Year 2008

VCE

Mathematical Methods and Mathematical Methods (CAS)

Trial Examination 1

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Victorian Certificate of Education
2008

STUDENT NUMBER

										Letter
Figures										
Words										

MATHEMATICAL METHODS
AND
MATHEMATICAL METHOD (CAS)

Trial Written Examination 1

Reading time: 15 minutes
Total writing time: 1 hour

QUESTION AND ANSWER BOOK

Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
12	12	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers.
- Students are NOT permitted to bring into the examination room: notes of any kind, blank sheets of paper, white out liquid/tape or a calculator of any type.

Materials supplied

- Question and answer book of 14 pages with a detachable sheet of miscellaneous formulas at the end of this booklet.
- Working space is provided throughout the booklet.

Instructions

- Detach the formula sheet from the end of this book during reading time.
- Write your **student number** in the space provided above on this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Instructions

Answer **all** questions in the spaces provided.
A decimal approximation will not be accepted if an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working must be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1

Let $f(x)=\dfrac{\sin(3x)}{3x^2}$. Find $f'(x)$.

2 marks

Question 2

Solve the equation $\tan^2(x)+\left(1-\sqrt{3}\right)\tan(x)=\sqrt{3}$ for $x\in[0,2\pi]$, giving exact values in terms of π.

3 marks

Question 3

Given the function $f(x) = \begin{cases} x^2 & \text{for } x < 0 \\ \cos(x) & \text{for } 0 \le x < \pi \\ \pi - x & \text{for } x \ge \pi \end{cases}$

a. State the gradient function, $f'(x)$ clearly indicating its domain.

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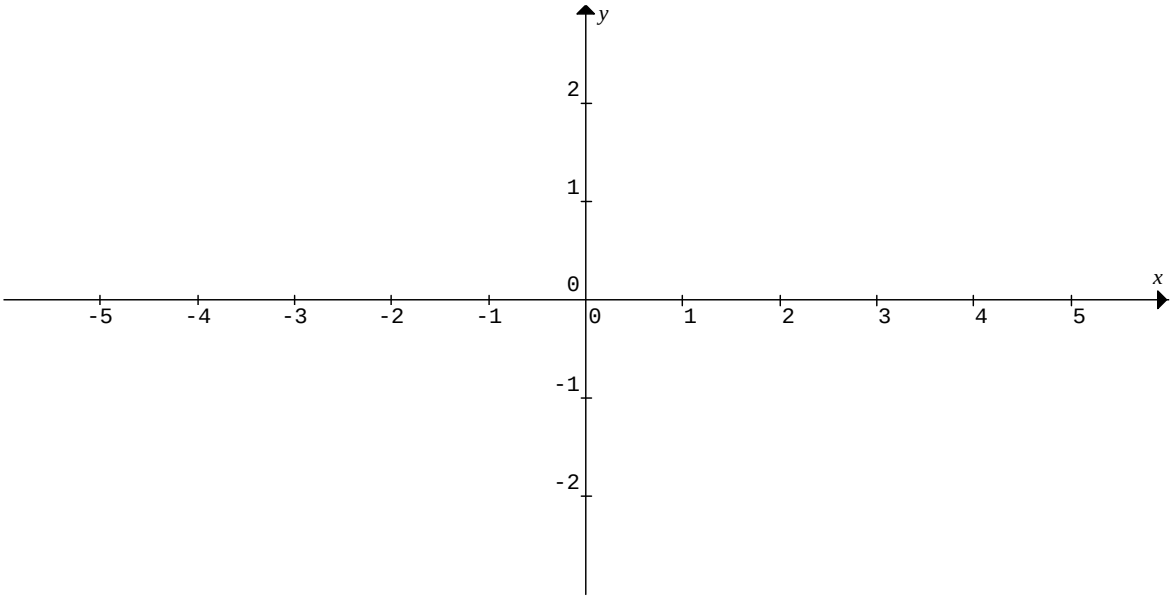
2 marks

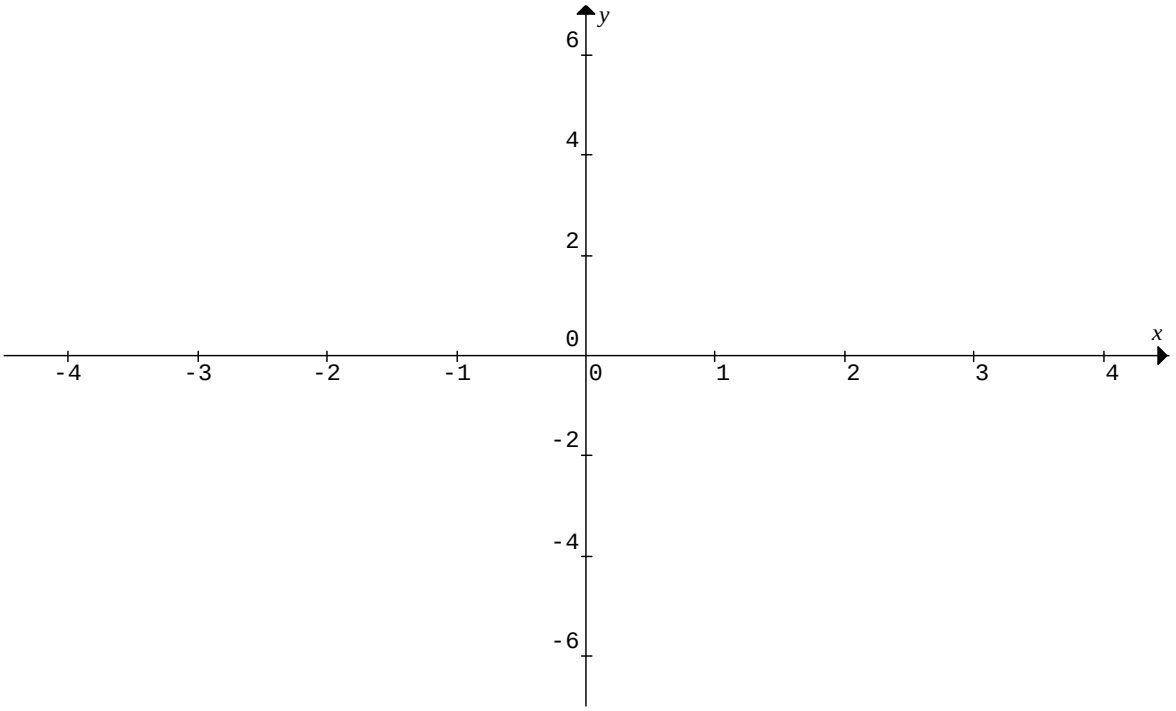
b. Sketch the graph of the gradient function, on the axes below.



2 marks

Question 4

Sketch the graph of the function $y = \left|x^2 - 4\right| - 5$, on the axes below, clearly showing the coordinates of all axial intercepts.



2 marks

Question 5

For the function $f:\{x:x<a\} \to R$, $f(x) = x^2 - 6x + 5$.

a. Find the largest value of the real number *a*, for which the function *f*, is a one-one decreasing function.

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1 mark

b. Find the inverse function f^{-1}.

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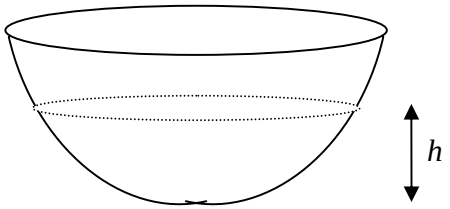
3 marks

Question 6

When a hemispherical bowl contains water to a depth of h cm, the volume V cm^3 of water in the bowl is given by $V = \frac{\pi h^2}{3}(120 - h)$



a. If the bowl is being filled with water at a rate of 200 cm^3 per minute, find the rate (in cm per minute) at which the depth of the water is increasing, when the depth is 10 cm.

2 marks

b. Find, using calculus, the approximate change in the volume of the water in the bowl, when the depth of the water in the bowl increases from 10 to 10.01 cm.

2 marks

Question 7

a. The probability density function of a continuous random variable X is given by

$$f(x) = \begin{cases} kx^2 & 0 \le x \le 2 \\ 0 & \text{otherwise} \end{cases}$$

i. Show that $k = \frac{3}{8}$.

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1 mark

ii. Find $E(X)$, giving your answer correct to one decimal place.

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1 mark

b. A discrete random variable X has a probability distribution given by

$p(x) = cx^2$, $x = 0, 1, 2$

i. Show that $c = \frac{1}{5}$.

1 mark

ii. Find $E(X)$, giving your answer correct to one decimal place.

1 mark

Question 8

Differentiate $x e^{-3x}$ and, hence, find $\int x e^{-3x}\, dx$.

3 marks

Question 9

Let $f(x) = \cos\left(\frac{1}{x}\right)$. Evaluate $f'\left(\frac{6}{\pi}\right)$.

3 marks

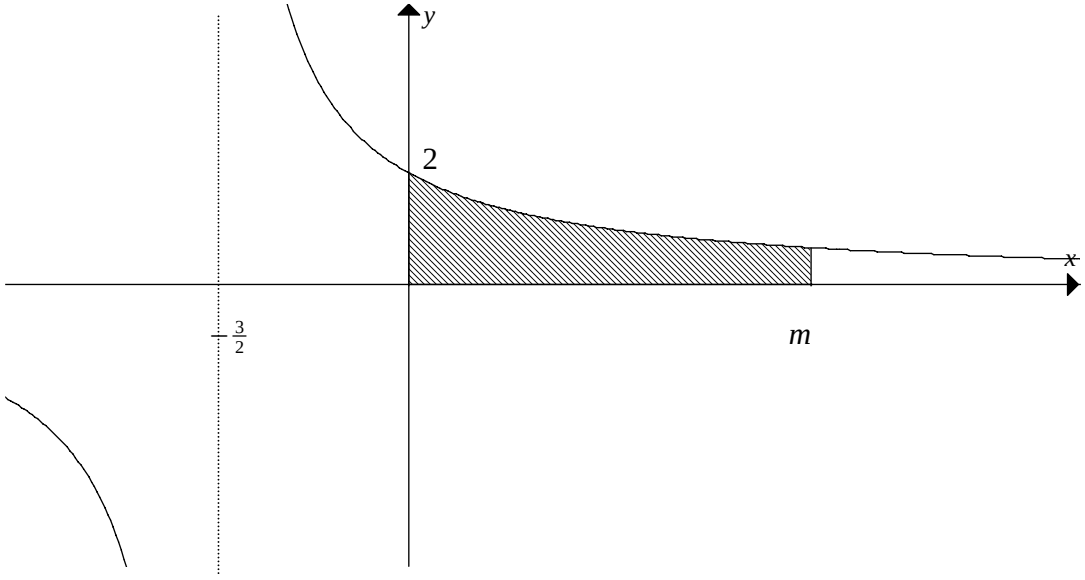
Question 10

The line $6y - x + d = 0$ is a normal to the curve $y = x^4 + px$ at the point $x = -1$.
Find the values of p and d.

4 marks

Question 11

Consider the graph of the function $f : R \setminus \{-\frac{3}{2}\} \to R,\ f(x) = \dfrac{b}{2x+a}$. The graph has a vertical asymptote at $x = -\dfrac{3}{2}$ and crosses the y-axis at $y = 2,$ as shown below.



The shaded area is the area bounded by the graph of $y = \dfrac{b}{2x+a}$, the coordinate axes and the line $x = m$. If the shaded area is equal to $\log_e(27)$ square units, find the values of *a*, *b* and *m*.

4 marks

Question 12

Ashley has either cereal or toast for breakfast every morning. If he has toast one morning, the probability he has cereal the next morning is 0.25. If he has cereal one morning, the probability he has toast the next morning is 0.4. Suppose he has toast on a Monday morning. What is the probability that from Monday to Wednesday inclusive he has toast for breakfast exactly twice?

3 marks

END OF QUESTION AND ANSWER BOOKLET

END OF EXAMINATION

MATHEMATICAL METHODS

Written examination 1

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time.
This formula sheet is provided for your reference.

Mathematical Methods and CAS Formulas

Mensuration

area of a trapezium: $\frac{1}{2}(a+b)h$

volume of a pyramid: $\frac{1}{3}Ah$

curved surface area of a cylinder: $2\pi rh$

volume of a sphere: $\frac{4}{3}\pi r^3$

volume of a cylinder: $\pi r^2 h$

area of triangle: $\frac{1}{2}bc\sin(A)$

volume of a cone: $\frac{1}{3}\pi r^2 h$

Calculus

$$\frac{d}{dx}\left(x^n\right) = nx^{n-1} \qquad \int x^n dx = \frac{1}{n+1}x^{n+1} + c \ , \ n \neq -1$$

$$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax} \qquad \int e^{ax} dx = \frac{1}{a}e^{ax} + c$$

$$\frac{d}{dx}\left(\log_e(x)\right) = \frac{1}{x} \qquad \int \frac{1}{x} dx = \log_e|x| + c$$

$$\frac{d}{dx}\left(\sin(ax)\right) = a\cos(ax) \qquad \int \sin(ax)\,dx = -\frac{1}{a}\cos(ax) + c$$

$$\frac{d}{dx}\left(\cos(ax)\right) = -a\sin(ax) \qquad \int \cos(ax)\,dx = \frac{1}{a}\sin(ax) + c$$

$$\frac{d}{dx}\left(\tan(ax)\right) = \frac{a}{\cos^2(ax)} = a\sec^2(ax)$$

product rule: $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$

quotient rule: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

Chain rule: $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$

approximation: $f(x+h) \approx f(x) + h\,f'(x)$

Probability

$\Pr(A) = 1 - \Pr(A')$

$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$

$\Pr(A/B) = \frac{\Pr(A \cap B)}{\Pr(B)}$

Mean: $\mu = E(X)$

variance: $\operatorname{var}(X) = \sigma^2 = E\left((X-\mu)^2\right) = E\left(X^2\right) - \mu^2$

probability distribution		mean	variance
discrete	$\Pr(X = x) = p(x)$	$\mu = E(X)$	$\sigma^2 = \sum (x-\mu)^2 p(x)$
continuous	$\Pr(a < X < b) = \int_a^b f(x)\,dx$	$\mu = \int_{-\infty}^{\infty} x\,f(x)\,dx$	$\sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)\,dx$