

Year 2008
VCE
Mathematical Methods
and
Mathematical Methods
(CAS)
Solutions
Trial Examination 1



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Question 1

$$y = \frac{\sin(3x)}{3x^2} \text{ differentiating using the quotient rule}$$

$$\text{let } u = \sin(3x) \quad v = 3x^2$$

$$\frac{du}{dx} = 3\cos(3x) \quad \frac{dv}{dx} = 6x \quad \text{M1}$$

$$\frac{dy}{dx} = \frac{9x^2 \cos(3x) - 6x \sin(3x)}{9x^4} = \frac{3x(3x \cos(3x) - 2 \sin(3x))}{9x^4}$$

$$\frac{dy}{dx} = \frac{1}{3x^3} (3x \cos(3x) - 2 \sin(3x)) \quad \text{A1}$$

Question 2

$$\tan^2(x) + (1 - \sqrt{3})\tan(x) - \sqrt{3} = 0$$

$$(\tan(x) - \sqrt{3})(\tan(x) + 1) = 0$$

$$\tan(x) = \sqrt{3} \quad \text{or} \quad \tan(x) = -1 \quad \text{A1}$$

$$x = \frac{\pi}{3}, \pi + \frac{\pi}{3} \quad \text{or} \quad x = \frac{3\pi}{4}, \pi + \frac{3\pi}{4} \quad \text{M1}$$

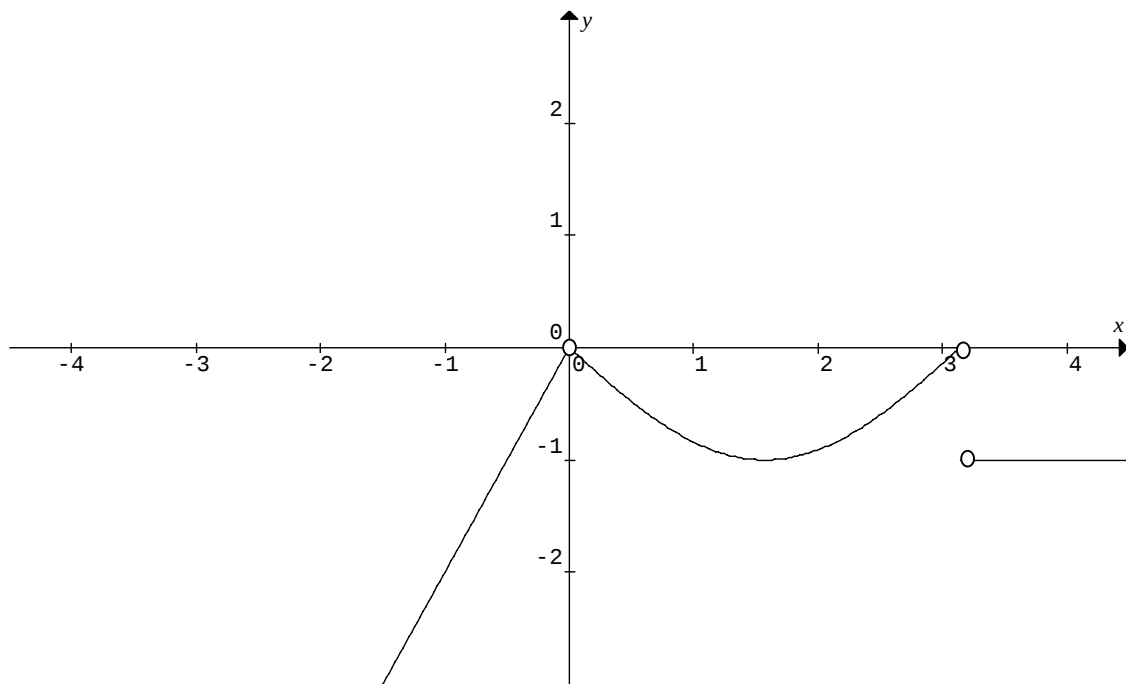
$$x = \frac{\pi}{3}, \frac{3\pi}{4}, \frac{4\pi}{3}, \frac{7\pi}{4} \quad \text{A1}$$

Question 3

$$\text{a.} \quad f'(x) = \begin{cases} 2x & \text{for } x < 0 \\ -\sin(x) & \text{for } 0 < x < \pi \\ -1 & \text{for } x > \pi \end{cases} \quad \text{A2}$$

$$\text{b.} \quad \text{note that the graph must have open circles at } x = 0 \text{ and } x = \pi$$

$$\text{the gradient function is not defined at } x = 0 \text{ or } x = \pi \quad \text{A2}$$

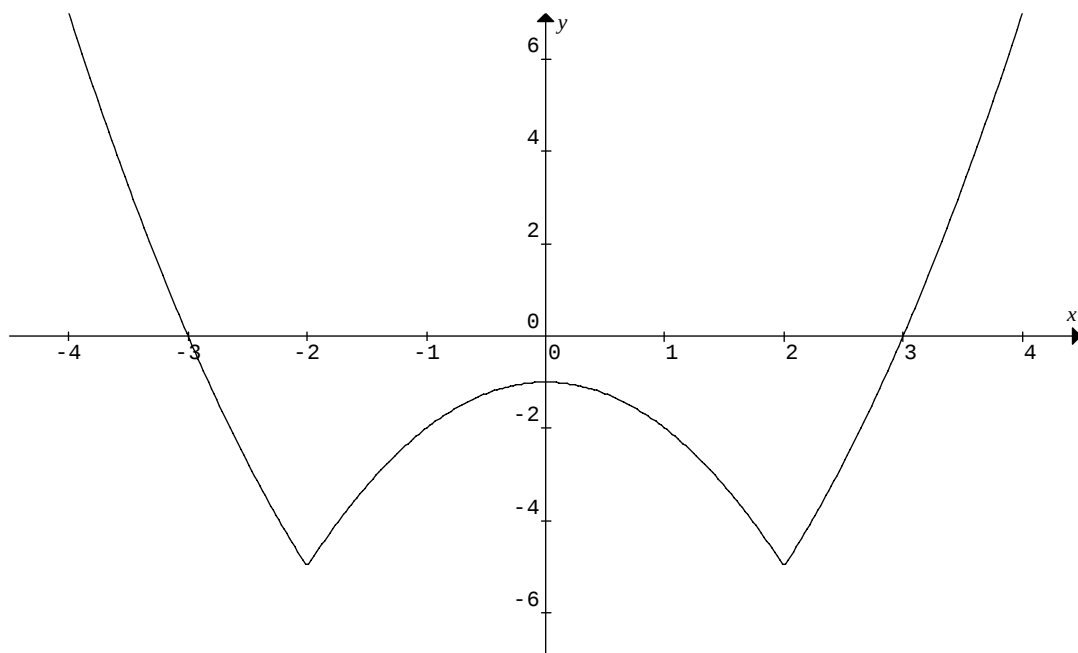


Question 4

$y = |x^2 - 4| - 5$, graph must pass through the y -intercept $(0, -1)$

and x -intercepts $(-3, 0)$ $(3, 0)$ and $(-2, -5)$, $(2, -5)$

A2



Question 5

- a.** $f(x) = x^2 - 6x + 5$
 $f(x) = (x^2 - 6x + 9) + 5 - 9$
 $f(x) = (x - 3)^2 - 4$
 for f to be a one-one decreasing function $a = 3$ A1
- b.** f $y = (x - 3)^2 - 4$ interchanging y and x
 f^{-1} $x = (y - 3)^2 - 4$ transposing to make y the subject M1
 f^{-1} $x + 4 = (y - 3)^2$
 f^{-1} $y - 3 = \pm\sqrt{x + 4}$
 the domain of f^{-1} , needs to be stated as we are asked for a function
 $\text{dom } f^{-1} = (-4, \infty)$ A1
 since $\text{ran } f^{-1} = \text{dom } f = (-\infty, 3)$, we need to take the negative in the square root
 $f^{-1}(x) = 3 - \sqrt{x + 4}$ A1

Question 6

- a.** Given $\frac{dV}{dt} = 200 \text{ cm}^3/\text{min}$ and $V = \frac{\pi}{3}(120h^2 - h^3)$
 $\frac{dV}{dh} = \frac{\pi}{3}(240h - 3h^2) = \pi(80h - h^2)$ A1
 By the chain rule $\frac{dh}{dt} = \frac{dh}{dV} \frac{dV}{dt} = \frac{200}{\pi(80h - h^2)}$
 when $h = 10$ $\left. \frac{dh}{dt} \right|_{h=10} = \frac{200}{\pi(800 - 100)} = \frac{2}{7\pi} \text{ cm/min}$ A1
- b.** when $h = 10$ $\Delta h = 0.01$ find ΔV
 $\frac{dV}{dh} = \pi(80h - h^2) \approx \frac{\Delta V}{\Delta h}$ M1
 $\Delta V \approx \pi(80h - h^2)\Delta h = \pi(800 - 100) \times 0.01 = 7\pi \text{ cm}^3$
 the volume increases by $7\pi \text{ cm}^3$ A1

Question 7

a.i. $f(x) = \begin{cases} kx^2 & 0 \leq x \leq 2 \\ 0 & \text{otherwise} \end{cases}$

$$\int_0^2 kx^2 dx = 1 \quad k \left[\frac{x^3}{3} \right]_0^2 = 1$$

M1

$$k \left[\frac{2^3}{3} - 0 \right] = 1$$

$$\frac{8k}{3} = 1$$

$$k = \frac{3}{8}$$

ii. $E(X) = \int_a^b x f(x) dx = \int_0^2 kx^3 dx$

$$E(X) = \frac{3}{8} \left[\frac{x^4}{4} \right]_0^2 = \frac{3}{32} (16 - 0)$$

$$E(X) = 1.5$$

A1

b.i. $p(x) = cx^2 \quad x = 0, 1, 2$

X	0	1	2
$\Pr(X = x)$	0	c	$4c$

Since $\sum \Pr(X = x) = 5c = 1$

$$c = \frac{1}{5}$$

A1

ii. $E(X) = \sum x \Pr(X = x) = 0 + c + 8c = 9c$

$$E(X) = \frac{9}{5} = 1.8$$

A1

Question 8

$$\frac{d}{dx}(xe^{-3x}) = e^{-3x} - 3xe^{-3x} \quad \text{product rule} \quad \text{A1}$$

$$\text{hence } \int (e^{-3x} - 3xe^{-3x}) dx = xe^{-3x} \quad \text{M1}$$

$$\begin{aligned} \int e^{-3x} dx - 3 \int xe^{-3x} dx &= xe^{-3x} \\ -3 \int xe^{-3x} dx &= xe^{-3x} - \int e^{-3x} dx = xe^{-3x} + \frac{1}{3}e^{-3x} \\ \int xe^{-3x} dx &= -\frac{1}{3}xe^{-3x} - \frac{1}{9}e^{-3x} + C = -\frac{e^{-3x}}{9}(3x+1) + C \quad \text{A1} \end{aligned}$$

Question 9

$$\text{let } f(x) = y = \cos\left(\frac{1}{x}\right) = \cos(u) \quad u = \frac{1}{x} = x^{-1} \quad \text{chain rule}$$

$$\frac{dy}{du} = -\sin(u) \quad \frac{du}{dx} = -x^{-2} = -\frac{1}{x^2}$$

$$\frac{dy}{dx} = \frac{1}{x^2} \sin\left(\frac{1}{x}\right) \quad \text{A1}$$

$$\frac{dy}{dx} \text{ when } x = \frac{6}{\pi} \quad \left. \frac{dy}{dx} \right|_{x=\frac{6}{\pi}} = f'\left(\frac{6}{\pi}\right) = \frac{1}{\left(\frac{6}{\pi}\right)^2} \sin\left(\frac{\pi}{6}\right) = \frac{\pi^2}{36} \times \frac{1}{2} \quad \text{M1}$$

$$f'\left(\frac{6}{\pi}\right) = \frac{\pi^2}{72} \quad \text{A1}$$

Question 10

The line $6y - x + d = 0$ $6y = x - d$ $y = \frac{x}{6} - \frac{d}{6}$

has a gradient of $\frac{1}{6}$ so $m_N = \frac{1}{6}$ so the tangent has a gradient of $m_T = -6$ A1

$$y = x^4 + px \Rightarrow \frac{dy}{dx} = 4x^3 + p = -6 \text{ at } x = -1$$
M1

$$4(-1)^3 + p = -4 + p = -6 \text{ so that } p = -2$$

The curve is $y = x^4 - 2x$ at the point $x = -1$

$$y(-1) = (-1)^4 - p = 1 + 2 = 3$$
M1

the point $P(-1, 3)$ is also on the line

$$6y - x + d = 0 \quad 18 + 1 + d = 0$$

$$\text{so } d = -19$$
A1

Question 11

since $x = -\frac{3}{2}$ is a vertical asymptote, the denominator is $2x + 3$, so that $a = 3$

since the y-intercept is $y = 2 = \frac{b}{a}$, it follows that $b = 6$ A1

$$\text{The area is } \int_0^m \frac{6}{2x+3} dx = \log_e(27)$$
A1

$$= 3[\log_e(2x+3)]_0^m = 3(\log_e(2m+3) - \log_e(3))$$
M1

$$= 3\log_e\left(\frac{2m+3}{3}\right) = \log_e(27) = 3\log_e(3)$$

$$\frac{2m+3}{3} = 3 \Rightarrow 2m+3 = 9 \Rightarrow 2m = 6$$

$$m = 3$$
A1

Question 12

$\Pr(\text{Toast on two days})$

$$= T T C \text{ or } T C T$$
M1

$$= 0.75 \times 0.25 + 0.25 \times 0.4$$

$$= \frac{3}{4} \times \frac{1}{4} + \frac{1}{4} \times \frac{2}{5} = \frac{3}{16} + \frac{1}{10} = \frac{15+8}{80}$$
A1

$$= \frac{23}{80}$$
A1

END OF SUGGESTED SOLUTIONS