



# Victorian Certificate of Education 2008

SUPERVISOR TO ATTACH PROCESSING LABEL HERE

## STUDENT NUMBER

Figures

Words

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## MATHEMATICAL METHODS (CAS)

### Written examination 1

Friday 7 November 2008

Reading time: 9.00 am to 9.15 am (15 minutes)

Writing time: 9.15 am to 10.15 am (1 hour)

### QUESTION AND ANSWER BOOK

#### Structure of book

| <i>Number of questions</i> | <i>Number of questions to be answered</i> | <i>Number of marks</i> |
|----------------------------|-------------------------------------------|------------------------|
| 10                         | 10                                        | 40                     |

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers.
- Students are NOT permitted to bring into the examination room: notes of any kind, blank sheets of paper, white out liquid/tape or a calculator of any type.

#### Materials supplied

- Question and answer book of 15 pages, with a detachable sheet of miscellaneous formulas in the centrefold.
- Working space is provided throughout the book.

#### Instructions

- Detach the formula sheet from the centre of this book during reading time.
- Write your **student number** in the space provided above on this page.
- All written responses must be in English.

**Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.**

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**Instructions**

Answer **all** questions in the spaces provided.

A decimal approximation will not be accepted if an **exact** answer is required to a question.

In questions where more than one mark is available, appropriate working must be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

**Question 1**

- a. Let  $y = (3x^2 - 5x)^5$ . Find  $\frac{dy}{dx}$ .

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- b. Let  $f(x) = xe^{3x}$ . Evaluate  $f'(0)$ .

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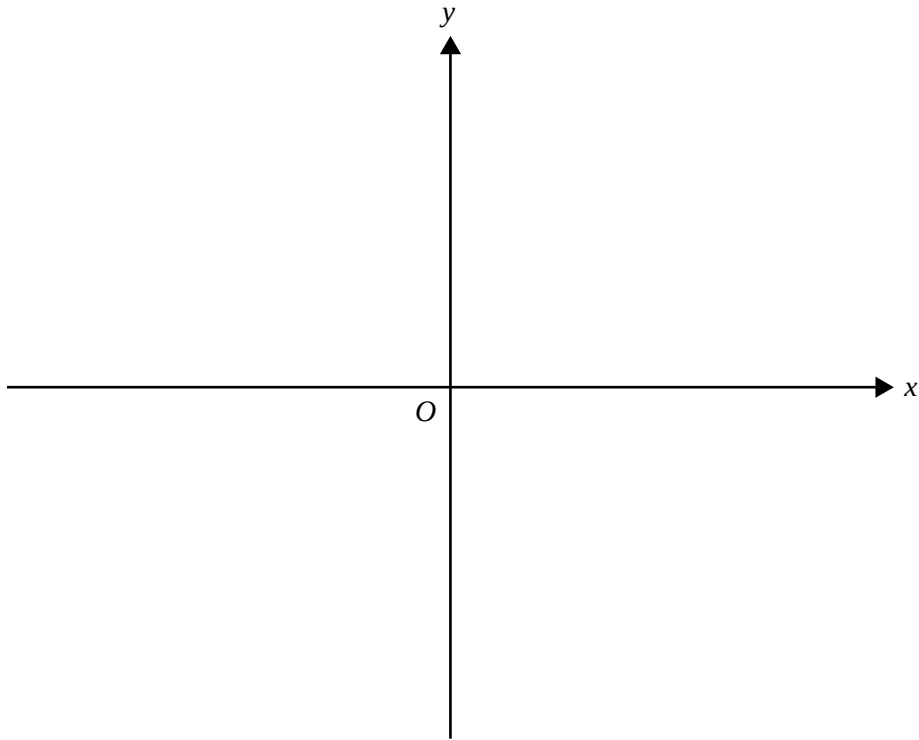
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2 + 3 = 5 marks

**TURN OVER**

**Question 2**

On the axes below, sketch the graph of  $f: \mathbb{R} \setminus \{-1\} \rightarrow \mathbb{R}$ ,  $f(x) = 2 - \frac{4}{x+1}$ .  
 Label all axis intercepts. Label each asymptote with its equation.




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4 marks

**Question 3**

Solve the equation  $\cos\left(\frac{3x}{2}\right) = \frac{1}{2}$  for  $x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ .

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2 marks

**Question 4**

The function

$$f(x) = \begin{cases} k \sin(\pi x) & \text{if } x \in [0, 1] \\ 0 & \text{otherwise} \end{cases}$$

is a probability density function for the continuous random variable  $X$ .

- a. Show that  $k = \frac{\pi}{2}$ .

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- b. Find  $\Pr\left(X \leq \frac{1}{4} \mid X \leq \frac{1}{2}\right)$ .

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2 + 3 = 5 marks

**TURN OVER**

**Question 5**

The area of the region bounded by the  $y$ -axis, the  $x$ -axis, the curve  $y = e^{2x}$  and the line  $x = C$ , where  $C$  is a positive real constant, is  $\frac{5}{2}$ . Find  $C$ .

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3 marks

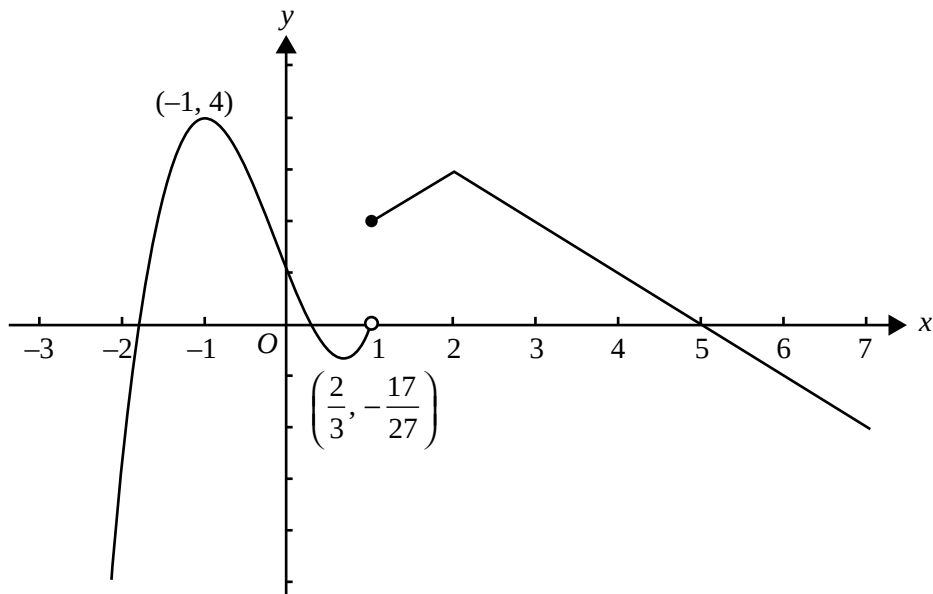
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**Question 6**

a. The graph of the function  $f$  is shown, where

$$f(x) = \begin{cases} 2x^3 + x^2 - 4x + 1 & \text{if } x \in (-\infty, 1) \\ -|x-2| + 3 & \text{if } x \in [1, \infty) \end{cases}$$



The stationary points of the function  $f$  are labelled with their coordinates.

Write down the domain of the derivative function  $f'$ .

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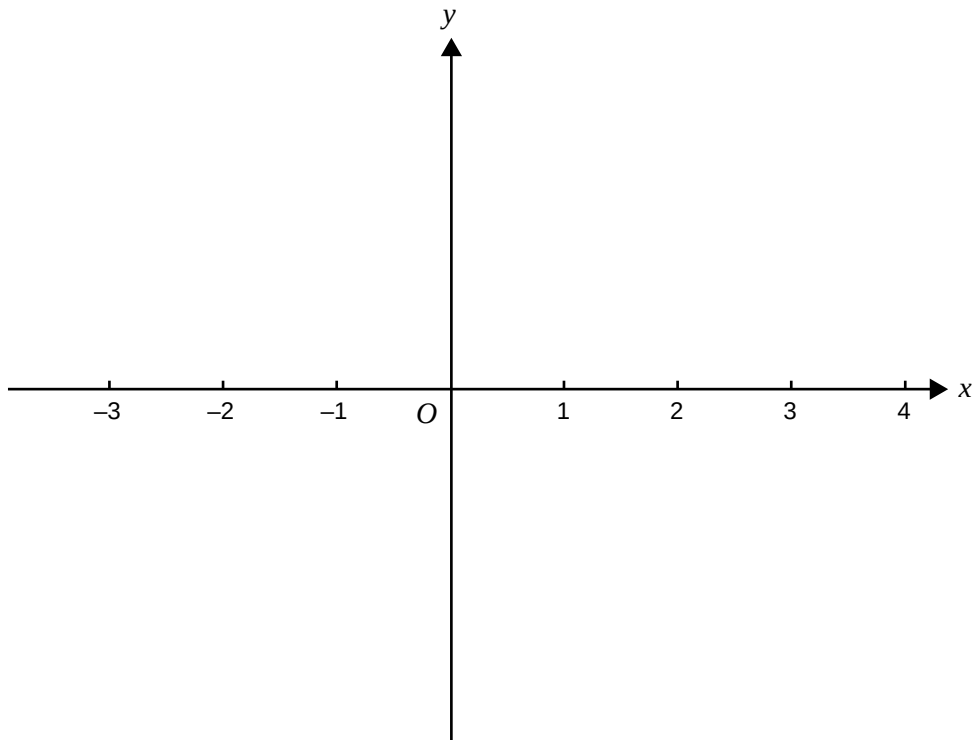


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- b.** By referring to the graph in part **a.**, sketch the graph of the function with rule  $y = |2x^3 + x^2 - 4x + 1|$ , for  $x < 1$ , on the set of axes below.

Label stationary points with their coordinates. (Do not attempt to find x-axis intercepts.)



1 + 2 = 3 marks

**Question 7**

Jane drives to work each morning and passes through three intersections with traffic lights. The number  $X$  of traffic lights that are red when Jane is driving to work is a random variable with probability distribution given by

|              |     |     |     |     |
|--------------|-----|-----|-----|-----|
| $x$          | 0   | 1   | 2   | 3   |
| $\Pr(X = x)$ | 0.1 | 0.2 | 0.3 | 0.4 |

- a. What is the mode of  $X$ ?

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- b. Jane drives to work on two consecutive days. What is the probability that the number of traffic lights that are red is the same on both days?

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1 + 2 = 3 marks

**Question 8**

Every Friday Jean-Paul goes to see a movie. He always goes to one of two local cinemas – the Dandy or the Cino.

If he goes to the Dandy one Friday, the probability that he goes to the Cino the next Friday is 0.5. If he goes to the Cino one Friday, then the probability that he goes to the Dandy the next Friday is 0.6.

On any given Friday the cinema he goes to depends only on the cinema he went to on the previous Friday.

If he goes to the Cino one Friday, what is the probability that he goes to the Cino on **exactly two** of the next three Fridays?

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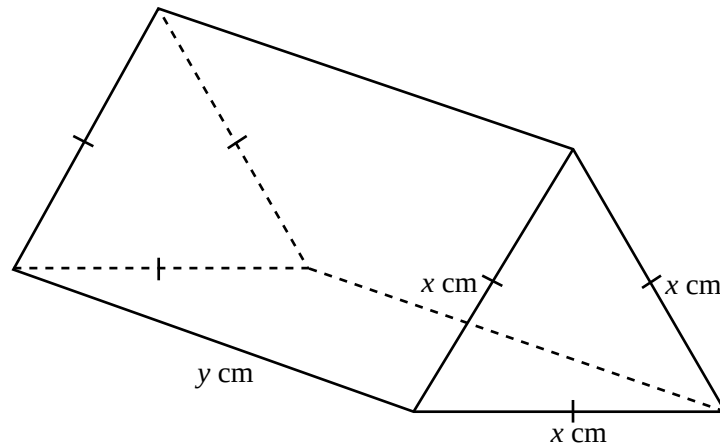
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3 marks

**Question 9**

A plastic brick is made in the shape of a right triangular prism. The triangular end is an equilateral triangle with side length  $x$  cm and the length of the brick is  $y$  cm.



The volume of the brick is  $1000 \text{ cm}^3$ .

- a. Find an expression for  $y$  in terms of  $x$ .

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- b. Show that the total surface area,  $A \text{ cm}^2$ , of the brick is given by

$$A = \frac{4000\sqrt{3}}{x} + \frac{\sqrt{3}x^2}{2}$$

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- c. Find the value of  $x$  for which the brick has minimum total surface area. (You do not have to find this minimum.)

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2 + 2 + 3 = 7 marks

**TURN OVER**

**Question 10**

Let  $f: \mathbb{R} \rightarrow \mathbb{R}$ ,  $f(x) = e^{2x} - 1$ .

- a. Find the rule and domain of the inverse function  $f^{-1}$ .

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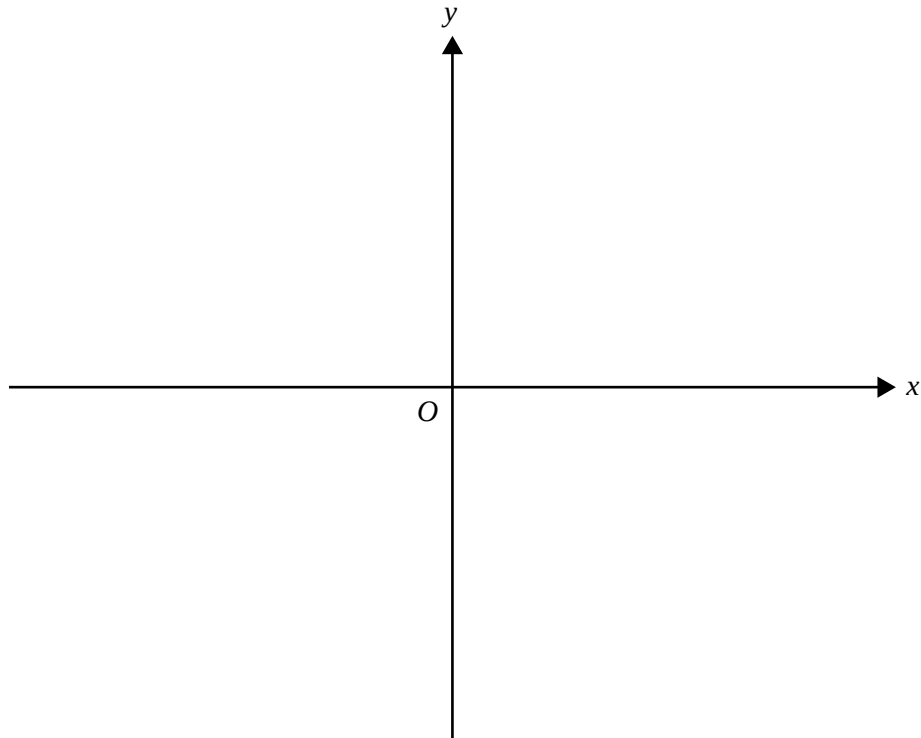
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- b. On the axes provided, sketch the graph of  $y = f(f^{-1}(x))$  for its maximal domain.



- c. Find  $f(f^{-1}(2x))$  in the form  $\frac{ax}{bx+c}$  where  $a$ ,  $b$  and  $c$  are real constants.

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2 + 1 + 2 = 5 marks

# **MATHEMATICAL METHODS AND MATHEMATICAL METHODS (CAS)**

## **Written examinations 1 and 2**

### **FORMULA SHEET**

#### **Directions to students**

Detach this formula sheet during reading time.

This formula sheet is provided for your reference.



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## Mathematical Methods and Mathematical Methods (CAS)

### Formulas

#### Mensuration

area of a trapezium:  $\frac{1}{2}(a+b)h$

curved surface area of a cylinder:  $2\pi rh$

volume of a cylinder:  $\pi r^2 h$

volume of a cone:  $\frac{1}{3}\pi r^2 h$

volume of a pyramid:  $\frac{1}{3}Ah$

volume of a sphere:  $\frac{4}{3}\pi r^3$

area of a triangle:  $\frac{1}{2}bc \sin A$

#### Calculus

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$$\frac{d}{dx}(e^{ax}) = ae^{ax}$$

$$\frac{d}{dx}(\log_e(x)) = \frac{1}{x}$$

$$\frac{d}{dx}(\sin(ax)) = a \cos(ax)$$

$$\frac{d}{dx}(\cos(ax)) = -a \sin(ax)$$

$$\frac{d}{dx}(\tan(ax)) = \frac{a}{\cos^2(ax)} = a \sec^2(ax)$$

product rule:  $\frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}$

chain rule:  $\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}$

$$\int x^n dx = \frac{1}{n+1} x^{n+1} + c, n \neq -1$$

$$\int e^{ax} dx = \frac{1}{a} e^{ax} + c$$

$$\int \frac{1}{x} dx = \log_e |x| + c$$

$$\int \sin(ax) dx = -\frac{1}{a} \cos(ax) + c$$

$$\int \cos(ax) dx = \frac{1}{a} \sin(ax) + c$$

quotient rule:  $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$

approximation:  $f(x+h) \approx f(x) + hf'(x)$

#### Probability

$$\Pr(A) = 1 - \Pr(A')$$

$$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$$

$$\Pr(A|B) = \frac{\Pr(A \cap B)}{\Pr(B)}$$

mean:  $\mu = E(X)$

variance:  $\text{var}(X) = \sigma^2 = E((X - \mu)^2) = E(X^2) - \mu^2$

| probability distribution |                                     | mean                                      | variance                                                 |
|--------------------------|-------------------------------------|-------------------------------------------|----------------------------------------------------------|
| discrete                 | $\Pr(X = x) = p(x)$                 | $\mu = \sum x p(x)$                       | $\sigma^2 = \sum (x - \mu)^2 p(x)$                       |
| continuous               | $\Pr(a < X < b) = \int_a^b f(x) dx$ | $\mu = \int_{-\infty}^{\infty} x f(x) dx$ | $\sigma^2 = \int_{-\infty}^{\infty} (x - \mu)^2 f(x) dx$ |