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Mathematical Methods

2009

Trial Examination 1

Instructions

Answer **all** questions. Do **not** use calculators.

A decimal approximation will not be accepted if an **exact** answer is required to a question.

In questions where more than one mark is available, appropriate working must be shown.

Unless otherwise indicated, the diagrams in this exam are **not** drawn to scale.

Question 1 The power function $(1-x)^n$ is expanded to form a polynomial function $P(x)$, where $n > 5$.

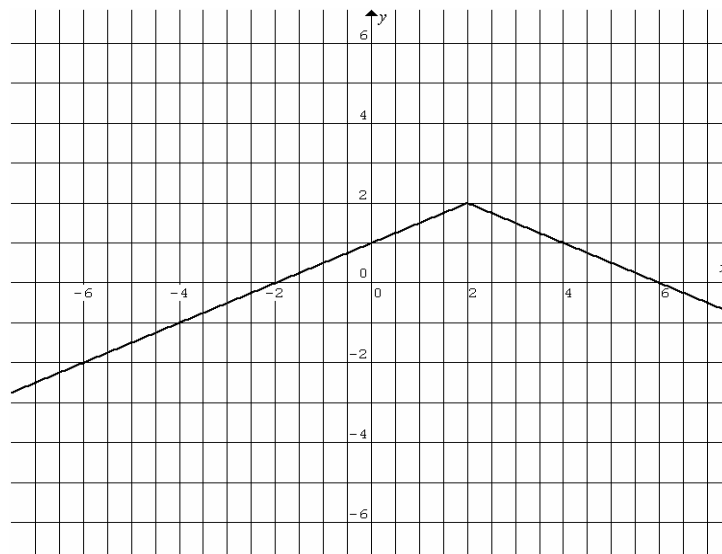
a. Determine the coefficient of the x^5 term in the polynomial in terms of n and combination C .

2 marks

b. Let $f(x) = \frac{P(x)}{\frac{d}{dx}P(x)}$, find $f(2)$ in terms of n .

3 marks

Question 2 The graph of the function $g(x)$ is shown below.



a. Determine the equation of the function $g(x)$.

2 marks

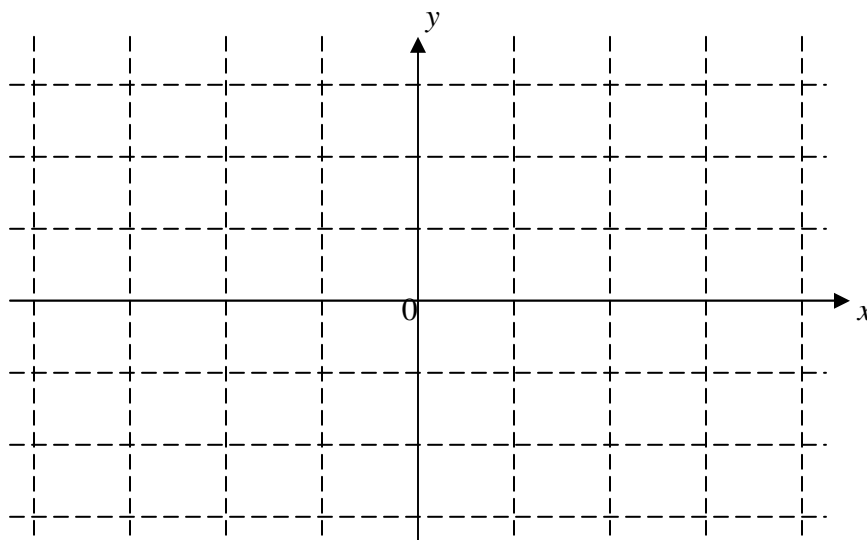
b. On the grid above sketch the graph of the inverse of $g(x)$.

1 mark

Question 3 Given $u(x) = \frac{1}{x^3 - x^2 + x - 1}$ and $v(x) = x^4 - 1$.

Sketch the graph of the **product** of $u(x)$ and $v(x)$.

3 marks



Question 4 Solve the equation $\cos\left(\frac{2x}{3}\right) = \sqrt{3} \sin\left(\frac{2x}{3}\right)$ for $x \in \left[-\frac{3\pi}{2}, \frac{3\pi}{2}\right]$.

3 marks

Question 5 The gradient of the tangent to the graph of a continuous and differentiable function at point $(2.7, 0.28)$ is -2.9 . $(p, -0.01)$ is another point on the graph very close to $(2.7, 0.28)$.

Use linear approximation to estimate a value for p .

3 marks

Question 6 Given $f(x) = \frac{1}{\sqrt{2x-1}}$ and $g(x) = e^{-x}$.

a. Determine the domain of $f(g(x))$ in exact form. 2 marks

b. Determine the range of $g(f(x))$. 2 marks

c. Find $\frac{d}{dx} \left(g \left((f(x))^{-1} \right) \right)^{-1}$. 4 marks

Question 7 Given $\int (\log_e 2x) dx - \log_e (2x)^e = \int e f(x) dx$, evaluate the exact value of $f(e)$. 3 marks

Question 8 Consider the binomial distribution $\text{Bi}(n, 0.5)$ for X and $\Pr(a < X < b) \approx 0.95$.

a. Find $\Pr(X \leq a) + \Pr(X \geq b)$ 1 mark

b. Estimate the smallest value of $b - a$ in terms of n . 2 marks

Question 9 Random variable X has normal distribution $N(125,9)$, $\Pr(X < a) = 0.9$ and $\Pr(X > b) = 0.2$.

a. Find $\Pr(X < a \mid X > b)$. 3 marks

b. Find $\Pr(X < b \mid X > a)$. 1 mark

Question 10 Random variable X has a probability distribution given by $f(x) = \begin{cases} \frac{1}{\pi} x \sin x & 0 \leq x \leq \pi \\ 0 & \text{elsewhere} \end{cases}$

a. Show that $\sin(m_o) + m_o \cos(m_o) = 0$, where m_o is the mode of X . 2 marks

b. Given $\frac{d}{dx}(x \cos x) = \cos x - x \sin x$, show that $\sin(m_e) - m_e \cos(m_e) = \frac{\pi}{2}$, where m_e is the median of X .

3 marks

End of exam 1