MATHEMATICAL METHODS (CAS)

Units 3 & 4 – Written examination 1

2008 Trial Examination

SOLUTIONS

Question 1

a. Reflection in x axis, dilation from x axis by factor 4, translation right 3 units, translation down 2 units

A1

b. Show asymptotes, intercepts, correct shape
Asymptotes $y = -2, x = 3$.

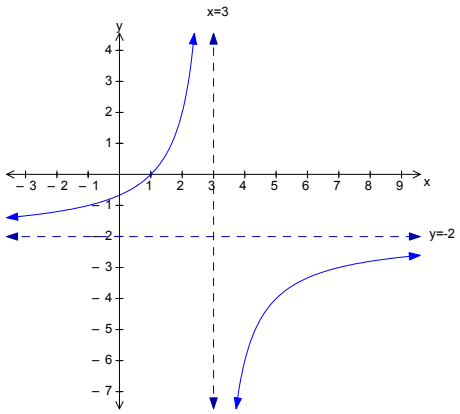
Let $x = 0$ $y = \frac{4}{3} - 2 = -\frac{2}{3}$

$$2 = -\frac{4}{x-3}$$

Let $y = 0$, $2x - 6 = -4$

$$2x = 2$$

$$x = 1$$



A3

Question 2

$\left(2^x\right)^2 - 2^x 2^3 + 2^4 = 0$

$let... 2^x = a$

$a^2 - 8a + 16 = 0$

$(a-4)^2 = 0$

$a = 4$

$2^x = 4 \therefore x = 2$

M2 + A1

Question 3

a.

$x = 1 - 2e^{y-1}$

$x - 1 = -2e^{y-1}$

$1 - x = 2e^{y-1}$

$\frac{1-x}{2} = e^{y-1}$

$\log_e\left|\frac{1-x}{2}\right| = y - 1$

$\log_e\left|\frac{1-x}{2}\right| + 1 = y \ldots \therefore f^{-1(x)} = \log_e\left|\frac{1-x}{2}\right| + 1$

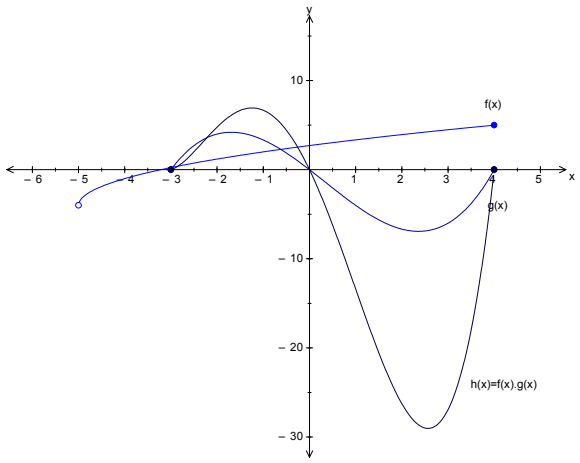
M3

b. Range R, domain $(-\infty, 1)$

A1

Question 4

a. Show shape
x-intercepts x = -3 , 0 , 4



M1 + A1

b. Domain [−3, 4]

A1

Question 5

a. $f'(x) = \frac{24x(3x^2 - 2)^3 \cos(x) + \sin(x)(3x^2 - 2)^4}{\cos^2(x)}$

This can be simplified considerably and re-expressed algebraically, but for one mark this answer is sufficient

A1

b. $\frac{dy}{dx} = 2xe^{2x} + 2x^2e^{2x}$

A1

c. $\int (\frac{1}{2} f'(x) + 1)dx = \frac{1}{2} f(x) + x + c = \frac{1}{2} x^2 e^{2x} + x + c$

A1

Question 6

a. $\frac{\sqrt{3}\sin(3x)}{\cos(3x)} = 1$ and $-3\pi \le x \le 3\pi$

$\tan(3x)$ is positive in 1st and 3rd quadrants

$\tan(3x) = \frac{1}{\sqrt{3}}$

$3x = -\frac{5\pi}{6}, -\frac{11\pi}{6}, -\frac{17\pi}{6}, \frac{\pi}{6}, \frac{7\pi}{6}, \frac{13\pi}{6}$

$x = -\frac{5\pi}{18}, -\frac{11\pi}{18}, -\frac{17\pi}{18}, \frac{\pi}{18}, \frac{7\pi}{18}, \frac{13\pi}{18}$

M2 + A1

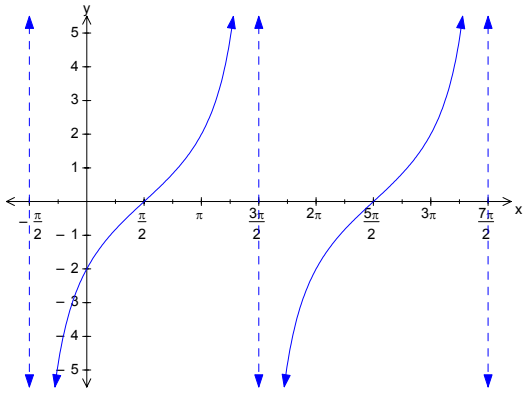
Question 7

a. period = $\pi \div \frac{1}{2} = 2\pi$, translated $\frac{\pi}{2}$ right

asymptotes: $x = \frac{\pi}{2} \pm \pi = -\frac{\pi}{2}, \frac{3\pi}{2}$

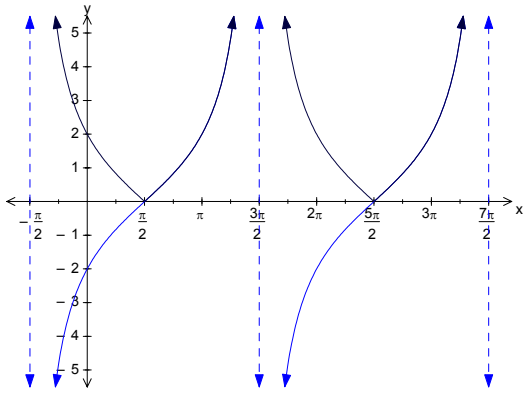
$and.. \frac{3\pi}{2} + 2\pi = \frac{7\pi}{2}$

let $x = 0, y = 2\tan(-\frac{\pi}{4}) = -2$



M3

b. correct shape and asymptotes



A1

Question 8

a. $\frac{dy}{dx} = \frac{3(2)}{2x-3}$, gradient at $x = 3$ $m = \frac{6}{6-3} = 2$

$sub..x = 3, y = 3\ln 3$

$y - 3\ln 3 = 2(x-3)$

$y = 2x + 3\ln 3 - 6$

M2 + A1

Question 9

a. Area can be calculated a few ways. Best method shown.
Area = area bounded by coordinate axes and $x = 2$ and $y = 9$ − area bounded by curve, x axis and $x = 0$ and $x = 2$

$A = 18 - \int_0^2 (x+1)^2 dx$

$= 18 - \int_0^2 x^2 + 2x + 1 dx$

$= 18 - \left[\frac{x^3}{3} + x^2 + x\right]_0^2$

$= 18 - \left[\frac{8}{3} + 4 + 2\right]$

$= 9\frac{1}{3} sq..units$

M2 + A1

Question 10

a.

$E(X) = 0 \times 0.2 + a + 2b + 3 \times 0.2 + 4 \times 0.2$

$2.1 = 1.4 + a + 2b$

$0.7 = a + 2b$

$a = 0.7 - 2b$.....(1)

$0.2 + a + b + 0.2 + 0.2 = 1$

$a + b = 0.4$.....(2)

$sub..(1)..in...(2),..0.7 - 2b + b = 0.4$

$..... - b = -0.3$

$sub..b = 0.3..in..(1)...a = 0.7 - 0.6$

$\therefore a = 0.1, b = 0.3$

M1 + A1

b.

$E(2X+1) = 2E(X)+1$

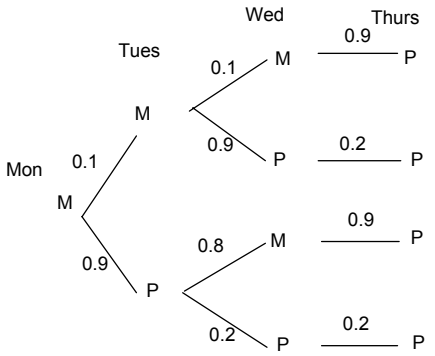
$\ldots\ldots\ldots\ldots = 2(2.1)+1$

$\ldots\ldots\ldots\ldots = 5.2$

A1

Question 11

a.



$$\Pr(Pellets.on.Thurs \mid Milk.on.Mon)$$

$$= \frac{1\times1\times9}{1000} + \frac{1\times9\times2}{1000} + \frac{9\times8\times9}{1000} + \frac{9\times2\times2}{1000}$$

$$= \frac{9+18+648+36}{1000}$$

$$= \frac{711}{1000}$$

M3 + A1

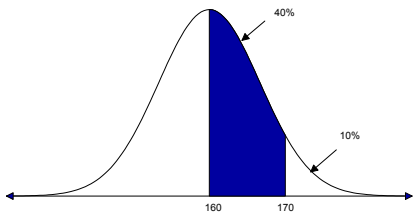
Question 12

a. For 95% data lies within $\mu \pm 2\sigma$

$\therefore 144 \le x \le 176cm$

A1

b.



$$\Pr(H < 170 \mid H > 160) = \frac{\Pr(H < 170 \cap H > 160)}{\Pr(H > 160)} = \frac{\Pr(160 < H < 170)}{\Pr(H > 160)}$$

$$= \frac{0.4}{0.5} = \frac{4}{5}$$

M1 + A1