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Student Name.....

MATHEMATICAL METHODS (CAS) UNITS 3 & 4
TRIAL EXAMINATION 1

2009

Reading Time: 15 minutes
Writing time: 1 hour

Instructions to students

This exam consists of 11 questions.
All questions should be answered in the spaces provided.
There is a total of 40 marks available.
The marks allocated to each of the questions are indicated throughout.
Students may **not** bring any calculators or notes into the exam.
Where an exact answer is required a decimal approximation will not be accepted.
Where more than one mark is allocated to a question, appropriate working must be shown.
Diagrams in this trial exam are not drawn to scale.
A formula sheet can be found on page 12 of this exam.

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Question 1

Let $f: R \rightarrow R, f(x) = x + 1$ and $g: (0, \infty) \rightarrow R, g(x) = \log_e(2x)$.

- a. Write down the rule of $f(g(x))$.

- b. Explain why the function $g(f(x))$ does not exist.

1 + 1 = 2 marks

Question 2

- a. Let $f(x) = x \log_e(x^2 + 5)$. Find $f'(x)$.

- b. Let $y = \frac{\tan(x)}{e^{2x}}$. Evaluate $\frac{dy}{dx}$ when $x = 0$.

2 + 3 = 5 marks

Question 3

Solve the equation $\sqrt{3} \tan(2x) = 1$ for $x \in [0, 2\pi]$.

2 marks

Question 4

From a particular event space, two events A and B are such that $\Pr(A) = 0.3$ and $\Pr(B) = 0.4$.

a. If $\Pr(A \cap B) = 0.15$, calculate

i. $\Pr(A' \cap B')$

ii. $\Pr(A|B)$

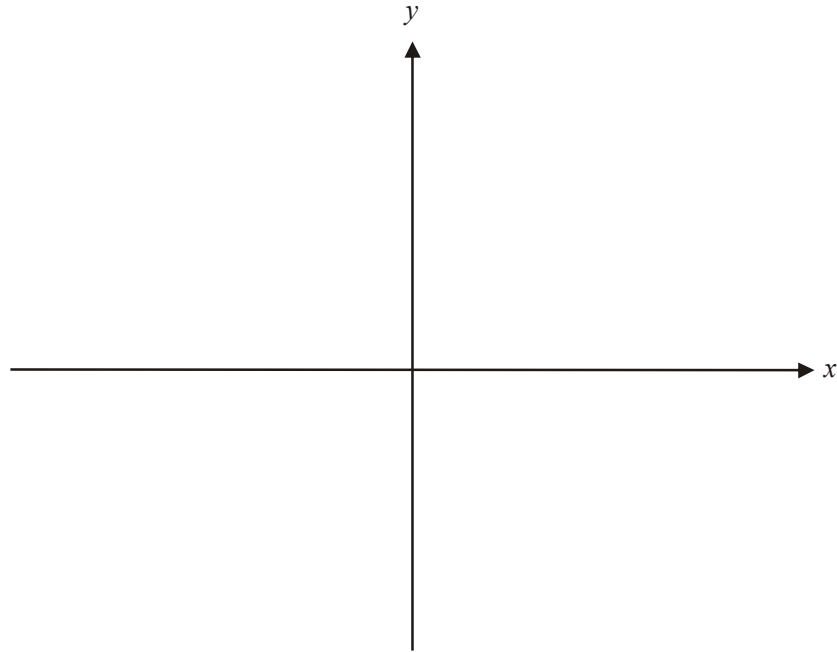
b. If A and B are mutually exclusive events find $\Pr(A|B)$.

1 + 1 + 1 = 3 marks

Question 5

Let $f: \mathbb{R} \rightarrow \mathbb{R}$, $f(x) = |x^2 - 6x + 5|$.

- a. Sketch the graph of $y = f(x)$ on the set of axes below. Indicate clearly any axes intercepts or turning points.



- b. Write down the domain of the derivative function f' .

- c. Write down the values of x for which $f'(x) > 0$.

2 + 1 + 1 = 4 marks

Question 6

The continuous random variable X has a probability density function given by

$$f(x) = \begin{cases} \frac{1}{2\sqrt{x}} & \text{if } x \in [1, 4] \\ 0 & \text{otherwise} \end{cases}$$

- a.** Find $\Pr(X < 2)$.

- b.** Find the mean value of X .

2 + 2 = 4 marks

Question 7

Let X be a random variable with a normal distribution.

The mean of X is 20 and the standard deviation is 5.

Let Z be a continuous random variable with a standard normal distribution.

- a.** Find m such that $\Pr(X > 20) = \Pr(Z < m)$.

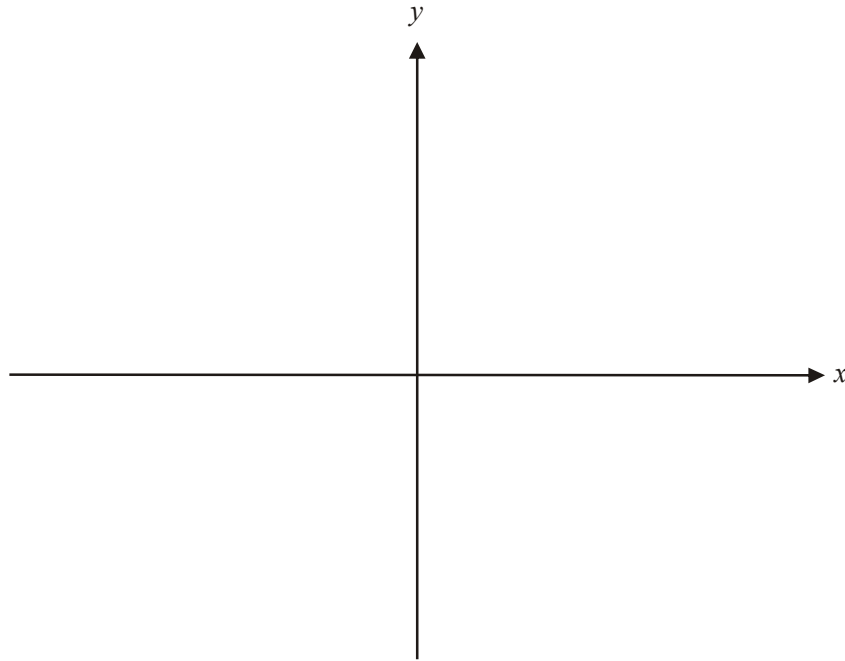
- b.** Find n such that $\Pr(X < 18) = \Pr(Z > n)$.

1 + 2 = 3 marks

Question 8

Let $h : (2, \infty) \rightarrow \mathbb{R}$, $h(x) = \frac{1}{x-2} + 1$.

- a.** On the axes below, sketch the graph of $y = h(x)$. Label any asymptotes with their equation.

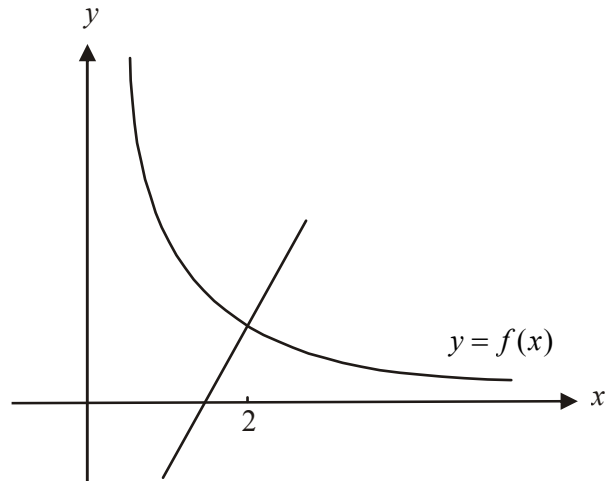


- b.** Find the rule and the domain of the inverse function h^{-1} .

2 + 2 = 4 marks

Question 9

The graph of $f: (0, \infty) \rightarrow \mathbb{R}, f(x) = \frac{2}{x}$ is shown below. The normal to the graph of f at the point where $x = 2$ is also shown.



- a.** Find the equation of the normal to the graph of f at the point where $x = 2$.

- b.** Find the area of the region enclosed by the graph of the normal described in part **a.**, the x -axis and the line $x = 2$.

2 + 2 = 4 marks

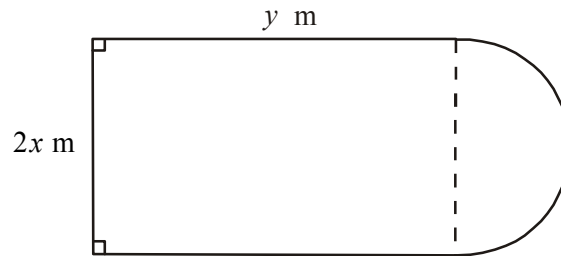
Question 10

Given that $f(x) = x\sqrt{1-x}$ and $f'(x) = \frac{-x}{2\sqrt{1-x}} + \sqrt{1-x}$, find an antiderivative of $\frac{x}{\sqrt{1-x}}$.

3 marks

Question 11

A pool complex is made up of a rectangular swimming pool with side lengths $2x$ m and y m attached to a semi-circular spa of radius x m.



The perimeter of the pool complex is 100m.

- a.** Express y in terms of x .

- b.** Show that the surface area of the pool complex is given by

$$A = 100x - \frac{x^2}{2}(\pi + 4).$$

- c.** Find the value of x for which the surface area of the pool complex is a maximum. It is not necessary to find this maximum surface area.

- d.** Using the result from part **b.** or otherwise, explain why the value of x found in part **c.** gives a maximum rather than a minimum surface area.

2 + 1 + 2 + 1 = 6 marks

END OF EXAM 1

Mathematical Methods and Mathematical Methods CAS Formulas

Mensuration

area of a trapezium:	$\frac{1}{2}(a+b)h$	volume of a pyramid:	$\frac{1}{3}Ah$
curved surface area of a cylinder:	$2\pi rh$	volume of a sphere:	$\frac{4}{3}\pi r^3$
volume of a cylinder:	$\pi r^2 h$	area of a triangle:	$\frac{1}{2}bc \sin A$
volume of a cone:	$\frac{1}{3}\pi r^2 h$		

Calculus

$\frac{d}{dx}(x^n) = nx^{n-1}$	$\int x^n dx = \frac{1}{n+1}x^{n+1} + c, n \neq -1$
$\frac{d}{dx}(e^{ax}) = ae^{ax}$	$\int e^{ax} dx = \frac{1}{a}e^{ax} + c$
$\frac{d}{dx}(\log_e(x)) = \frac{1}{x}$	$\int \frac{1}{x} dx = \log_e x + c$
$\frac{d}{dx}(\sin(ax)) = a \cos(ax)$	$\int \sin(ax) dx = -\frac{1}{a} \cos(ax) + c$
$\frac{d}{dx}(\cos(ax)) = -a \sin(ax)$	$\int \cos(ax) dx = \frac{1}{a} \sin(ax) + c$
$\frac{d}{dx}(\tan(ax)) = \frac{a}{\cos^2(ax)} = a \sec^2(ax)$	
product rule: $\frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}$	quotient rule: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$
chain rule: $\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}$	approximation: $f(x+h) \approx f(x) + hf'(x)$

Probability

$\Pr(A) = 1 - \Pr(A')$	$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$
$\Pr(A/B) = \frac{\Pr(A \cap B)}{\Pr(B)}$	
mean: $\mu = E(X)$	variance: $\text{var}(X) = \sigma^2 = E((X - \mu)^2) = E(X^2) - \mu^2$

probability distribution		mean	variance
discrete	$\Pr(X = x) = p(x)$	$\mu = \sum x p(x)$	$\sigma^2 = \sum (x - \mu)^2 p(x)$
continuous	$\Pr(a < X < b) = \int_a^b f(x) dx$	$\mu = \int_{-\infty}^{\infty} x f(x) dx$	$\sigma^2 = \int_{-\infty}^{\infty} (x - \mu)^2 f(x) dx$

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