

**THE  
HEFFERNAN  
GROUP**

P.O. Box 1180  
Surrey Hills North VIC 3127  
Phone 03 9836 5021  
Fax 03 9836 5025  
info@theheffernangroup.com.au  
www.theheffernangroup.com.au

Student Name.....

**MATHEMATICAL METHODS UNITS 3 & 4**

**TRIAL EXAMINATION 1**

**2009**

Reading Time: 15 minutes

Writing time: 1 hour

**Instructions to students**

This exam consists of 11 questions.  
All questions should be answered in the spaces provided.  
There is a total of 40 marks available.  
The marks allocated to each of the questions are indicated throughout.  
Students may **not** bring any calculators or notes into the exam.  
Where an exact answer is required a decimal approximation will not be accepted.  
Where more than one mark is allocated to a question, appropriate working must be shown.  
Diagrams in this trial exam are not drawn to scale.  
A formula sheet can be found on page 12 of this exam.

*This paper has been prepared independently of the Victorian Curriculum and Assessment Authority to provide additional exam preparation for students. Although references have been reproduced with permission of the Victorian Curriculum and Assessment Authority, the publication is in no way connected with or endorsed by the Victorian Curriculum and Assessment Authority.*

**© THE HEFFERNAN GROUP 2009**

This Trial Exam is licensed on a non transferable basis to the purchasing school. It may be copied by the school which has purchased it. This license does not permit distribution or copying of this Trial Exam by any other party.

**Question 1**

Let  $f: R \rightarrow R, f(x) = x + 1$  and  $g: (0, \infty) \rightarrow R, g(x) = \log_e(2x)$ .

- a. Write down the rule of  $f(g(x))$ .

---



---

- b. Explain why the function  $g(f(x))$  does not exist.

---



---

1 + 1 = 2 marks

**Question 2**

- a. Let  $f(x) = x \log_e(x^2 + 5)$ . Find  $f'(x)$ .

---



---



---



---

- b. Let  $y = \frac{\tan(x)}{e^{2x}}$ . Evaluate  $\frac{dy}{dx}$  when  $x = 0$ .

---



---



---



---



---



---



---



---



---



---

2 + 3 = 5 marks

**Question 3**

Solve the equation  $\sqrt{3} \tan(2x) = 1$  for  $x \in [0, 2\pi]$ .

---



---



---



---



---



---

2 marks

**Question 4**

From a particular event space, two events  $A$  and  $B$  are such that  $\Pr(A) = 0.3$  and  $\Pr(B) = 0.4$ .

**a.** If  $\Pr(A \cap B) = 0.15$ , calculate

**i.**  $\Pr(A' \cap B')$

---



---



---



---

**ii.**  $\Pr(A|B)$

---



---



---

**b.** If  $A$  and  $B$  are mutually exclusive events find  $\Pr(A|B)$ .

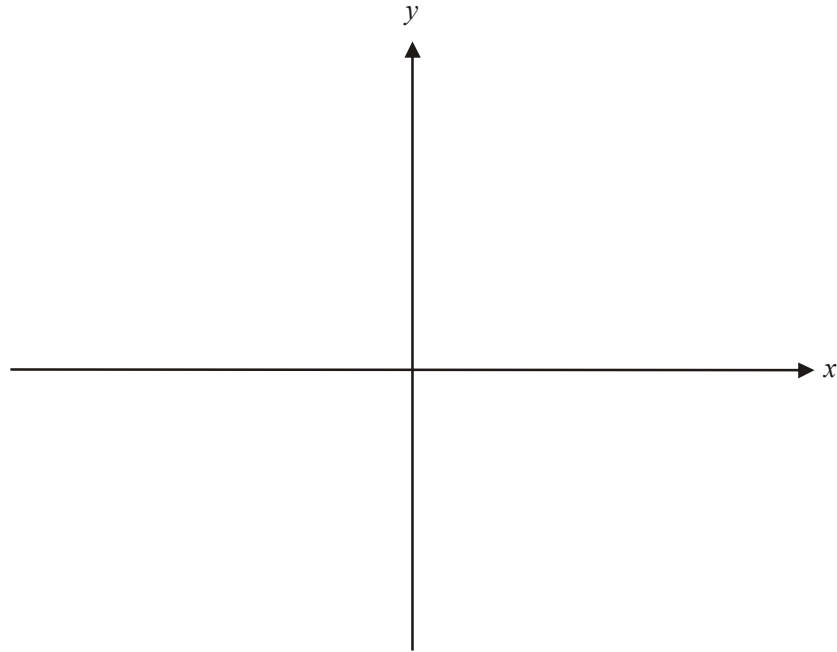
---

1 + 1 + 1 = 3 marks

**Question 5**

Let  $f: \mathbb{R} \rightarrow \mathbb{R}$ ,  $f(x) = |x^2 - 6x + 5|$ .

- a. Sketch the graph of  $y = f(x)$  on the set of axes below. Indicate clearly any axes intercepts or turning points.



- b. Write down the domain of the derivative function  $f'$ .

---



---

- c. Write down the values of  $x$  for which  $f'(x) > 0$ .

---



---

2 + 1 + 1 = 4 marks

**Question 6**

The continuous random variable  $X$  has a probability density function given by

$$f(x) = \begin{cases} \frac{1}{2\sqrt{x}} & \text{if } x \in [1, 4] \\ 0 & \text{otherwise} \end{cases}$$

- a.** Find  $\Pr(X < 2)$ .

---



---



---



---



---

- b.** Find the mean value of  $X$ .

---



---



---



---



---

2 + 2 = 4 marks

**Question 7**

Let  $X$  be a random variable with a normal distribution.

The mean of  $X$  is 20 and the standard deviation is 5.

Let  $Z$  be a continuous random variable with a standard normal distribution.

- a.** Find  $m$  such that  $\Pr(X > 20) = \Pr(Z < m)$ .

---

---

---

- b.** Find  $n$  such that  $\Pr(X < 18) = \Pr(Z > n)$ .

---

---

---

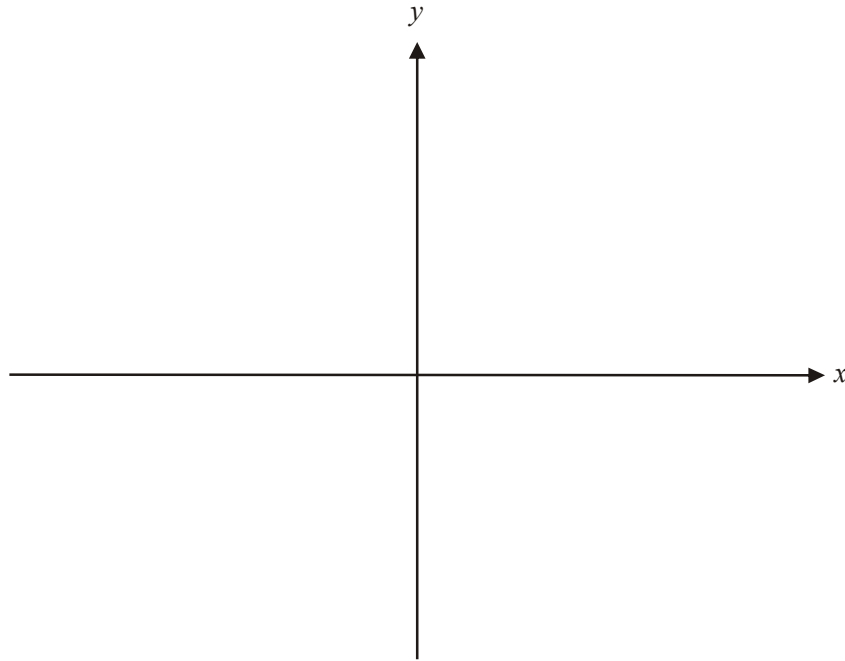
---

1 + 2 = 3 marks

**Question 8**

Let  $h : (2, \infty) \rightarrow \mathbb{R}$ ,  $h(x) = \frac{1}{x-2} + 1$ .

- a.** On the axes below, sketch the graph of  $y = h(x)$ . Label any asymptotes with their equation.



- b.** Find the rule and the domain of the inverse function  $h^{-1}$ .

---



---



---



---



---

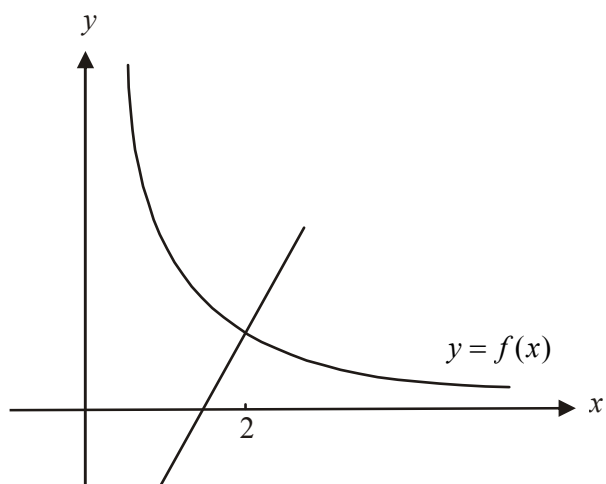


---

2 + 2 = 4 marks

**Question 9**

The graph of  $f: (0, \infty) \rightarrow \mathbb{R}, f(x) = \frac{2}{x}$  is shown below. The normal to the graph of  $f$  at the point where  $x = 2$  is also shown.



- a.** Find the equation of the normal to the graph of  $f$  at the point where  $x = 2$ .

---



---



---



---



---



---

- b.** Find the area of the region enclosed by the graph of the normal described in part **a.**, the  $x$ -axis and the line  $x = 2$ .

---



---



---



---



---



---

2 + 2 = 4 marks



**Question 10**

Given that  $f(x) = x\sqrt{1-x}$  and  $f'(x) = \frac{-x}{2\sqrt{1-x}} + \sqrt{1-x}$ , find an antiderivative of  $\frac{x}{\sqrt{1-x}}$ .

---

---

---

---

---

---

---

---

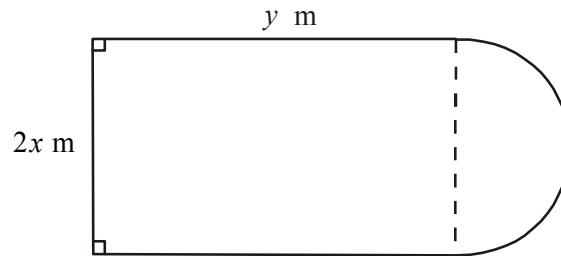
---

---

3 marks

**Question 11**

A pool complex is made up of a rectangular swimming pool with side lengths  $2x$  m and  $y$  m attached to a semi-circular spa of radius  $x$  m.



The perimeter of the pool complex is 100m.

- a.** Express  $y$  in terms of  $x$ .

---



---



---



---



---

- b.** Show that the surface area of the pool complex is given by

$$A = 100x - \frac{x^2}{2}(\pi + 4).$$

---



---



---



---



---

- c.** Find the value of  $x$  for which the surface area of the pool complex is a maximum. It is not necessary to find this maximum surface area.

---

---

---

---

---

---

- d.** Using the result from part **b.** or otherwise, explain why the value of  $x$  found in part **c.** gives a maximum rather than a minimum surface area.

---

---

---

2 + 1 + 2 + 1 = 6 marks

END OF EXAM 1

## Mathematical Methods and Mathematical Methods CAS Formulas

### Mensuration

|                                    |                        |                      |                        |
|------------------------------------|------------------------|----------------------|------------------------|
| area of a trapezium:               | $\frac{1}{2}(a+b)h$    | volume of a pyramid: | $\frac{1}{3}Ah$        |
| curved surface area of a cylinder: | $2\pi rh$              | volume of a sphere:  | $\frac{4}{3}\pi r^3$   |
| volume of a cylinder:              | $\pi r^2 h$            | area of a triangle:  | $\frac{1}{2}bc \sin A$ |
| volume of a cone:                  | $\frac{1}{3}\pi r^2 h$ |                      |                        |

### Calculus

|                                                                      |                                                                                                       |
|----------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------|
| $\frac{d}{dx}(x^n) = nx^{n-1}$                                       | $\int x^n dx = \frac{1}{n+1}x^{n+1} + c, n \neq -1$                                                   |
| $\frac{d}{dx}(e^{ax}) = ae^{ax}$                                     | $\int e^{ax} dx = \frac{1}{a}e^{ax} + c$                                                              |
| $\frac{d}{dx}(\log_e(x)) = \frac{1}{x}$                              | $\int \frac{1}{x} dx = \log_e x  + c$                                                                 |
| $\frac{d}{dx}(\sin(ax)) = a \cos(ax)$                                | $\int \sin(ax) dx = -\frac{1}{a} \cos(ax) + c$                                                        |
| $\frac{d}{dx}(\cos(ax)) = -a \sin(ax)$                               | $\int \cos(ax) dx = \frac{1}{a} \sin(ax) + c$                                                         |
| $\frac{d}{dx}(\tan(ax)) = \frac{a}{\cos^2(ax)} = a \sec^2(ax)$       |                                                                                                       |
| product rule: $\frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}$ | quotient rule: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$ |
| chain rule: $\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}$            | approximation: $f(x+h) \approx f(x) + hf'(x)$                                                         |

### Probability

|                                           |                                                                        |
|-------------------------------------------|------------------------------------------------------------------------|
| $\Pr(A) = 1 - \Pr(A')$                    | $\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$                      |
| $\Pr(A/B) = \frac{\Pr(A \cap B)}{\Pr(B)}$ |                                                                        |
| mean: $\mu = E(X)$                        | variance: $\text{var}(X) = \sigma^2 = E((X - \mu)^2) = E(X^2) - \mu^2$ |

| probability distribution |                                     | mean                                      | variance                                                 |
|--------------------------|-------------------------------------|-------------------------------------------|----------------------------------------------------------|
| discrete                 | $\Pr(X = x) = p(x)$                 | $\mu = \sum x p(x)$                       | $\sigma^2 = \sum (x - \mu)^2 p(x)$                       |
| continuous               | $\Pr(a < X < b) = \int_a^b f(x) dx$ | $\mu = \int_{-\infty}^{\infty} x f(x) dx$ | $\sigma^2 = \int_{-\infty}^{\infty} (x - \mu)^2 f(x) dx$ |

*Reproduced with permission of the Victorian Curriculum and Assessment Authority, Victoria, Australia.*

*This formula sheet has been copied in 2009 from the VCAA website [www.vcaa.vic.edu.au](http://www.vcaa.vic.edu.au)  
The VCAA publish an exam issue supplement to the VCAA bulletin.*