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**MATHS METHODS 3 & 4
TRIAL EXAMINATION 2
SOLUTIONS
2009**

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|------|-------|-------|
| 1. C | 9. B | 17. E |
| 2. A | 10. D | 18. A |
| 3. D | 11. B | 19. D |
| 4. E | 12. D | 20. B |
| 5. A | 13. C | 21. E |
| 6. D | 14. B | 22. E |
| 7. A | 15. C | |
| 8. A | 16. A | |

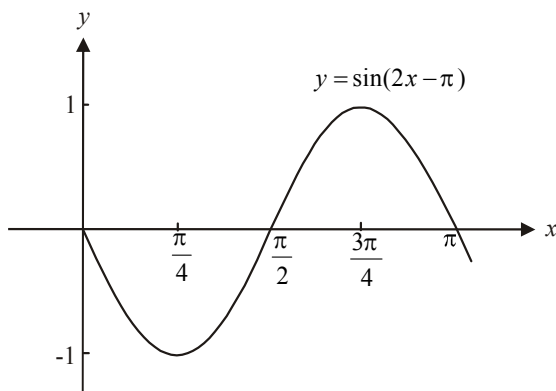
Section 1 – Multiple-choice solutions

Question 1

$E(X)$ = mean of X
 $= -2 \times 0.3 + -1 \times 0.2 + 0 \times 0.4 + 1 \times 0.1$
 $= -0.7$
 The answer is C.

Question 2

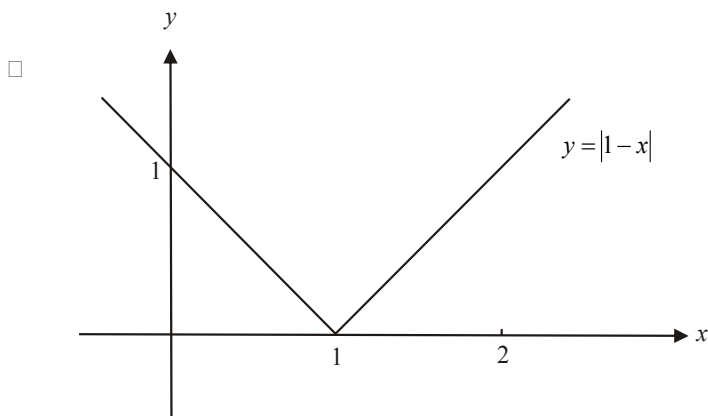
f^{-1} exists if f is a 1:1 function.
 Sketch the graph of $y = \sin(2x - \pi)$ for $x \geq 0$.



If $a = \frac{\pi}{4}$, then f is a 1:1 function and f^{-1} exists.
 The answer is A.

Question 3

Sketch the function $y = |1 - x|$.



The rate of change is the gradient of the function. At $x = 2$, the gradient = 1.
The answer is D.

Question 4

□ $y = \sin(e^{2x})$

This is a composite function so use the chain rule.

□ Method 1 – fast way

$$y = \sin(e^{2x})$$

$$\frac{dy}{dx} = 2e^{2x} \cos(e^{2x})$$

The answer is E.

□ Method 2

$$y = \sin(e^{2x}) \quad \text{Let } u = e^{2x}$$

$$= \sin(u) \quad \frac{du}{dx} = 2e^{2x}$$

$$\frac{dy}{du} = \cos(u)$$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} \quad (\text{Chain rule})$$

$$= \cos(u) \cdot 2e^{2x}$$

$$= 2e^{2x} \cos(e^{2x})$$

The answer is E.

Question 5

This is a binominal distribution with $n = 8$ and $p = 0.1$.

$$\begin{aligned}
 \Pr(X \geq 2) &= 1 - \Pr(X < 2) \\
 &= 1 - \{\Pr(X = 0) + \Pr(X = 1)\} \\
 &= 1 - \{ {}^8C_0 (0.1)^0 (0.9)^8 + {}^8C_1 (0.1)^1 (0.9)^7 \} \\
 &= 1 - (0.430467... + 0.382637...) \\
 &= 0.186896... \\
 &= 0.1869 \text{ (to four decimal places)}
 \end{aligned}$$

The answer is A.

Question 6

$$\begin{aligned}
 &\int \left(\frac{2}{x+1} + \sin(2x) \right) dx \\
 &= 2 \log_e |x+1| - \frac{1}{2} \cos(2x) + c
 \end{aligned}$$

The answer is D.

Question 7

$$\begin{aligned}
 \int_0^4 (2 - 5f(x)) dx &= \int_0^4 2 dx - 5 \int_0^4 f(x) dx \\
 &= [2x]_0^4 - 5 \times 3 \\
 &= 8 - 0 - 15 \\
 &= -7
 \end{aligned}$$

The answer is A.

Question 8

Sketch the function.

$$\text{At } x = \frac{\pi}{12},$$

$$4 \cos(2x)$$

$$= 4 \cos\left(\frac{\pi}{6}\right)$$

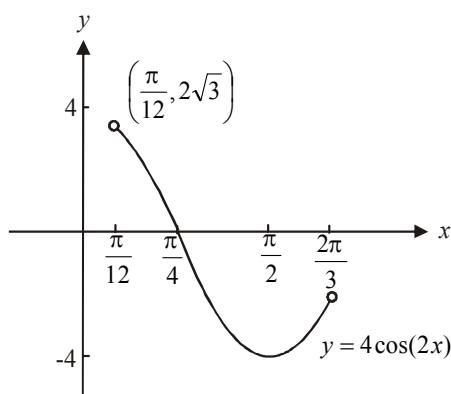
$$= 4 \times \frac{\sqrt{3}}{2}$$

$$= 2\sqrt{3}$$

From the diagram,

$$r_f = [-4, 2\sqrt{3})$$

The answer is A.



□

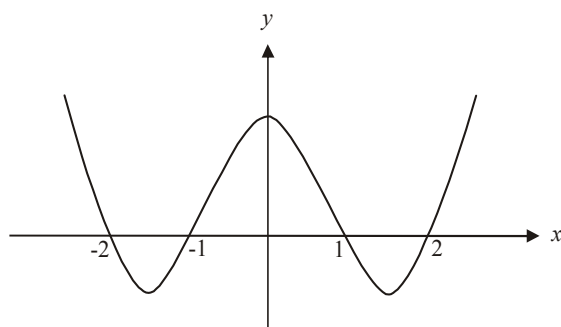
Question 9

$$\begin{aligned}
 \Pr(X < 5 | X < 10) &= \frac{\Pr(X < 5 \cap X < 10)}{\Pr(X < 10)} \\
 &= \frac{\Pr(X < 5)}{0.5} \\
 &= \frac{0.00621}{0.5} \\
 &= 0.01242
 \end{aligned}$$

The answer is B.

Question 10

$$\begin{aligned}
 y &= x^4 - 5x^2 + 4 \\
 &= (x^2 - 4)(x^2 - 1) \\
 &= (x - 2)(x + 2)(x - 1)(x + 1)
 \end{aligned}$$



The graph is symmetrical about the y -axis.

Use your calculator to find the minimum turning points.

They occur at $(-1.58114, -2.25)$ and, by symmetry at $(1.58114, -2.25)$.

The gradient is positive for $x \in (-1.5811\dots, 0) \cup (1.5811\dots, \infty)$

The closest answer is D.

Question 11

$$\begin{aligned}
 \text{Average rate of change} &= \frac{f(2) - f(1)}{2 - 1} \\
 &= \log_e(5) - \log_e(3) \\
 &= \log_e\left(\frac{5}{3}\right)
 \end{aligned}$$

The answer is B.

Question 12

Option A is incorrect because h is discontinuous at $x = 1$ and $x = 3$.

Option B is incorrect because h does not exist at $x = 3$.

Option C is incorrect because $h(x) \leq 0$ for $x \in [-3, 0]$.

Option D is correct because $h(x)$ exists at $x = 1$.

Option E is incorrect because $h'(1)$ does not exist. This is because the limits for $h'(x)$ from the left and right hand side of $x = 1$ are not equal.

The answer is D.

Question 13

$$e^{2x} - 2e^x = 0$$

$$e^x(e^x - 2) = 0$$

$$e^x = 0 \quad \text{or} \quad e^x - 2 = 0$$

For $e^x = 0$, there is no real solution.

For $e^x = 2$

$$x = \log_e(2)$$

The answer is C.

Question 14

$$\frac{dV}{dt} = 5$$

$$V = \frac{4}{3}\pi r^3 \quad (\text{from formula sheet})$$

$$\frac{dV}{dr} = 4\pi r^2$$

$$\frac{dr}{dt} = \frac{dr}{dV} \cdot \frac{dV}{dt}$$

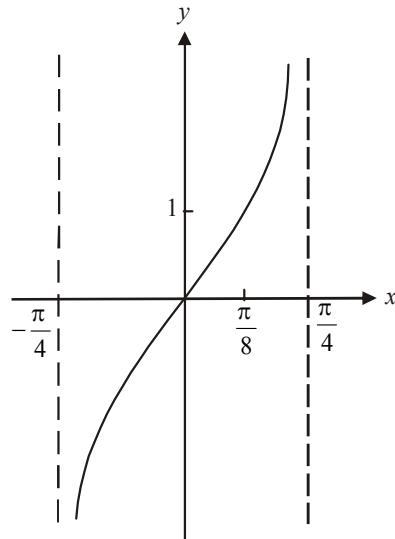
$$= \frac{1}{4\pi r^2} \cdot 5$$

$$= \frac{5}{4\pi r^2}$$

The answer is B.

Question 15

Do a quick sketch.



A dilation by a factor of 3 from the x -axis changes the rule $y = \tan(2x)$ to become

$$\frac{y}{3} = \tan(2x)$$

$$y = 3 \tan(2x)$$

□

A reflection in the x -axis changes the rule to

$$-y = 3 \tan(2x)$$

$$y = -3 \tan(2x)$$

Note that the domain is not affected by these two transformations.

The answer is C.

Question 16

Since we have a probability density function,

$$\int_{-\infty}^{\infty} f(x) dx = 1$$

$$\int_0^1 \frac{x^2}{k} dx = 1$$

$$\frac{1}{k} \left[\frac{x^3}{3} \right]_0^1 = 1$$

$$\left[\frac{x^3}{3} \right]_0^1 = k$$

$$\frac{1}{3} - 0 = k$$

$$k = \frac{1}{3}$$

The answer is A.

Question 17

$$f(x) = \frac{h(x)}{\log_e(2x)}, x > 0$$

$$f'(x) = \frac{\log_e(2x)h'(x) - \frac{1}{x}h(x)}{(\log_e(2x))^2} \quad (\text{Quotient rule})$$

The answer is E.

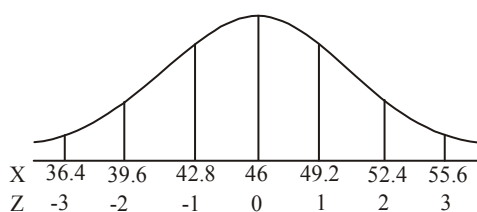
Question 18

Let w = minimum weight required.

$$\Pr(X < w) = 0.1$$

$$w = 41.899$$

The answer is A.



□

Question 19

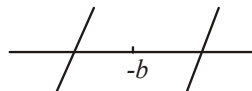
$$\begin{aligned}
& \int_0^{\frac{\pi}{8}} (\cos(2x) - \sec^2(2x)) dx \\
&= \left[\frac{1}{2} \sin(2x) - \frac{1}{2} \tan(2x) \right]_0^{\frac{\pi}{8}} \\
&= \frac{1}{2} \left\{ \left(\sin\left(\frac{\pi}{4}\right) - \tan\left(\frac{\pi}{4}\right) \right) - ((\sin(0) - \tan(0))) \right\} \\
&= \frac{1}{2} \left\{ \left(\frac{1}{\sqrt{2}} - 1 \right) - 0 \right\} \\
&= \frac{1}{2} \left(\frac{1}{\sqrt{2}} - 1 \right)
\end{aligned}$$

The answer is D.

Question 20

For a stationary point of inflection to occur $f'(x) = 0$. This only occurs at $x = -b$ and at $x = d$.

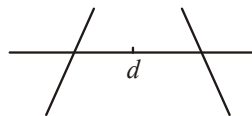
To the left of $x = -b$, $f'(x) > 0$ and to the right of $x = -b$, $f'(x) < 0$ so we have a stationary point of inflection at $x = -b$.



Note that to the left of $x = d$, $f'(x) > 0$ and to the right of $x = d$, $f'(x) < 0$

So we have a local maximum at $x = d$.

The answer is B.



□

Question 21

$f(x) = \frac{1}{\sqrt{x-1}}$, has a maximal domain.

That maximal domain is given by

$$x-1 > 0$$

$$x > 1$$

So $d_f = (1, \infty)$.

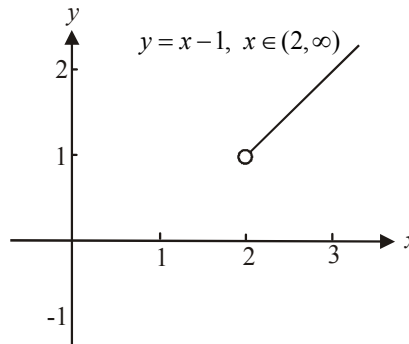
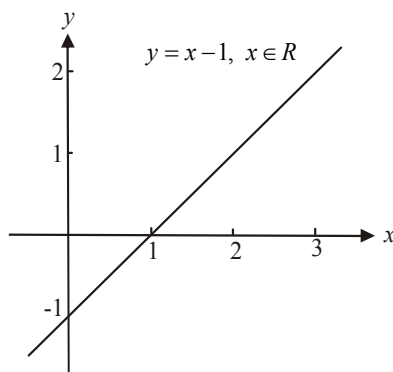
$f(g(x))$ exists iff $r_g \subseteq d_f$

So we require $r_g \subseteq (1, \infty)$

Method 1

The graph of $y = x - 1$ for $x \in \mathbb{R}$ is shown in the diagram on the left below.

In order to restrict the range to $(1, \infty)$, we are going to have to restrict the domain to $x \in (2, \infty)$ as shown on the graph on the right below.



Since $r_g \subseteq (1, \infty)$, $d_g \subseteq (2, \infty)$.

So $a \neq -2, -1, 0$ or 1

So $a = 2$ is the only possible answer.

The answer is E.

Method 2

We require $r_g \subseteq (1, \infty)$

$$r_g = (a - 1, \infty)$$

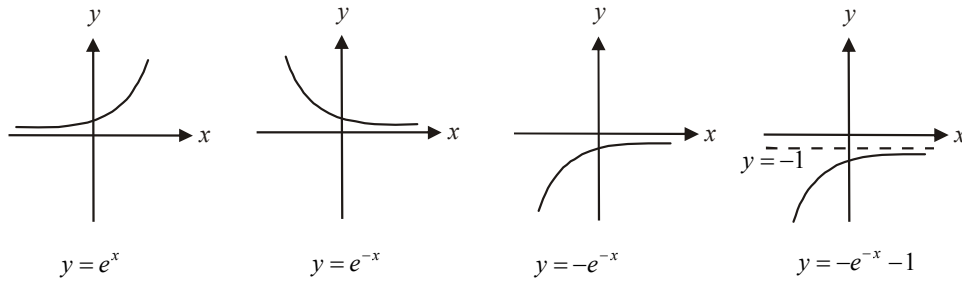
$$a - 1 \geq 1$$

$$a \geq 2$$

So $a = 2$ is the only possible answer.

The answer is E.

Question 22



The only possible graph is E.
The answer is E.

SECTION 2

Question 1

- a. The graph of $y = f(x)$ passes through $(0,0)$

$$\text{so, } 0 = e^{ax \cdot 0} - b$$

$$b = e^0$$

$$b = 1 \text{ as required}$$

(1 mark)

- b. $f(x) = e^{ax} - 1$

$$f'(x) = ae^{ax}$$

(1 mark)

$$f'(1) = ae^a$$

$$\text{Since } f'(1) = 2e^2 \quad (\text{given})$$

$$\text{then } a = 2$$

(1 mark)

- c. i. Note that the equation of the asymptote of $y = e^{2x} - 1$ is $y = -1$.

Method 1

The graph of $y = f(x)$ has undergone a reflection in the x -axis followed by a translation of 1 unit down. (1 mark) – reflection (1 mark) – translation

Method 2

The graph of $y = f(x)$ has undergone a translation of 1 unit up followed by a reflection in the x -axis. (1 mark) – reflection (1 mark) – translation

- ii. Method 1

After the reflection in the x -axis, the rule $y = e^{2x} - 1$ becomes $-y = e^{2x} - 1$ so

$$y = 1 - e^{2x}.$$

(1 mark)

After the translation of 1 unit down, $y = 1 - e^{2x}$ becomes

$$y = 1 - e^{2x} - 1 = -e^{2x}. \text{ So } g(x) = -e^{2x} \text{ as required.}$$

(1 mark)

Method 2

After the translation of 1 unit up, the rule $y = e^{2x} - 1$ becomes $y = e^{2x} - 1 + 1$

$$\text{so } y = e^{2x}.$$

(1 mark)

After the reflection in the x -axis, $y = e^{2x}$ becomes $-y = e^{2x}$ so $y = -e^{2x}$.

$$\text{So } g(x) = -e^{2x} \text{ as required.}$$

(1 mark)

- d. The graphs of $y = f(x)$ and $y = g(x)$ intersect when

$$f(x) = g(x)$$

$$e^{2x} - 1 = -e^{2x}$$

(1 mark)

$$2e^{2x} = 1$$

$$e^{2x} = \frac{1}{2}$$

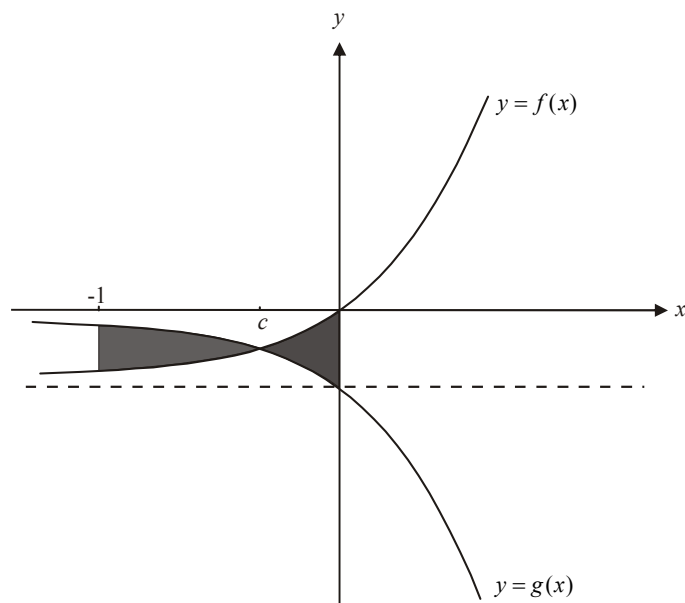
$$\log_e \left(\frac{1}{2} \right) = 2x$$

$$x = \frac{1}{2} \log_e (2^{-1})$$

$$\text{So } c = -\frac{1}{2} \log_e (2)$$

(1 mark)

e.



$$\text{Area required} = \int_{-1}^c (g(x) - f(x)) dx + \int_c^0 (f(x) - g(x)) dx$$

$$\text{OR} \quad \int_{-1}^{-\frac{1}{2} \log_e(2)} (g(x) - f(x)) dx + \int_{-\frac{1}{2} \log_e(2)}^0 (f(x) - g(x)) dx$$

(1 mark) for first integrand and terminals

(1 mark) – for second integrand and terminals

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f.

$$f(x) + e^x - 1 = 0$$

$$e^{2x} - 1 + e^x - 1 = 0$$

$$e^{2x} + e^x - 2 = 0$$

$$\text{Let } a = e^x$$

$$a^2 + a - 2 = 0$$

$$(a + 2)(a - 1) = 0$$

Sub back

$$(e^x + 2)(e^x - 1) = 0$$

(1 mark)

$$\text{For } e^x + 2 = 0$$

$$e^x = -2$$

no real solutions exist.

$$\text{For } e^x - 1 = 0$$

$$e^x = 1$$

$$x = 0$$

(1 mark)

Total 13 marks

Question 2

- a. i. $\Pr(\text{rides on next 5 days})$
 $= 0.6^5$
 $= 0.0778$ (correct to 4 decimal places) (1 mark)

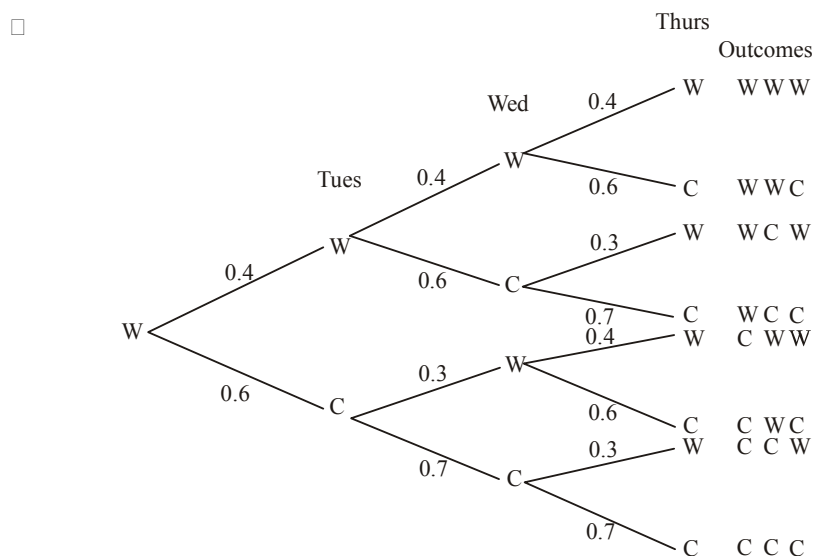
- ii. This is a binomial distribution with $n = 5, x = 2$ and $p = 0.6$.
 $\Pr(\text{rides 2 out of next 5 days})$
 $= \Pr(X = 2)$
 $= {}^5C_2 (0.6)^2 (0.4)^3$ (1 mark)
 $= 0.2304$ (1 mark)

- b. i. $\Pr(wwww) = 0.4^4$
 $= 0.0256$ (1 mark)

- ii. $\Pr(wcc) + \Pr(cwc) + \Pr(ccw)$ (1 mark)
 $= 0.4 \times 0.6 \times 0.7 + 0.6 \times 0.3 \times 0.6 + 0.6 \times 0.7 \times 0.3$ (1 mark)
 $= 0.402$ (1 mark)

- iii. Method 1
 $\Pr(\text{walked on at least 1 of next 3})$
 $= 1 - \Pr(\text{walked zero times out of next 3})$
 $= 1 - \Pr(ccc)$ (1 mark)
 $= 1 - (0.6 \times 0.7 \times 0.7)$
 $= 0.706$ (1 mark)

Method 2 - use a tree diagram



Pr(walked on at least 1 of next 3)

$$= 1 - \Pr(ccc)$$

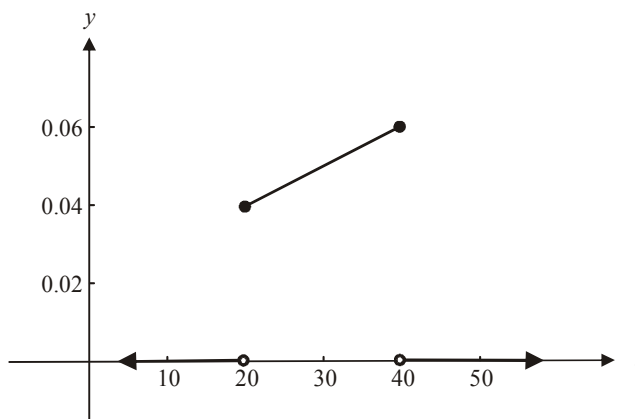
$$= 1 - (0.6 \times 0.7 \times 0.7)$$

$$= 0.706$$

(1 mark)

(1 mark)

c. ☐ i.



(1 mark) correct linear function and included endpoints for $20 \leq t \leq 40$

(1 mark) correct marking of function along t -axis

ii. From the graph, we see that the mode is 40; that is, the value of t with the highest probability; that is, the highest value of $f(t)$.

(1 mark)

iii. Let m = median

$$\int_{20}^m f(t) dt = 0.5$$

(1 mark)

☐

$$\int_{20}^m \frac{1}{1000} (t + 20) dt = 0.5$$

$$\frac{1}{1000} \left[\frac{t^2}{2} + 20t \right]_{20}^m = 0.5$$

(1 mark)

$$\left(\frac{m^2}{2} + 20m \right) - (200 + 400) = 500$$

$$\frac{m^2}{2} + 20m - 1100 = 0$$

(1 mark)

$$\left. \begin{array}{l} m = -70.9902... \\ \text{or } m = 30.9902... \end{array} \right\}$$

Since $20 \leq m \leq 40$, $m = 31$ minutes (to the nearest minute).

(1 mark) correct answer

Total 16 marks

☐

Question 3

a. $f(x) = \cos(2x)$
 $f'(x) = -2\sin(2x)$

(1 mark)

- b. The gradient of the tangent is -1 when
 $f'(x) = -1$

$$-2\sin(2x) = -1$$

$$\sin(2x) = \frac{1}{2}$$

$$x \in \left[0, \frac{\pi}{2}\right]$$

S	A
T	C

$$2x = \frac{\pi}{6}, \frac{5\pi}{6}$$

$$2x \in [0, \pi]$$

$$x = \frac{\pi}{12}, \frac{5\pi}{12}$$

$$f\left(\frac{\pi}{12}\right) = \cos\left(\frac{\pi}{6}\right) \quad f\left(\frac{5\pi}{12}\right) = \cos\left(\frac{5\pi}{6}\right)$$

(1 mark)

$$= \frac{\sqrt{3}}{2}$$

$$= -\frac{\sqrt{3}}{2}$$

$$\text{Required points are } \left(\frac{\pi}{12}, \frac{\sqrt{3}}{2}\right) \text{ and } \left(\frac{5\pi}{12}, -\frac{\sqrt{3}}{2}\right).$$

□

(1 mark)

- c. i. Tangent passes through $\left(\frac{\pi + \sqrt{3}}{6}, 0\right)$ and $\left(0, \frac{\sqrt{3}\pi + 3}{6}\right)$.

Method 1

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \left(\frac{\sqrt{3}\pi + 3}{6} - 0\right) \div \left(0 - \frac{\pi + \sqrt{3}}{6}\right)$$

$$= -\left(\frac{\sqrt{3}(\pi + \sqrt{3})}{\pi + \sqrt{3}}\right)$$

$$= -\sqrt{3} \text{ as required}$$

(1 mark)Method 2

Note that the gradient must be negative since the tangent is sloping up to the left.

Gradient of tangent is given by

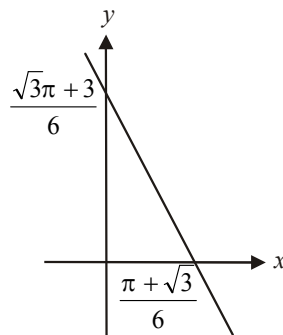
$$\frac{\text{rise}}{\text{run}}$$

$$= -\left(\frac{\sqrt{3}\pi + 3}{6} \div \frac{\pi + \sqrt{3}}{6}\right)$$

$$= -\left(\frac{\sqrt{3}\pi + 3}{6} \times \frac{6}{\pi + \sqrt{3}}\right)$$

$$= -\left(\frac{\sqrt{3}(\pi + \sqrt{3})}{\pi + \sqrt{3}}\right)$$

$$= -\sqrt{3} \text{ as required}$$

(1 mark)

ii. $f'(x) = -2\sin(2x) = -\sqrt{3} \quad x \in \left[0, \frac{\pi}{2}\right] \quad (1 \text{ mark})$

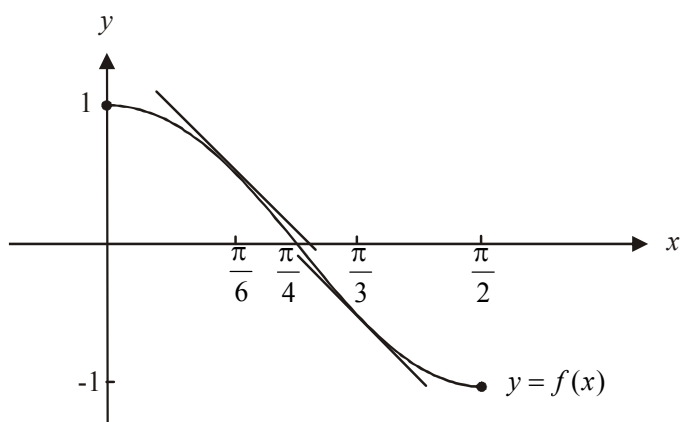
$$\sin(2x) = \frac{\sqrt{3}}{2} \quad 2x \in [0, \pi]$$

$$2x = \frac{\pi}{3}, \frac{2\pi}{3}$$

$$x = \frac{\pi}{6}, \frac{\pi}{3}$$

(1 mark)

iii.



The x -intercept of the tangent we require is $\frac{\pi + \sqrt{3}}{6} = 0.8122\dots$

Now $\frac{\pi}{4} = 0.7853\dots$

From the diagram above, we can see that the tangent we require must pass

through the point where $x = \frac{\pi}{6}$. The other possible tangent which passes

through the point where $x = \frac{\pi}{3}$, has an x -intercept which is less than $\frac{\pi}{4}$ and

therefore less than $\frac{\pi + \sqrt{3}}{6}$.

(1 mark)

$$f(x) = \cos(2x)$$

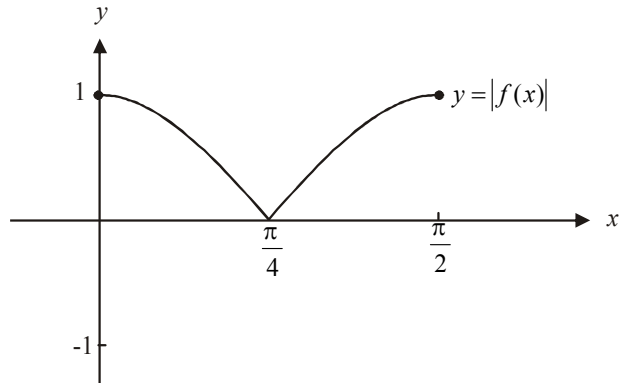
$$f\left(\frac{\pi}{6}\right) = \cos\left(\frac{\pi}{3}\right)$$

$$= \frac{1}{2}$$

Point of tangency is $\left(\frac{\pi}{6}, \frac{1}{2}\right)$.

(1 mark)

- d. Do a quick sketch of the graph of $y = |f(x)|$.



The minimum value of $|f(x)| = 0$.

(1 mark)

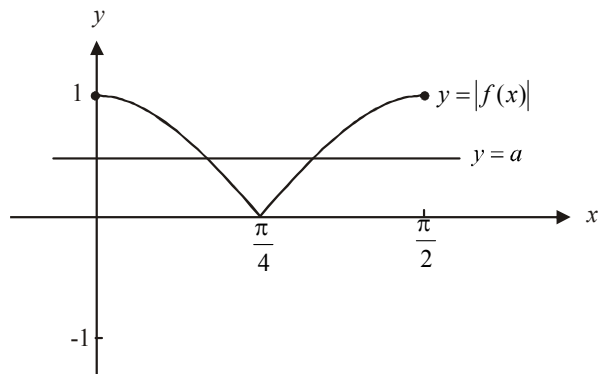
The maximum value of $|f(x)| = 1$.

(1 mark)

- e. $|f(x)| - a = 0$ ☐
 $|f(x)| = a$

The graphs of $y = |f(x)|$ and $y = a$ are shown below.

☐ ☐



- i. For exactly one solution we see that $a = 0$.

(1 mark)

- ii. For at least one solution $a \in [0, 1]$.

(1 mark)

f. Method 1

$$|f(x)| = \frac{1}{2}$$

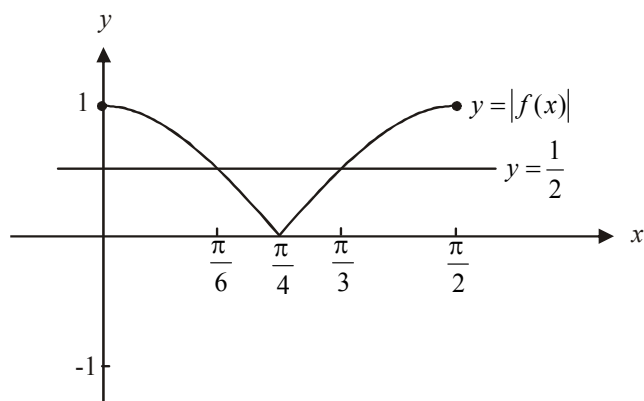
$$\cos(2x) = \frac{1}{2}$$

□

$$2x = \frac{\pi}{3}$$

$$x = \frac{\pi}{6} \quad (1 \text{ mark})$$

□



From the graphs of $y = |f(x)|$ and $y = \frac{1}{2}$ we see that there is a point of intersection at $x = \frac{\pi}{6}$. The graph of $y = |f(x)|$ is symmetrical about the line $x = \frac{\pi}{4}$.

Now $\frac{\pi}{4} - \frac{\pi}{6} = \frac{3\pi}{12} - \frac{2\pi}{12} = \frac{\pi}{12}$ so there is another point of intersection at the point

□

where $x = \frac{\pi}{4} + \frac{\pi}{12} = \frac{3\pi}{12} + \frac{\pi}{12} = \frac{4\pi}{12} = \frac{\pi}{3}$.

□

So $f(x) = \frac{1}{2}$ for $x = \frac{\pi}{6}, \frac{\pi}{3}$.

(1 mark)

Method 2

$$|f(x)| = \frac{1}{2}$$

$$f(x) = \pm \frac{1}{2}$$

$$\cos(2x) = \pm \frac{1}{2}$$

$$x \in \left[0, \frac{\pi}{2}\right]$$

(1 mark)

$$2x = \frac{\pi}{3}, \frac{2\pi}{3}$$

$$2x \in [0, \pi]$$

$$x = \frac{\pi}{6}, \frac{\pi}{3}$$

(1 mark)

Total 14 marks

Question 4

- a. Solve $f(x) = 0$
 $5 \log_e(x-10) = 0$
 $e^0 = x-10$
 $1 = x-10$
 $x = 11$
 So $a = 11$ as required.

(1 mark)

- b. $f(x) = 5 \log_e(x-10)$
 Let $y = 5 \log_e(x-10)$
 Swap x and y for inverse.
 $x = 5 \log_e(y-10)$

$$\frac{x}{5} = \log_e(y-10)$$

$$e^{\frac{x}{5}} = y-10$$

$$y = e^{\frac{x}{5}} + 10$$

$$f^{-1}(x) = e^{\frac{x}{5}} + 10$$

(1 mark) – correct rule

$$d_f = [11, 50]$$

$$r_f = [0, f(50)]$$

$$= [0, 18.44\dots]$$

So $d_{f^{-1}} = r_f$
 $= [0, 18.4]$ (correct to 1 decimal place)

(1 mark) – correct domain

- c. Since the graph of $y = f^{-1}(x)$ is a reflection of the graph of $y = f(x)$ in the line $y = x$, the floodlit areas north and east are the same.

$$\text{Total area} = 2 \int_{11}^{50} f(x) dx$$

(1 mark)

$$= 10 \int_{11}^{50} \log_e(x-10) dx$$

$$= 1086 \text{m}^2 \text{ (to nearest square metre)}$$

(1 mark)

- d. Find the x-coordinates of the points of intersection between $y = 15$ and $y = f(x)$ and between $y = 15$ and $y = f^{-1}(x)$.

Method 1 – by calculator

$y = 15$ and $y = 5 \log_e(x - 10)$ intersect when $x = 30.09$ (to 2 decimal places)

$y = 15$ and $y = e^{\frac{x}{5}} + 10$ intersect when $x = 8.05$ (to 2 decimal places)

So $b \in (8.05, 30.09)$ or $8.05 < b < 30.09$.

(1 mark) correct values

(1 mark) correct brackets or inequality signs

Method 2 – by hand

$$15 = 5 \log_e(x - 10) \qquad 15 = e^{\frac{x}{5}} + 10$$

$$3 = \log_e(x - 10) \qquad 5 = e^{\frac{x}{5}}$$

$$e^3 = x - 10 \qquad \log_e(5) = \frac{x}{5}$$

$$x = 10 + e^3 \qquad x = 5 \log_e(5)$$

$$= 30.09 \text{ (to 2 dec. places)} \qquad = 8.05 \text{ (to 2 dec. places)}$$

So $b \in (8.05, 30.09)$ or $8.05 < b < 30.09$.

(1 mark) correct values

(1 mark) correct bracket or inequality signs

e. i. $T = \sqrt{x^2 + 2500} + \frac{50 - x}{2}$

$$\frac{dT}{dx} = \frac{1}{2}(x^2 + 2500)^{-\frac{1}{2}} \times 2x - \frac{1}{2}$$

$$= \frac{x}{\sqrt{x^2 + 2500}} - \frac{1}{2}$$

$$= \frac{2x - \sqrt{x^2 + 2500}}{2\sqrt{x^2 + 2500}}$$

(1 mark)

$$\frac{dT}{dx} = 0 \text{ for minimum.}$$

$$\text{So } 2x - \sqrt{x^2 + 2500} = 0$$

(1 mark)

$$2x = \sqrt{x^2 + 2500}$$

$$4x^2 = x^2 + 2500$$

$$3x^2 = 2500$$

$$x = \pm 28.8675$$

$$\text{but } x \geq 0$$

$$\text{So } x = 28.87 \text{ (correct to 2 decimal places)}$$

(1 mark)

ii. $T = 68.3$ seconds correct to 1 decimal place.

(1 mark)

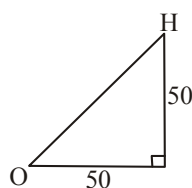
- f. To take the quickest path, Victoria runs to $P(28 \cdot 87, 50)$. The straight line from $O(0,0)$ to $P(28 \cdot 87, 50)$ is given by $y = \frac{50}{28 \cdot 87}x$. **(1 mark)**
 Check whether this intersects with the perimeter of the floodlit area to the north given by $f^{-1}(x) = e^{\frac{x}{5}} + 10$.

Graph the two functions.

They intersect at the point $(10 \cdot 3475..., 17 \cdot 9209...)$ and at the point $(11 \cdot 226152, 19 \cdot 44259...)$ so Victoria does enter the floodlit area.

(1 mark)

- g. The dogs have to run $\sqrt{50^2 + 50^2} = 50\sqrt{2}$ m.
 They run at 7m/sec.
 It takes the dogs $50\sqrt{2}\text{m} \div \frac{7\text{m}}{\text{sec}}$
 $= \frac{50\sqrt{2}}{7}\text{secs}$



$= 10 \cdot 1015... \text{secs}$

to get to H from O .

(1 mark)

The dogs leave 60 secs after Victoria so they arrive $70 \cdot 1015... \text{secs}$ after she leaves $O(0,0)$.

From part e. ii., the shortest time it takes Victoria to get from O to H is 68.3 seconds (to 1 decimal place).

So Victoria escapes the dogs; but only just!

(1 mark)

Total 15 marks