.2 + 2^2 \times 0.4 + 3^2 \times 0.2 + 4^2 \times 0.1 - 2^2 = 1.2$

Q8 $f(x) = e^x + k$, $k \in R$, $f'(x) = e^x$

Gradient of tangent at $x = a$ is $f'(a) = e^a$

Equation of tangent: $y = e^a x$ since it passes through $(0,0)$

At $x = a$, $f(a) = e^a + k = e^a a$

$\therefore k = e^a(a-1)$

Q9 For $2\log_e x - \log_e(x+3) = \log_e \frac{1}{2}$ to be defined, $x > 0$

$\log_e \frac{x^2}{x+3} = \log_e \frac{1}{2}$, $\frac{x^2}{x+3} = \frac{1}{2}$, $(2x-3)(x+1) = 0$

Since $x > 0$, $\therefore x+1 \neq 0$

$\therefore 2x-3 = 0$, $x = \frac{3}{2}$

Q10a $f(x+h) \approx f(x) + hf'(x)$

Let $f(x) = \sqrt[3]{x} = x^{\frac{1}{3}}$, $f'(x) = \frac{1}{3x^{\frac{2}{3}}}$

Let $x = 8$, $\sqrt[3]{8.06} \approx \sqrt[3]{8} + 0.06 \times \frac{1}{3 \times 8^{\frac{2}{3}}} = 2 + 0.005 = 2.005$

Q10b $f'(x) = \frac{1}{3x^{\frac{2}{3}}}$ is a decreasing function for $x > 0$. As x

increases from a point, say at $x = a > 0$, the tangent at $x = a$ is always higher than $f(x)$.

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