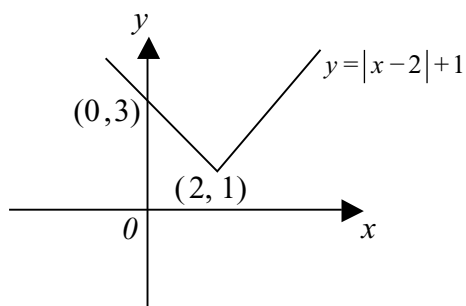


The Mathematical Association of Victoria
MATHEMATICAL METHODS AND MATHEMATICAL METHODS(CAS)
2008 Trial written examination 1 – Solutions

Question 1

a.



Shape [1A]

Cusp and intercept with correct coordinates. [1A]

b. $|x - 2| + 1 = 2$

$$x - 2 + 1 = 2$$

$$x = 3$$

[1A]

OR

$$-x + 2 + 1 = 2$$

$$x = 1$$

[1A]

c. f is not a one to one function.
Hence f^{-1} does not exist.

[1A]

Question 2

a. $3 \log_2(2x) - 2 \log_2(x) = 1$

$$\log_2(2x)^3 - \log_2(x)^2 = \log_2(2)$$

$$\log_2\left(\frac{8x^3}{x^2}\right) = \log_2(2) \quad [1M]$$

$$\log_2(8x) = \log_2(2)$$

$$8x = 2$$

$$x = \frac{1}{4} \quad [1A]$$

b. $A(t) = 8e^{-\frac{t}{10}}$

When the area is halved,

$$4 = 8e^{-\frac{t}{10}}$$

$$e^{-\frac{t}{10}} = \frac{1}{2} \quad [1M]$$

$$-\frac{t}{10} = \log_e\left(\frac{1}{2}\right)$$

$$t = -10 \log_e\left(\frac{1}{2}\right) \quad [1A]$$

Alternatively

$$t = 10 \log_e(2)$$

Question 3

a. $y = \frac{\tan(x)}{x}$

Apply the quotient rule.

[1M]

Let $u = \tan(x)$ and $v = x$

$$\frac{du}{dx} = \sec^2(x) \text{ and } \frac{dv}{dx} = 1$$

$$\frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

$$\frac{dy}{dx} = \frac{x \sec^2(x) - \tan(x)}{x^2}$$

[1A]

(or an equivalent form of the above)

b. $f(x) = \sin(x)$ and $g(x) = \log_e(x)$

$$f(g(x)) = \sin(\log_e(x))$$

[1A]

Apply the chain rule

$$\frac{d}{dx} \left(f(g(x)) \right) = \cos(\log_e(x)) \times \frac{1}{x}$$

$$\frac{d}{dx} \left(f(g(1)) \right) = \cos(\log_e(1))$$

$$= \cos(0)$$

$$= 1$$

[1A]

Question 4

a. $f: [0, a] \rightarrow \mathbb{R}, f(x) = 2\cos(\pi x) - 1$

$$\text{Period} = \frac{2\pi}{\pi} = 2$$

To cover one period the domain must be $[0, 2]$.

Hence $a = 2$.

[1M]

b. $f(2x) = 0$

$$2\cos(\pi x) - 1 = 0$$

$$\cos(2\pi x) = \frac{1}{2}$$

$$2\pi x = \frac{\pi}{3} \text{ or } \frac{5\pi}{3} \text{ or } \frac{7\pi}{3} \text{ or } \frac{11\pi}{3} \text{ or } \frac{13\pi}{3} \dots$$

$$x = \frac{1}{6} \text{ or } \frac{5}{6} \text{ or } \frac{7}{6} \text{ or } \frac{11}{6} \text{ or } \frac{13}{6} \dots$$

[1M]

Since $x \in [0, 2]$,

$$x = \frac{1}{6} \text{ or } \frac{5}{6} \text{ or } \frac{7}{6} \text{ or } \frac{11}{6}$$

[1A]

Question 5

- a. Let X be the number of students using a Tasmania Implements calculator.

$$X \sim Bi\left(3, \frac{3}{4}\right)$$

$$\Pr(X = 2) = {}^3C_2 \left(\frac{3}{4}\right)^2 \left(\frac{1}{4}\right) \quad [1M]$$

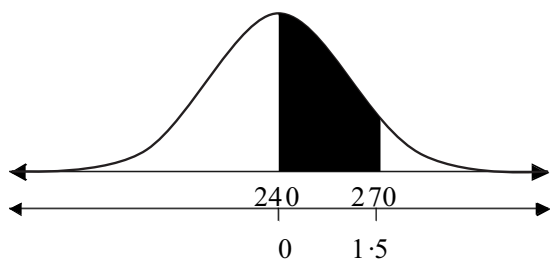
$$\begin{aligned} \Pr(X = 2) &= {}^3C_2 \left(\frac{3}{4}\right)^2 \left(\frac{1}{4}\right) \\ &= 3 \times \frac{9}{16} \times \frac{1}{4} \\ &= \frac{27}{64} \end{aligned} \quad [1A]$$

$$\begin{aligned} \text{b. } \Pr(X = 2 | X \geq 1) &= \frac{\Pr(X = 2 \cap X \geq 1)}{\Pr(X \geq 1)} \\ &= \frac{\Pr(X = 2)}{1 - \Pr(X = 0)} \end{aligned} \quad [1M]$$

$$\begin{aligned} \Pr(X = 2 | X \geq 1) &= \frac{\frac{27}{64}}{1 - \left(\frac{1}{4}\right)^3} \\ &= \frac{\frac{27}{64}}{\frac{63}{64}} \\ &= \frac{27}{63} = \frac{3}{7} \end{aligned} \quad [1A]$$

Question 6

a. $\Pr(240 < X < 270) = \Pr(0 < Z < 1.5)$ [1M]
 $= 0.5 - 0.067$
 $= 0.433$ [1A]



b. $\int_0^1 \left(x^{\frac{4}{3}}\right) dx = \left[\frac{3}{7}x^{\frac{7}{3}}\right]_0^1$ [1M]
 $= \frac{3}{7}$

For a probability density function,

$$\int_{-\infty}^{\infty} f(x) dx = 1. \text{ Therefore}$$

$$\int_1^a (x) dx = \frac{4}{7} \quad [1M]$$

$$\left[\frac{x^2}{2}\right]_1^a = \frac{4}{7}$$

$$\frac{a^2}{2} - \frac{1}{2} = \frac{4}{7} \quad [1M]$$

$$a^2 = \frac{15}{7}$$

$$a = \sqrt{\frac{15}{7}} \quad [1A]$$

Question 7

a. $V = \frac{1}{3}\pi r^2 h$

However, $r = \frac{h}{2}$

$$V = \frac{1}{3}\pi \left(\frac{h}{2}\right)^2 h \quad \dots \text{equation 1}$$

When $h = 6$ m

$$V = \frac{1}{3}\pi \left(\frac{6}{2}\right)^2 \times 6$$

Cancel as appropriate to get

$$V = 18\pi \text{ m}^3 \quad [1A]$$

b. $\frac{dV}{dt} = 2 \text{ m}^3/\text{s}$ and require $\frac{dh}{dt}$

$$\frac{dV}{dt} = \frac{dV}{dh} \times \frac{dh}{dt} \quad \dots \text{equation 2} \quad [1M]$$

From **equation 1** in a. above,

$$V = \frac{1}{12}\pi h^3, \text{ therefore}$$

$$\frac{dV}{dh} = \frac{1}{4}\pi h^2 \quad \dots \text{equation 3}$$

Substitute **equation 3** in **equation 2**

$$2 = \frac{1}{4}\pi h^2 \times \frac{dh}{dt} \quad [1M]$$

$$\frac{dh}{dt} = \frac{8}{\pi h^2}$$

When $h = 6$ m

$$\frac{dh}{dt} = \frac{8}{36\pi} \text{ m/s} = \frac{2}{9\pi} \text{ m/s} \quad [1A]$$

Question 8

a. $f(x) = (\sin(x))^3$

Apply the chain rule

$$f'(x) = 3(\sin(x))^2 \times \cos(x)$$

[1M]

$$f'(x) = 3\sin^2(x)\cos(x)$$

[1A]

b. i. $\frac{d}{dx}(\sin^3(x)) = 3\cos(x)\sin^2(x)$

Take $\int dx$ of both sides of the equation

$$\sin^3(x) + c_1 = 3 \int (\cos(x)\sin^2(x)) dx$$

$$\int (\cos(x)\sin^2(x)) dx = \frac{1}{3} \sin^3(x) + c$$

[1A]

(c and c_1 are real constants)

ii. The gradient of the tangent is given by

$$f'\left(\frac{\pi}{4}\right) = 3\sin^2\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{4}\right)$$

$$= 3 \times \left(\frac{1}{\sqrt{2}}\right)^2 \times \frac{1}{\sqrt{2}}$$

$$= \frac{3}{2\sqrt{2}}$$

[1M]

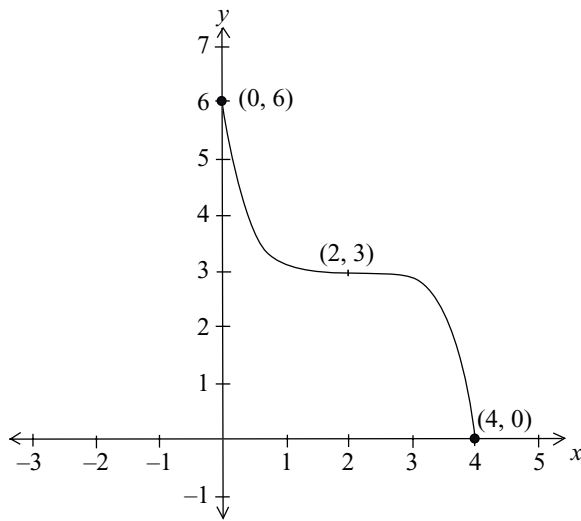
$$\text{Gradient of normal} = -\frac{1}{f'\left(\frac{\pi}{4}\right)}$$

$$= -\frac{2\sqrt{2}}{3}$$

[1A]

Question 9

a. $y = -3\left(\frac{1}{2}x - 1\right)^5 + 3$



Correct shape with (2, 3) [1A]

Correct intercepts (0, 6) and (4, 0) and closed circles [1A]

b. $\int_0^4 \left(-3\left(\frac{1}{2}x - 1\right)^5 + 3 \right) dx$

[1M]

$$= \left[-\left(\frac{1}{2}x - 1\right)^6 + 3x \right]_0^4$$

[1M]

$$= -1 + 12 + 1$$

$$= 12 \text{ units}^2$$

[1A]