

2008 VCAA Mathematical Methods Exam 2 Solutions

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SECTION 1: Multiple-choice questions

1	2	3	4	5	6	7	8	9	10	11
E	C	D	A	E	A	D	D	D	C	D

12	13	14	15	16	17	18	19	20	21	22
B	C	D	E	C	B	E	B	B	D	C

Q1 The quotient rule:

$$\frac{d}{dx} \left(\frac{\log_e(2x)}{2x} \right) = \frac{(2x) \left(\frac{1}{x} \right) - 2 \log_e(2x)}{4x^2} = \frac{1 - \log_e(2x)}{2x^2} \quad \text{E}$$

Q2 Translate $y = |x|$ to the left by 2 units and down by 2 units to obtain $y = |x+2| - 2$. C

Q3 $3 \log_e(2x-3) = 6$, $\log_e(2x-3) = 2$, $2x-3 = e^2$,
 $x = \frac{1}{2}(e^2 + 3)$. D

Q4 $\int_1^3 (2f(x) - 3) dx = 2 \int_1^3 f(x) dx - \int_1^3 3 dx = 2 \times 5 - [3x]_1^3 = 4$ A

Q5 $\mu = np = 1.2$, $\sigma^2 = np(1-p)$, $0.72 = 1.2(1-p)$,
 $p = 0.4$ and $n = 3$. E

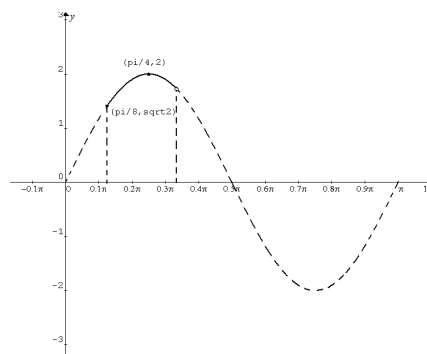
Q6 $\int \left(e^{3(x-2)} + \frac{2}{2-x} \right) dx = \frac{1}{3} e^{3(x-2)} - 2 \log_e |2-x| + c$
 $= \frac{1}{3} e^{3(x-2)} - 2 \log_e |x-2| + c$ A

Q7 $y = \frac{1}{\sqrt{x}} - 3$, inverse is $x = \frac{1}{\sqrt{y}} - 3$, $\sqrt{y} = \frac{1}{x+3}$,
 $y = \frac{1}{(x+3)^2}$, $f^{-1}(x) = \frac{1}{(x+3)^2}$. D

Q8 $f(x) = \frac{x-3}{2-x} = -\left(\frac{x-3}{x-2} \right) = -\left(\frac{x-2-1}{x-2} \right) = -\left(1 - \frac{1}{x-2} \right)$
 $= \frac{1}{x-2} - 1$. Asymptotes: $x = 2$, $y = -1$. D

Q9 $\int_1^3 \left(6x^2 + \frac{3a}{x^2} \right) dx = \left[2x^3 - \frac{3a}{x} \right]_1^3 = (54-a) - (2-3a)$
 $= 52 + 2a$. D

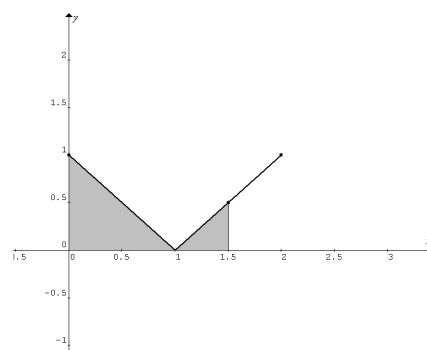
Q10



The range is $[\sqrt{2}, 2]$.

C

Q11



$\Pr(X < 1.5) = \frac{1}{2} \times 1 \times 1 + \frac{1}{2} \times 0.5 \times 0.5 = 0.625$ D

Q12 The quotient rule:
 $f'(x) = \frac{2\pi g(x) \cos(2\pi x) - \sin(2\pi x) g'(x)}{[g(x)]^2}$ B

Q13 Binomial distribution: $n = 10$, $p = 0.30$.
 $\Pr(X \geq 7) = 1 - \Pr(X \leq 6) = 1 - \text{binomcdf}(10, 0.30, 6) = 0.0106$ C

Q14 $\Pr(\text{all heads}) < 0.0005$, $0.5^n < 0.0005$,
 $n > \frac{\log_e 0.0005}{\log_e 0.5} = 10.97$, $\therefore$ minimum n is 11. D

Note: $<$ is changed to $>$ because $\log_e 0.5$ has a negative value.

Q15 $\Pr(\{1, 2\} \cap \{2, 4, 6\}) = \Pr(\{2\}) = \frac{1}{6}$,
 $\Pr(\{1, 2\}) \Pr(\{2, 4, 6\}) = \frac{2}{6} \times \frac{3}{6} = \frac{1}{6}$.
 $\therefore \{1, 2\}$ and $\{2, 4, 6\}$ are independent. E

Q16 $V = \pi r^2 h = \pi 2^2 h = 4\pi h$, $\frac{dV}{dt} = 4\pi \frac{dh}{dt}$,
 $\therefore \frac{dh}{dt} = \frac{1}{4\pi} \times \frac{dV}{dt} = \frac{1}{4\pi} \times 2 = \frac{1}{2\pi}$. C

Q17 $e^{2x} - 2 = e^x$, $e^{2x} - e^x - 2 = 0$, $(e^x)^2 - e^x - 2 = 0$,
 $(e^x - 2)(e^x + 1) = 0$.

Since $e^x + 1 > 0$, $\therefore e^x - 2 = 0$, $x = \log_e 2$.

B

Q18 $\sin(4x) + 1 \rightarrow -(\sin(4x) + 1) \rightarrow -\left(\sin 4\left(\frac{x}{4}\right) + 1\right) = -\sin x - 1$.

The domain of the transformed function is $\left[0, \frac{\pi}{2} \times 4\right]$,

i.e. $[0, 2\pi]$.

E

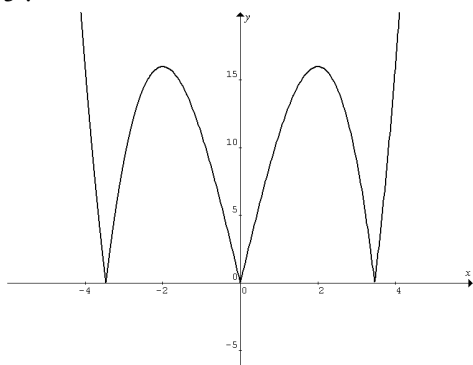
Q19 The gradient of the antiderivative graph B is the graph of function f .

B

Q20 Domain $B = \left(\frac{1}{2}, \infty\right)$ makes f a one-to-one function for it to have an inverse function.

B

Q21 $y = x^3 - 12x = x(x^2 - 12) = x(x - 2\sqrt{3})(x + 2\sqrt{3})$,
 $x = 0, \pm 2\sqrt{3}$.



Positive gradient: $x \in (-2\sqrt{3}, -2) \cup (0, 2) \cup (2\sqrt{3}, \infty)$

D

Q22 For $x = 2$, $f(x) \neq 0$.

C

SECTION 2:

Q1ai $\Pr(SSSSSSSS) = (\Pr(S))^8 = 0.80^8 \approx 0.1678$

Q1aii Binomial: $n = 8$, $p = 0.80$,

$\Pr(X = 6) = \text{binompdf}(8, 0.80, 6) = 0.2936$.

Q1aiii Conditional probability:

Let A be the event that the first 4 are successful, and B the event that exactly 6 of the first 8 are successful.

$\Pr(A|B) = \frac{\Pr(A \cap B)}{\Pr(B)} = \frac{0.8^4 \times \text{binompdf}(4, 0.8, 2)}{0.2936} \approx 0.214$.

Q1bi

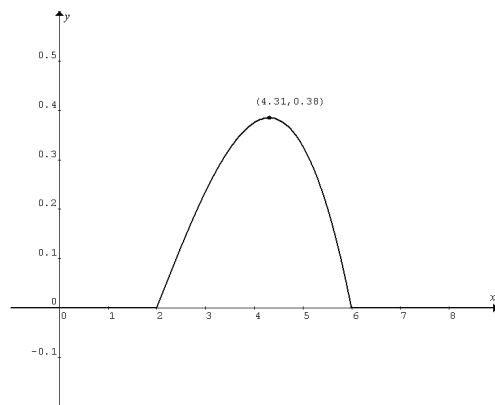
$S \xrightarrow{0.84} S$, $S \xrightarrow{0.16} S'$, $S' \xrightarrow{0.64} S$, $S' \xrightarrow{0.36} S'$.

$\Pr(SSSSSSS) = 0.84^7 = 0.2951$.

Q1bii $\Pr(2 \text{ of next } 3) = \Pr(SSSS') + \Pr(SSS'S) + \Pr(SS'SS)$
 $= 0.84 \times 0.84 \times 0.16 + 0.84 \times 0.16 \times 0.64 + 0.16 \times 0.64 \times 0.84 = 0.2849$

Q1ci $y = f(x) = \begin{cases} \frac{1}{64}(6-x)(x-2)(x+2) & \text{if } 2 \leq x \leq 6 \\ 0 & \text{elsewhere} \end{cases}$

Local maximum $(4.31, 0.38)$, graphics calculator.



Q1cii $\Pr(X < 3) = \int_2^3 f(x) dx = 0.1211$, graphics calculator.

Q1ciii Mean time $= \int_2^6 xf(x) dx = 4.1333$, graphics calculator.

Q2ai $f(1) = 7$, $f(a) = \frac{7}{a}$.

Gradient of CA $= \frac{\frac{7}{a} - 7}{a - 1} = \frac{7(1-a)}{a(a-1)} = -\frac{7}{a}$.

Q2aii Gradient of tangent $= f'(x) = -\frac{7}{x^2} = -\frac{7}{a}$ for $x > 0$.

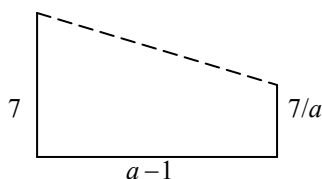
$\therefore x^2 = a$, $x = \sqrt{a}$.

Q2bi $\int_1^e \frac{7}{x} dx = [7 \log_e x]_1^e = 7 \log_e e - 7 \log_e 1 = 7$

Q2bii $\int_b^1 \frac{7}{x} dx = [7 \log_e x]_b^1 = 7 \log_e 1 - 7 \log_e b = -7 \log_e b = 7$.

$\therefore \log_e b = -1$, $b = e^{-1} = \frac{1}{e}$.

Q2ci



$$\text{Area of trapezium} = \frac{1}{2} \left(7 + \frac{7}{a} \right) (a-1) = \frac{7}{2} \left(1 + \frac{1}{a} \right) (a-1).$$

$$\text{Q2cii } \frac{7}{2} \left(1 + \frac{1}{a} \right) (a-1) = 7, \quad \left(1 + \frac{1}{a} \right) (a-1) = 2.$$

Expand and simplify to $a^2 - 2a - 1 = 0$, where $a > 1$.

Use the quadratic formula to find $a = 1 + \sqrt{2}$.

Q2ciii $\int_1^a f(x) dx < 7$, i.e. less than the area of the trapezium, because the function $f(x)$ is below the line CA between $x = 1$ and $x = 1 + \sqrt{2}$.

$$\text{From Q2bi, } \int_1^e f(x) dx = 7, \quad \int_1^a f(x) dx < \int_1^e f(x) dx, \quad \therefore a < e.$$

$$\text{Q3a } 50 \log_e(1+2t) < 100, \quad \log_e(1+2t) < 2, \quad 1+2t < e^2,$$

$$t < \frac{1}{2}(e^2 - 1) \approx 3.19453 \text{ hours}$$

or 3 hours and 12 minutes (to the nearest minute as required by the question). Tasmania would be killed by then.

$$\text{Q3b Time required} = \frac{18}{5} = 3.6 \text{ hours} > 3.19453 \text{ hours.}$$

$$\text{Q3c } NY = XM = \sqrt{3^2 + x^2} = \sqrt{9 + x^2}.$$

$$T = \frac{2\sqrt{9+x^2}}{5} + \frac{18-2x}{13} = 2 \left(\frac{\sqrt{9+x^2}}{5} + \frac{9-x}{13} \right).$$

$$\text{Q3d } \frac{dT}{dx} = 2 \left(\frac{x}{5\sqrt{9+x^2}} - \frac{1}{13} \right) = 0 \text{ for } x > 0,$$

$$\frac{x}{\sqrt{9+x^2}} = \frac{5}{13}, \quad \frac{x^2}{9+x^2} = \frac{25}{169}, \quad 144x^2 = 225, \quad \therefore x = \frac{5}{4}.$$

$$\text{Q3e Minimum time} = 2 \left(\frac{\sqrt{9 + \left(\frac{25}{16}\right)^2}}{5} + \frac{9 - \frac{5}{4}}{13} \right) = 2.4923 < 3.19453$$

$$\text{Q3f Curve AB: } z = \frac{16}{d+1}.$$

$$\text{Point A: } d = 0, \quad z = 16, \quad (0, 16).$$

$$\text{Point B: } d = 1, \quad z = 8.$$

$$\text{Point C: } d = 1, \quad z = 8 + 16 = 24, \quad (1, 24).$$

Q3g Curve CD: Curve AB is translated to the right by 1 unit and upwards by 8 units. $z = \frac{16}{(d-1)+1} + 8,$

$$\text{i.e. } z = \frac{16}{d} + 8, \text{ where } d \in [1, 2].$$

Q3h

Day	z
1	16 to 8
2	24 to 16
3	32 to 24
4	40 to 32
5	48 to 40
6	56 to 48

$\therefore 6$ days.

$$\text{Q4ai } f(x) = \tan\left(\frac{x}{2}\right), \quad f'(x) = \frac{1}{2} \sec^2\left(\frac{x}{2}\right),$$

$$f'\left(\frac{\pi}{2}\right) = \frac{1}{2} \sec^2\left(\frac{\pi}{4}\right) = 1.$$

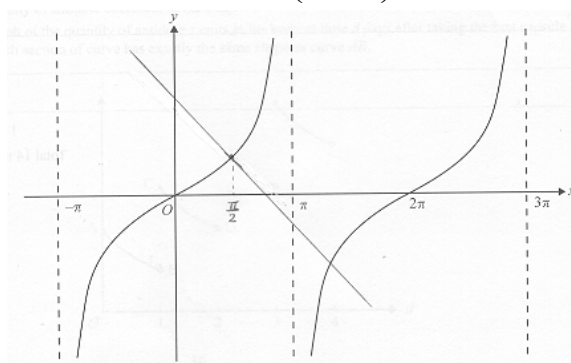
$$\text{Q4aii At } x = \frac{\pi}{2}, \quad y = \tan\left(\frac{\pi}{4}\right) = 1, \quad \left(\frac{\pi}{2}, 1\right).$$

Gradient of the normal = -1.

$$\text{Equation: } y - 1 = -1 \left(x - \frac{\pi}{2} \right), \quad y = -x + \frac{\pi}{2} + 1.$$

$$\text{Q4aiii } x\text{-intercepts: } y = 0, \quad x = \frac{\pi}{2} + 1, \quad \left(\frac{\pi}{2} + 1, 0\right).$$

$$y\text{-intercepts: } x = 0, \quad y = \frac{\pi}{2} + 1, \quad \left(0, \frac{\pi}{2} + 1\right).$$



Q4b $f'(x) = f'\left(\frac{\pi}{2}\right)$ when $x = -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}$.

Q4c $-1 < a < 1$, $0 < \frac{1-a}{2} < 1$ and

$$g(1) = f(1-a) = \tan\left(\frac{1-a}{2}\right) = 1.$$

$$\therefore \frac{1-a}{2} = \frac{\pi}{4}, a = 1 - \frac{\pi}{2}.$$

Q4di $h(x) = \sin\left(\frac{x}{2}\right) + \tan\left(\frac{x}{2}\right) + 2$,

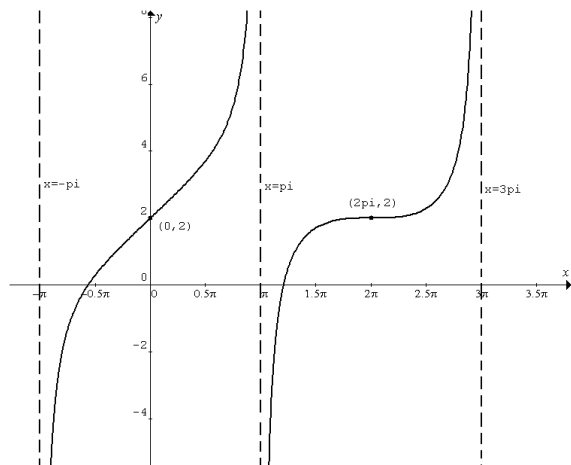
$$h'(x) = \frac{1}{2} \left(\cos\left(\frac{x}{2}\right) + \sec^2\left(\frac{x}{2}\right) \right).$$

Q4dii $h'(x) = \frac{1}{2} \left(\cos\left(\frac{x}{2}\right) + \sec^2\left(\frac{x}{2}\right) \right) = 0$,

$$\cos\left(\frac{x}{2}\right) + \sec^2\left(\frac{x}{2}\right) = 0, \cos\left(\frac{x}{2}\right) + \frac{1}{\cos^2\left(\frac{x}{2}\right)} = 0,$$

$$\cos^3\left(\frac{x}{2}\right) = -1, \therefore \cos\left(\frac{x}{2}\right) = -1, \frac{x}{2} = \pi, x = 2\pi.$$

Q4e



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