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VCE

Mathematical Methods

CAS Solutions

Trial Examination 2



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SECTION 1

ANSWERS

1	A	B	C	D	E
2	A	B	C	D	E
3	A	B	C	D	E
4	A	B	C	D	E
5	A	B	C	D	E
6	A	B	C	D	E
7	A	B	C	D	E
8	A	B	C	D	E
9	A	B	C	D	E
10	A	B	C	D	E
11	A	B	C	D	E
12	A	B	C	D	E
13	A	B	C	D	E
14	A	B	C	D	E
15	A	B	C	D	E
16	A	B	C	D	E
17	A	B	C	D	E
18	A	B	C	D	E
19	A	B	C	D	E
20	A	B	C	D	E
21	A	B	C	D	E
22	A	B	C	D	E

SECTION 1

Question 1 Answer C

the average value is $\bar{y} = \frac{1}{b-a} \int_a^b y \, dx$

$$\bar{y} = \frac{1}{\frac{\pi}{2}-0} \int_0^{\frac{\pi}{2}} 3\sin^2(2x) \, dx = \frac{2}{\pi} \times \frac{3\pi}{4} = 1.5$$

Question 2 Answer E

Let $f(x) = |\sin(x^2)|$ then $f(x+h) = |\sin((x+h)^2)|$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \quad \text{and} \quad f'(k) = \lim_{h \rightarrow 0} \frac{|\sin((k+h)^2)| - |\sin(k^2)|}{h}$$

Question 3 Answer E

The function $f: [0, c] \rightarrow R$ $f(x) = c - 2x$ is a straight line, now

$f(0) = c$ and $f(c) = -c$ so that $\text{ran } f = [-c, c] = \text{dom } f^{-1}$

interchanging x and y the inverse is $f^{-1} \quad x = c - 2y$

transposing to make y the subject $2y = c - x \quad y = f^{-1}(x) = \frac{c-x}{2}$

Question 4 Answer A

the graph of $y = \sqrt{x+3}$ when reflected in the y -axis becomes $y = \sqrt{3-x}$

then translated 2 units to the left becomes $y = \sqrt{1-x}$

then translated 3 units up becomes $y = \sqrt{1-x} + 3$

Question 5 Answer C

$N(t) = 20e^{0.2t}$, the average rate is $\frac{N(3) - N(0)}{3-0}$

$$\frac{N(3) - N(0)}{3-0} = \frac{20e^{0.6} - 20}{3} = 5.48$$

Question 6

Answer B

$$f(x) = x^3 \quad g(x) = \cos(x) \quad \text{and} \quad h(x) = \sqrt{x}$$

$$g(h(x)) = \cos(h(x)) = \cos(\sqrt{x})$$

$$f(g(h(x))) = f(\cos(\sqrt{x})) = (\cos(\sqrt{x}))^3 = \cos^3(\sqrt{x})$$

Question 7

Answer B

$$f(x) = \sqrt[3]{x}, \quad \text{and} \quad x = -64 \quad h = 0.5 \quad \text{now} \quad -63.5 = x + h$$

$$\text{using } f(x+h) = f(x) + h f'(x)$$

$$\sqrt[3]{-63.5} = f(-64) + 0.5 f'(-64)$$

Question 8

Answer D

this is just the chain rule, in function form

$$\frac{d}{dx}(f(g(x))) = g'(x) f'(g(x))$$

Question 9

Answer C

$$f: [0, \pi] \rightarrow \mathbb{R} \quad \text{where} \quad f(x) = 300 \tan(3x) = \frac{300 \sin(3x)}{\cos(3x)}$$

The graph crosses the x-axis when $\sin(3x) = 0 \Rightarrow x = 0, \frac{\pi}{3}, \frac{2\pi}{3}, \pi$ Option **A.** is true

The graph has asymptotes when $\cos(3x) = 0 \Rightarrow x = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}$ Option **B.** is true

The range of the graph is $\mathbb{R}$. Option **C.** is false

The period of $\tan(3x)$ is $\frac{\pi}{3}$ so the graph has three cycles in $[0, \pi]$ Option **D.** is true

The graph has a domain of $[0, \pi]$ is its restricted domain. Option **E.** is true

Question 10

Answer B

$$f(0) = 1 \quad f(a) = 1 \quad \text{and} \quad f\left(\frac{a}{2}\right) = 1 - a \sin\left(\frac{\pi}{2}\right) = 1 - a$$

so the range is $[1, 1-a]$

Question 11

Answer E

$$y = x^4 - 4x^2$$

for turning points

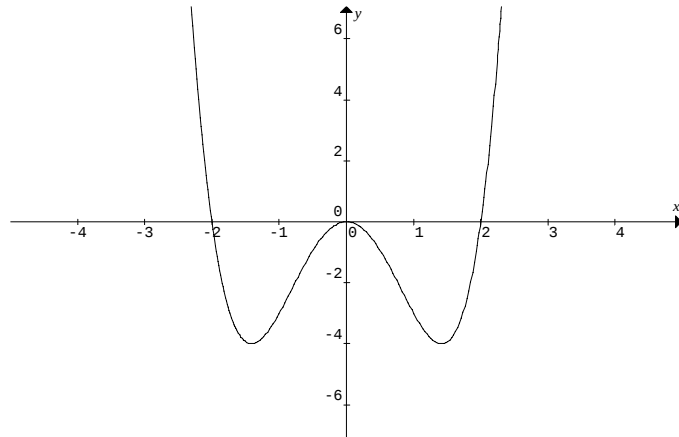
$$\frac{dy}{dx} = 4x^3 - 8x = 0$$

$$4x(x^2 - 2) = 0$$

$$\text{at } x = 0 \quad x = \pm\sqrt{2}$$

from the graph, the gradient is negative for

$$(-\infty, -\sqrt{2}) \cup (0, \sqrt{2})$$



Question 12

Answer D

$$(1) 2x - 3y = q \quad (2) px + 6y = 10$$

A. is true if $p = -4$, there is no unique solution.

B. is true if $p \neq -4$, there is a unique solution, the two lines intersect in a unique point.

C. is true, if $p = -4$ and $q = -5$ the two equations are the same line, so that there is an infinite number of solutions.

E. is true, if $p = -4$ and $q \neq -5$ the two equations, represent straight parallel lines, with different y-intercepts, so there is no solution.

D. is false.

Question 13

Answer D

$$\int_a^0 (1 - f(x)) dx = [x]_a^0 - \int_a^0 f(x) dx = (0 - a) + \int_0^a f(x) dx = A - a$$

Question 14

Answer C

$$\frac{dy}{dx} = 4 \sin(2x) \Rightarrow y = \int 4 \sin(2x) dx = -2 \cos(2x) + c \quad \text{to find } c, \text{ use } y\left(\frac{5\pi}{3}\right) = 0$$

$$0 = -2 \cos\left(\frac{10\pi}{3}\right) + c = 1 + c = 0 \Rightarrow c = -1$$

$$y = -2 \cos(2x) - 1 \quad \text{now when } x = 0 \quad y = -2 \cos(0) - 1 = -3$$

Question 15

Answer C

let $y_1 = kx$ and $y_2 = x^2 + bx + c^2$ if $y_1 = y_2$ $kx = x^2 + bx + c^2$

$x^2 + (b-k)x + c^2 = 0$ the number of roots, depends upon the discriminant

$$\Delta = (b-k)^2 - 4c^2$$

A. and **B.** for the graphs to touch or to be a tangent, $\Delta = 0 \Rightarrow b-k = \pm 2c \quad b = k \pm 2c$

both **A.** and **B.** are true.

For the graphs to intersect at two distinct points $\Delta > 0$

$$\Delta > 0 \Rightarrow (b-k)^2 > 4c^2 \Rightarrow b-k > 2c \text{ and } b-k < -2c$$

$k > b+2c$ and $k < b-2c$ option **D.** and **E.** are true.

For the graphs to not intersect $\Delta < 0$

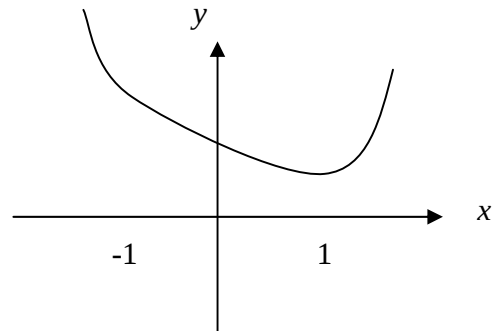
$$\Delta < 0 \Rightarrow (b-k)^2 < 4c^2 \Rightarrow b-k < 2c \text{ and } b-k > -2c \text{ or}$$

$k < b+2c$ and $k > b-2c$ or $b-2c < k < b+2c$ **C.** is false.

Question 16

Answer B

- $f'(x) = 0$ at $x = -1$ and $x = 1$
- $f'(x) < 0$ for $x < -1$ and $-1 < x < 1$
- $f'(x) > 0$ for $x > 1$



The graph has a stationary point of inflexion at $x = -1$ and a minimum at $x = 1$.

Question 17

Answer E

$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$ this is always true.

$$\Pr(A \cap B) = \frac{p}{2} \text{ then } \Pr(\bar{A} \cap \bar{B}) = 1 - \frac{3p}{2} = \frac{2-3p}{2}$$

Option **A.** is true

If A and B are mutually exclusive then $\Pr(A \cap B) = 0$

so that $\Pr(A \cup B) = \Pr(A) + \Pr(B) = 2p$ Option **B.** is true.

If A and B are independent then $\Pr(A \cap B) = \Pr(A) \cdot \Pr(B)$

$$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A) \Pr(B) = 2p - p^2 = p(2-p) \text{ Option C. is true.}$$

If A and B are independent then $\Pr(A/B) = \frac{\Pr(A \cap B)}{\Pr(B)} = \frac{\Pr(A) \cdot \Pr(B)}{\Pr(B)} = p$ Option **D.** is true.

If A and B are mutually exclusive then $\Pr(A/B) = \frac{\Pr(A \cap B)}{\Pr(B)} = 0$ Option **E.** is false.

	A	$\bar{A}$	
B	$\frac{p}{2}$	$\frac{p}{2}$	p
$\bar{B}$	$\frac{p}{2}$	$1 - \frac{3p}{2}$	1-p
	p	1-p	

Question 18

Answer D

Since it is a discrete random variable, the probabilities add to one, so that $a + b = 1$

$$E(X) = \sum x \Pr(X = x) = (-1)a + (1)b = b - a$$

$$E(X^2) = \sum x^2 \Pr(X = x) = (-1)^2 a + (1)^2 b = a + b$$

$$\text{VAR}(X) = E(X^2) - (E(X))^2 = (a + b) - (b - a)^2 \quad \text{since } a + b = 1 \text{ and } b = 1 - a$$

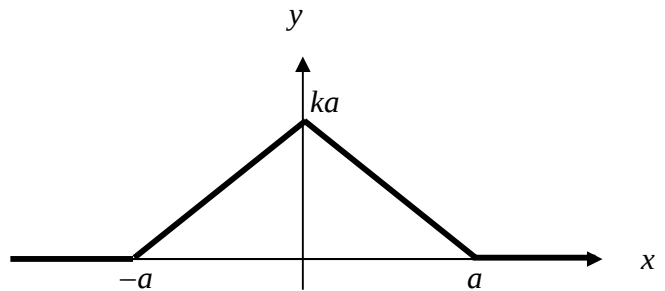
$$\text{VAR}(X) = 1 - (1 - 2a)^2 = 1 - (1 - 4a + 4a^2)$$

$$\text{VAR}(X) = 4a - 4a^2 = 4a(1 - a)$$

Question 19

Answer A

$$f(x) = \begin{cases} k(a - x) & \text{for } 0 \leq x \leq a \\ k(a + x) & \text{for } -a \leq x \leq 0 \\ 0 & \text{elsewhere} \end{cases}$$



The total area under the two triangles is one.

$$A = 2 \left(\frac{1}{2} a \cdot ka \right) = ka^2 = 1 \quad k = \frac{1}{a^2}$$

Question 20

Answer A

$$\text{Given that } \Pr(|Z| < c) = \Pr(-c < Z < c) = a$$

$$\Pr(0 < Z < c) = \Pr(-c < Z < 0) = \frac{a}{2}$$

$$\Pr(Z \geq -c) = \Pr(Z \leq c) = \frac{a}{2} + 0.5 = \frac{a+1}{2}$$

Question 21

Answer A

The higher probabilities $\Pr(X = 8) \approx 0$ $\Pr(X = 9) \approx 0$ $\Pr(X = 10) \approx 0$ are small, and the graph is right (or positively skewed) so the probability of a success on any one trial p is very small, $p \ll 0.5$, $p = 0.3$ is the correct choice.

Question 22

Answer D

want BR or RB

$$\Pr(BR + RB) = \frac{b}{(b+r)} \times \frac{r}{(b+r-1)} + \frac{r}{(b+r)} \times \frac{b}{(b+r-1)} = \frac{2br}{(b+r)(b+r-1)}$$

END OF SECTION 1 SUGGESTED ANSWERS

SECTION 2

Question 1

- a. $f(x) = x^2$ $f(2) = 4$, A is the point $(2, 4)$,

also on $y = ax^3 + bx^2 + cx + d$ substitute

$$(1) \quad 4 = 8a + 4b + 2c + d \quad \text{A1}$$

C is the point $(4, 0)$

$$(2) \quad 0 = 64a + 16b + 4c + d \quad \text{A1}$$

at A the join is smooth, the gradients are equal, so that

$$\frac{dy}{dx} = 2x = 3ax^2 + 2bx + c \quad \text{at } x = 2$$

$$(3) \quad 4 = 12a + 4b + c \quad \text{A1}$$

at B , we have a maximum $\frac{dy}{dx} = 0$ at $x = 3$

$$(4) \quad 0 = 27a + 6b + c \quad \text{A1}$$

- b. the four equations become the one matrix equation $AX = C$ where

$$A = \begin{bmatrix} 8 & 4 & 2 & 1 \\ 64 & 16 & 4 & 1 \\ 12 & 4 & 1 & 0 \\ 27 & 6 & 1 & 0 \end{bmatrix} \quad X = \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} \quad C = \begin{bmatrix} 4 \\ 0 \\ 4 \\ 0 \end{bmatrix} \quad \text{the solution for } X \text{ is} \quad \text{M1}$$

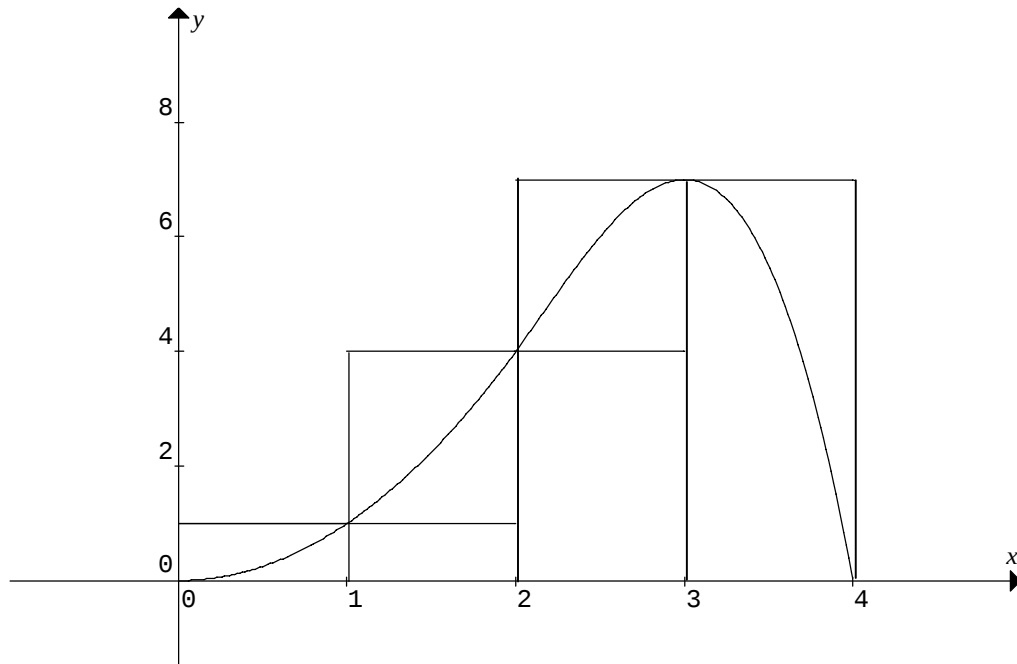
$$X = A^{-1}C = \begin{bmatrix} 8 & 4 & 2 & 1 \\ 64 & 16 & 4 & 1 \\ 12 & 4 & 1 & 0 \\ 27 & 6 & 1 & 0 \end{bmatrix}^{-1} \begin{bmatrix} 4 \\ 0 \\ 4 \\ 0 \end{bmatrix} = \begin{bmatrix} -2 \\ 13 \\ 24 \\ 16 \end{bmatrix} \quad \text{A1}$$

so that $a = -2$ $b = 13$ $c = -24$ $d = 16$

- c. $f(x) = \begin{cases} x^2 & \text{for } 0 \leq x \leq 2 \\ -2x^3 + 13x^2 - 24x + 16 & \text{for } 2 \leq x \leq 4 \end{cases}$

at B $x = 3$ $f(3) = 7$, completing the table of values A1

x	0	1	2	3	4
y	0	1	4	7	0



using left (end-point) rectangles $L = h(y_0 + y_1 + y_2 + y_3) = 1(0 + 1 + 4 + 9) = 14$

using right (end-point) rectangles $R = h(y_1 + y_2 + y_3 + y_4) = 1(1 + 4 + 9 + 16) = 30$

both give the area of the grassed region as 12 m^2 A1

the area of the rectangular region is 28 m^2 , so that,

16 m^2 is the area of the trees and shrubs, the ratio of areas $12:16$ or $3:4$ A1

d.i. $A = \int_0^2 x^2 dx + \int_2^4 (-2x^3 + 13x^2 - 24x + 16) dx$ A1

ii. $A = 13\frac{1}{3} \text{ m}^2$ A1

e.i. the point P is on $y = x^2$ and $0 < p < 2$, so that P has coordinates $P(p, p^2)$ and D is the point $D(3, 0)$ A1

Let s be the distance from P to D , the path length

$$s = d(PD) = \sqrt{(3-p)^2 + p^4}$$
 A1

ii. for the path length s , to be a minimum $\frac{ds}{dp} = \frac{4p^3 - 2(3-p)}{2\sqrt{(3-p)^2 + p^4}} = 0$ M1

$$4p^3 - 6 - 2p = 0 \quad \text{since } 0 < p < 2$$

$$p = 1$$
 A1

$$s_{\min} = \sqrt{5}$$
 A1

Question 2

a. i. $f'(x) = 2 + 4\cos\left(\frac{x}{2}\right)$ A1

ii. maximum value of the gradient is 6 and occurs when

$\cos\left(\frac{x}{2}\right) = 1$ M1

$x = 0, 4\pi$ now, $f(0) = 0$ and $f(4\pi) = 8\pi$ the coordinates are
(0,0) and (4π,8π) A1

b. for stationary points $f'(x) = 0$

$4\cos\left(\frac{x}{2}\right) = -2$

$\cos\left(\frac{x}{2}\right) = -\frac{1}{2}$

$x = \frac{4\pi}{3}, \frac{8\pi}{3}$

$f\left(\frac{4\pi}{3}\right) = \frac{8\pi}{3} + 8\sin\left(\frac{2\pi}{3}\right) = \frac{8\pi}{3} + 4\sqrt{3}$

the maximum coordinate is $\left(\frac{4\pi}{3}, \frac{8\pi}{3} + 4\sqrt{3}\right)$ A1

$f\left(\frac{8\pi}{3}\right) = \frac{16\pi}{3} + 8\sin\left(\frac{4\pi}{3}\right) = \frac{16\pi}{3} - 4\sqrt{3}$

the minimum coordinate is $\left(\frac{8\pi}{3}, \frac{16\pi}{3} - 4\sqrt{3}\right)$ A1

c. at $x = 2\pi$ $f(2\pi) = 4\pi + \sin(\pi) = 4\pi$

$f'(2\pi) = 2 + 4\cos(\pi) = -2$ A1

the equation of the tangent is

$y - 4\pi = -2(x - 2\pi)$

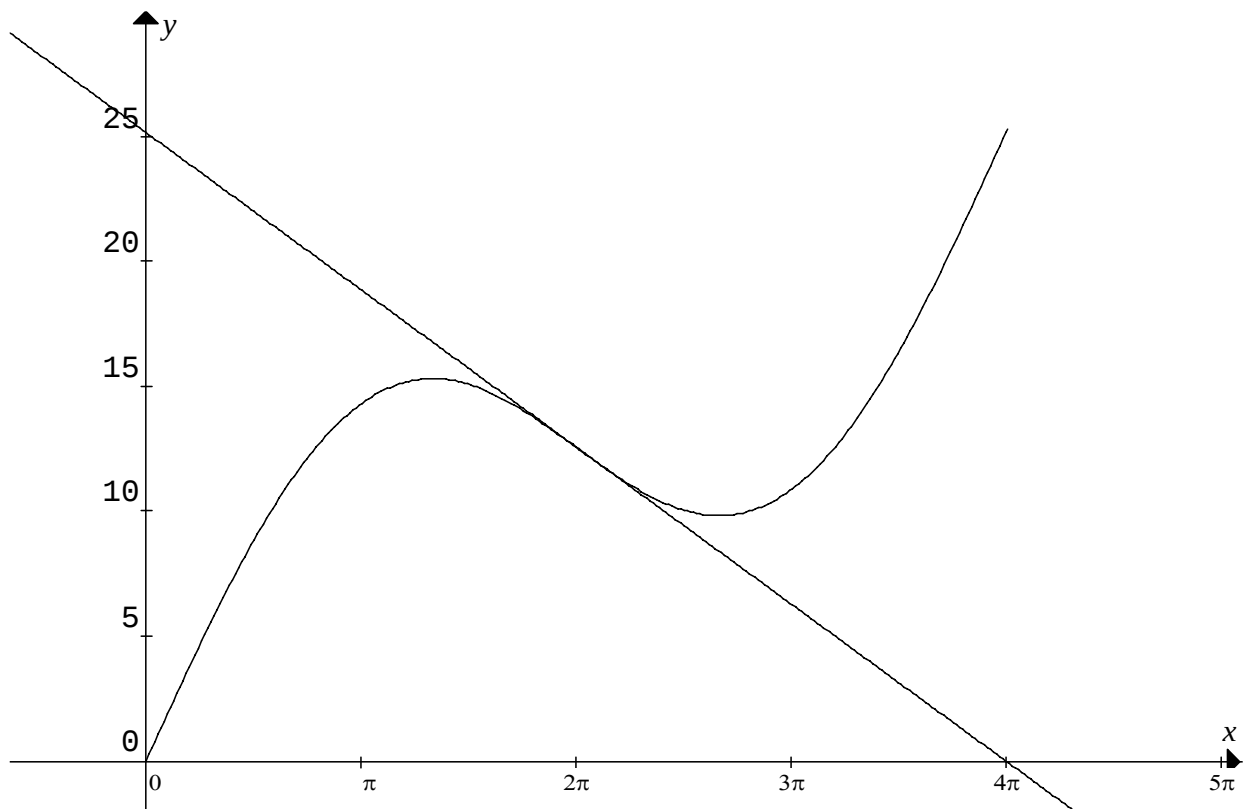
$y = -2x + 8\pi$ A1

d. correct graph, shape, restricted domain $[0, 4\pi]$,

maximum at $\left(\frac{4\pi}{3}, \frac{8\pi}{3} + 4\sqrt{3}\right) \approx (4.2, 15.3)$

minimum at $\left(\frac{8\pi}{3}, \frac{16\pi}{3} - 4\sqrt{3}\right) \approx (8.4, 9.8)$ A1

correct tangent, on either side of the curve and passing through $(4\pi, 0)$ A1



e. $\cos\left(\frac{x}{2}\right) = -\frac{1}{2}$

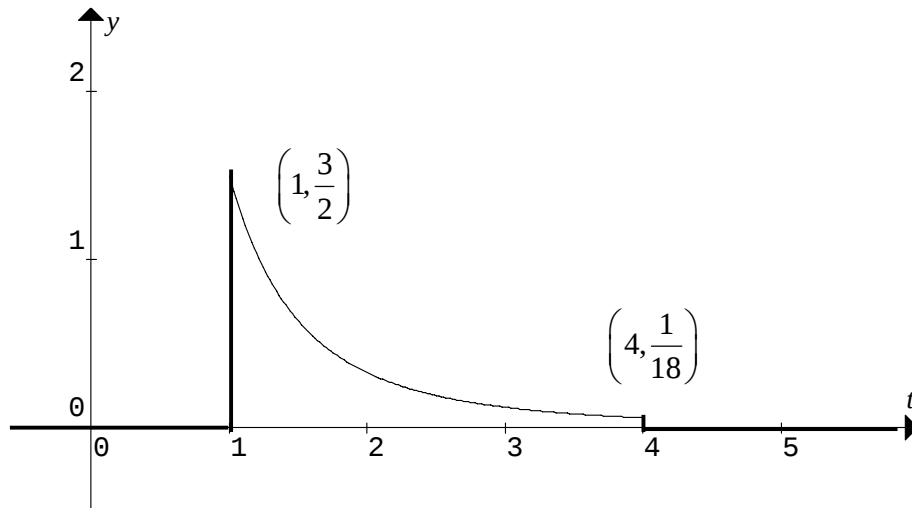
$$\frac{x}{2} = 2k\pi \pm \cos^{-1}\left(-\frac{1}{2}\right) = 2k\pi \pm \frac{2\pi}{3}$$

A1

$$x = 4k\pi \pm \frac{4\pi}{3} \quad \text{where } k \in J$$

Question 3

- a. correct graph and $y = 0$ for $(-\infty, 1]$ and $[4, \infty)$ A1



- b. want $\Pr(T > 3 | T > 2) = \frac{\Pr(T > 3)}{\Pr(T > 2)}$ M1

$$\Pr(T > 2) = \int_2^4 \frac{81}{2(2t+1)^3} dt = \frac{7}{25} \quad \text{A1}$$

$$\Pr(T > 3) = \int_3^4 \frac{81}{2(2t+1)^3} dt = \frac{4}{49}$$

$$\Pr(T > 3 | T > 2) = \frac{100}{343} \quad \text{A1}$$

- c. $Y \sim \text{Bi}\left(n=4, p=\frac{7}{25}\right)$
 $\Pr(Y \geq 1) = 1 - \Pr(Y = 0)$ M1

$$\Pr(Y \geq 1) = 1 - \left(1 - \frac{7}{25}\right)^4$$

$$\Pr(Y \geq 1) = 0.731 \quad \text{A1}$$

- d. $E(T) = \int_1^4 \frac{81t}{2(2t+1)^3} dt$ A1

$$E(T) = 1.75 \quad \text{A1}$$

e. $\int_1^m \frac{81}{2(2t+1)^3} dt = \frac{1}{2}$ A1

$$\left[-\frac{81}{8(2t+1)^2} \right]_1^m = \frac{1}{2}$$

$$\frac{1}{(2m+1)^2} - \frac{1}{9} = -\frac{4}{81} \quad \text{since } 1 < m < 4 \quad \text{M1}$$

$$m = \frac{1}{2} \left(\frac{9}{\sqrt{5}} - 1 \right)$$

$$m = 1.5 \text{ years} \quad \text{A1}$$

f. $X \sim N(\mu = ?, \sigma^2 = ?^2)$ times in months

$$\Pr(X < 25) = 0.18$$

$$\frac{25 - \mu}{\sigma} = -0.915 \quad \text{M1}$$

$$(1) \quad -0.915 \sigma = 25 - \mu$$

$$\Pr(X > 57) = 0.04$$

$$\frac{57 - \mu}{\sigma} = 1.75 \quad \text{M1}$$

$$(2) \quad 1.75 \sigma = 57 - \mu$$

now subtract equations (2) – (1)

$$2.665 \sigma = 32$$

$$\sigma = 12 \text{ months} \quad \text{A1}$$

$$\text{substituting gives } \mu = 57 - 1.75 \times 12 = 36 \text{ months} \quad \text{A1}$$

Question 4

a. we require $1 + \frac{x}{2} > 0 \Rightarrow x > -2$

domain $D = (-2, \infty)$ A1

b. $y = 3 \log_e \left(1 + \frac{x}{2} \right) \quad \frac{dy}{dx} = \frac{3}{2+x}$

since $\frac{dy}{dx} \neq 0 \Rightarrow$ no turning points A1

c. $f(x) = 3 \log_e \left(\frac{1}{2}(x+2) \right)$

• dilation by a factor of 3 parallel to the y-axis (or away from the x-axis) A1

• dilation by a factor of 2 parallel to the x-axis (or away from the y-axis) A1

• translation by 2 units to the left parallel to the x-axis A1
(or away from the y-axis)

d.i. $f(u-2) = 3 \log_e \left(1 + \frac{u-2}{2} \right) = 3 \log_e \left(\frac{u}{2} \right)$ and

$$f(v-2) = 3 \log_e \left(1 + \frac{v-2}{2} \right) = 3 \log_e \left(\frac{v}{2} \right)$$

$$f(u-2) + f(v-2) = 3 \log_e \left(\frac{u}{2} \right) + 3 \log_e \left(\frac{v}{2} \right) = 3 \log_e \left(\frac{uv}{4} \right) \quad \text{M1}$$

since $u > 0$ and $v > 0$

$$f(auv+b) = 3 \log_e \left(1 + \frac{auv+b}{2} \right) = 3 \log_e \left(\frac{uv}{4} \right)$$

$$1 + \frac{b}{2} + \frac{auv}{2} = \frac{uv}{4}$$

$$b = -2 \quad a = \frac{1}{2} \quad \text{A1}$$

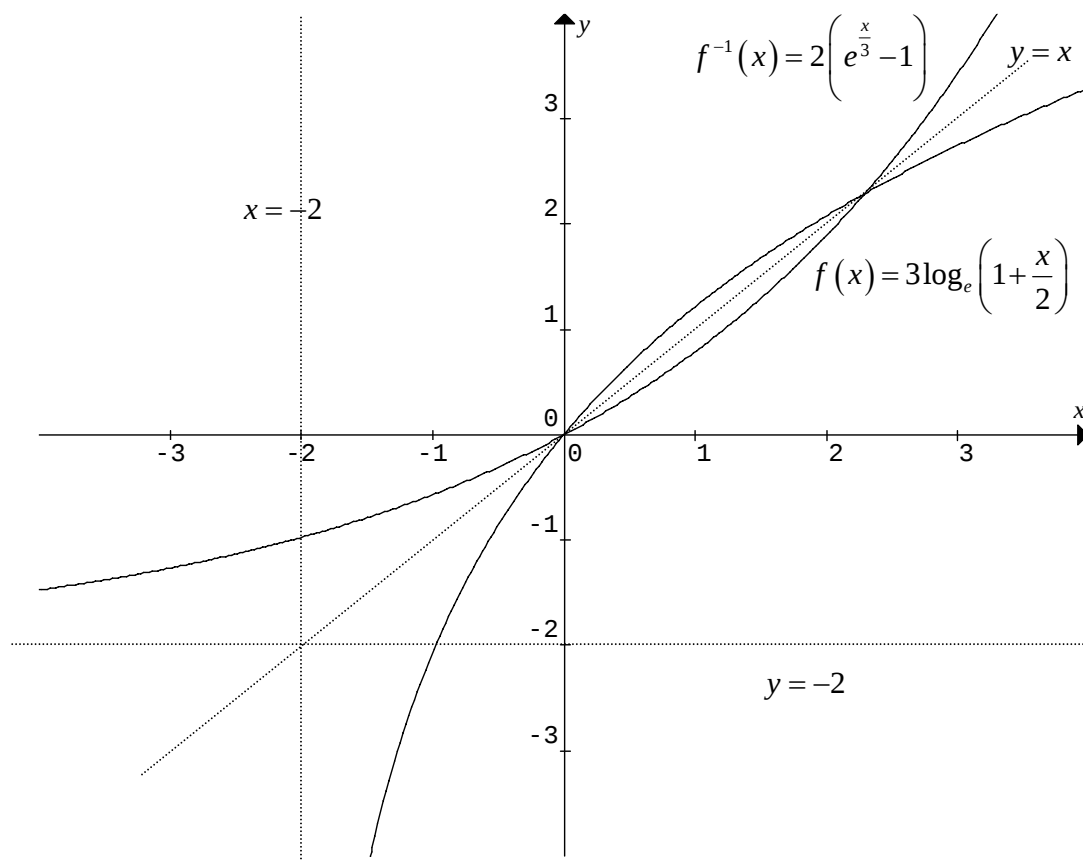
ii. $f(u) + f(-u) = 3 \log_e \left(1 + \frac{u}{2} \right) + 3 \log_e \left(1 - \frac{u}{2} \right)$

$$f(u) + f(-u) = 3 \log_e \left(\left(1 + \frac{u}{2} \right) \left(1 - \frac{u}{2} \right) \right) = 3 \log_e \left(1 - \frac{u^2}{4} \right) = f \left(-\frac{u^2}{4} \right) \quad \text{M1}$$

provided that $1 - \frac{u^2}{4} > 0$ or $u^2 < 4$

$$|u| < 2 \quad \text{or} \quad u \in (-2, 2) \quad \text{A1}$$

- e. $f \quad y = 3\log_e\left(1 + \frac{x}{2}\right)$ interchanging x and y
- $f^{-1} \quad x = 3\log_e\left(1 + \frac{y}{2}\right)$ rearranging for y M1
- $f^{-1}(x) = 2\left(e^{\frac{x}{3}} - 1\right)$ A1
- $\text{dom } f^{-1} = \text{ran } f = \mathbb{R}$ must give domain since a function is required
- f. both graphs pass through the origin $(0,0)$, the graphs are reflection in the line $y = x$
- the graph of f has $x = -2$ as a vertical asymptote A1
- the graph of f^{-1} has $y = -2$ as a horizontal asymptote A1



g. i. the coordinate is (2.288, 2.288)

since p satisfies $f^{-1}(x) = f(x) = x$ or

$$3\log_e\left(1 + \frac{p}{2}\right) = 2\left(e^{\frac{p}{3}} - 1\right) = p$$

so that $p = 2.288$

A1

g. ii. let the area bounded by the graph of f , the x -axis and the line $x = p$ be

$$A_1 = \int_0^p f(x) dx = \int_0^p 3\log_e\left(1 + \frac{x}{2}\right) dx$$

let the area bounded by the graph of f^{-1} , the x -axis and the line $x = p$ be

$$A_2 = \int_0^p f^{-1}(x) dx = \int_0^p 2\left(e^{\frac{x}{3}} - 1\right) dx$$

but $A_1 + A_2 = p^2$

A1

(the area of the square of side length p)

$$A_1 = p^2 - A_2 = p^2 - \int_0^p f^{-1}(x) dx$$

$$A_1 = p^2 - 2 \int_0^p \left(e^{\frac{x}{3}} - 1\right) dx$$

A1

$$A_1 = p^2 - 2 \left[\left(3e^{\frac{p}{3}} - p\right) - 3 \right]$$

$$A_1 = p^2 - 6e^{\frac{p}{3}} + 2p + 6$$

$$A_1 = p^2 - 3 \left(2 \left(e^{\frac{p}{3}} - 1 \right) \right) + 2p$$

A1

$$A_1 = p^2 - 3p + 2p$$

$$A_1 = p^2 - p$$

END OF SECTION 2 SUGGESTED ANSWERS