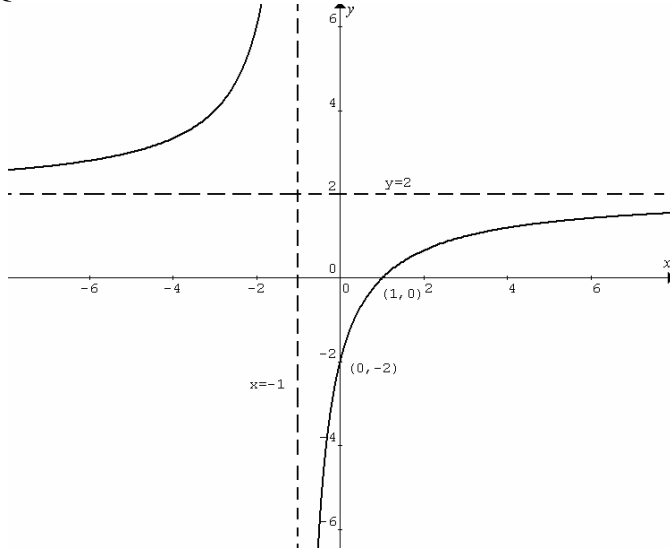


Q1a The chain rule: $\frac{dy}{dx} = 5(3x^2 - 5x)^4 (6x - 5)$

Q1b The product rule: $f'(x) = 3xe^{3x} + e^{3x} = e^{3x}(3x + 1)$,
 $f'(0) = e^0(1) = 1$

Q2



Q3 $\cos\left(\frac{3x}{2}\right) = \frac{1}{2}$, $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$, $\therefore -\frac{3\pi}{4} \leq \frac{3x}{2} \leq \frac{3\pi}{4}$.
 $\therefore \frac{3x}{2} = -\frac{\pi}{3}, \frac{\pi}{3}$, $\therefore x = -\frac{2\pi}{9}, \frac{2\pi}{9}$.

Q4a $\int_{-\infty}^{\infty} f(x)dx = 1$, $\therefore \int_0^1 k \sin(\pi x)dx = 1$, $\left[\frac{-k \cos(\pi x)}{\pi}\right]_0^1 = 1$,
 $\therefore \frac{-k \cos \pi}{\pi} + \frac{k \cos 0}{\pi} = 1$, $\frac{k}{\pi} + \frac{k}{\pi} = 1$, $\frac{2k}{\pi} = 1$, $\therefore k = \frac{\pi}{2}$.

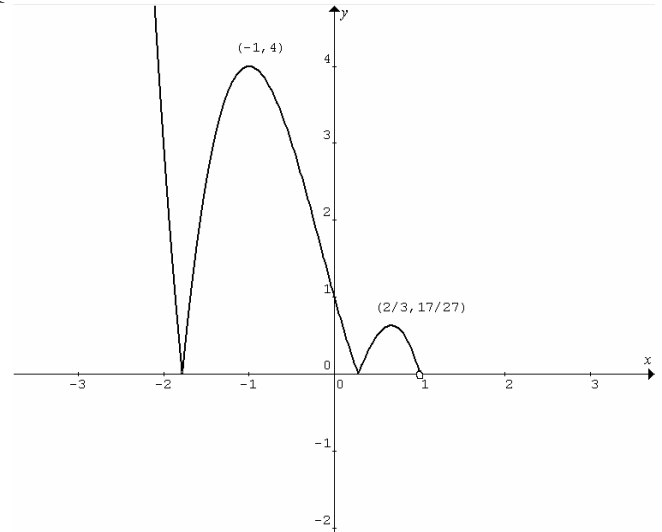
Q4b $\Pr\left(X \leq \frac{1}{4} \mid X \leq \frac{1}{2}\right) = \frac{\Pr\left(X \leq \frac{1}{4} \cap X \leq \frac{1}{2}\right)}{\Pr\left(X \leq \frac{1}{2}\right)} = \frac{\Pr\left(X \leq \frac{1}{4}\right)}{\Pr\left(X \leq \frac{1}{2}\right)}$

$= \frac{\int_{-\frac{1}{2}}^{\frac{1}{4}} \frac{\pi}{2} \sin(\pi x)dx}{\int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{\pi}{2} \sin(\pi x)dx} = 1 - \frac{\sqrt{2}}{2}$.

Q5 $\int_0^C e^{2x} dx = \frac{5}{2}$, $\left[\frac{e^{2x}}{2}\right]_0^C = \frac{5}{2}$, $\frac{e^{2C}}{2} - \frac{1}{2} = \frac{5}{2}$, $\therefore e^{2C} = 6$,
 $C = \frac{1}{2} \log_e 6$ or $\log_e \sqrt{6}$.

Q6a Domain of f' is $(-\infty, 1) \cup (1, 2) \cup (2, \infty)$.

Q6b



Q7a The mode of X is 3.

Q7b $\Pr = 0.1^2 + 0.2^2 + 0.3^2 + 0.4^2 = 0.3$

Q8 $\Pr(DCC) + \Pr(CDC) + \Pr(CCD)$
 $= 0.6 \times 0.5 \times 0.4 + 0.4 \times 0.6 \times 0.5 + 0.4 \times 0.4 \times 0.6 = 0.336$

Q9a Area of equilateral triangle $= \frac{1}{2} x^2 \sin 60^\circ = \frac{\sqrt{3}}{4} x^2$.
 $\therefore V = \frac{\sqrt{3}}{4} x^2 y = 1000$. Hence $y = \frac{4000}{x^2 \sqrt{3}}$.

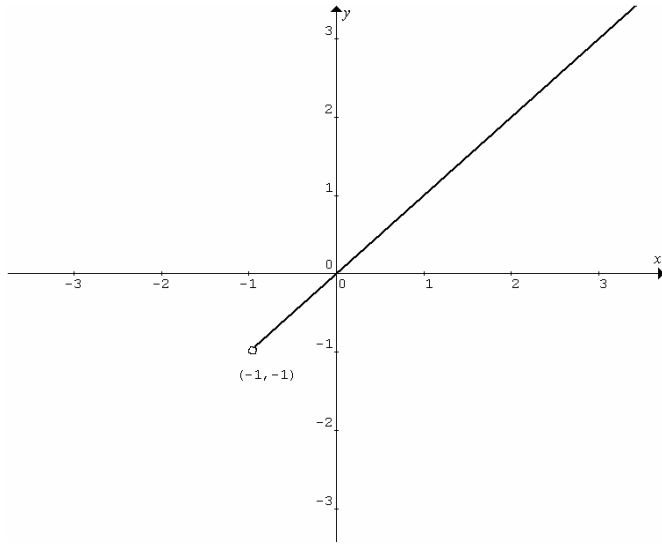
Q9b Total surface area $A = 3xy + 2\left(\frac{\sqrt{3}}{4} x^2\right) = \frac{4000\sqrt{3}}{x} + \frac{\sqrt{3}x^2}{2}$

Q9c Let $\frac{dA}{dx} = -\frac{4000\sqrt{3}}{x^2} + \sqrt{3}x = 0$,
 $\therefore x^3 = 4000$, $x = 10 \times \sqrt[3]{4}$ or $10 \times 2^{\frac{2}{3}}$.

Q10a The range of f is $(-1, \infty)$, it becomes the domain of f^{-1} .
The equation of f is $y = e^{2x} - 1$, the equation of f^{-1} is
 $x = e^{2y} - 1$. $\therefore e^{2y} = x + 1$, $y = \frac{1}{2} \log_e (x + 1)$.

Hence $f^{-1}(x) = \frac{1}{2} \log_e (x + 1)$, $x \in (-1, \infty)$.

Q10b $y = f(f^{-1}(x)) = x, x \in (-1, \infty).$



Q10c $f^{-1}(x) = \frac{1}{2} \log_e(x+1), -f^{-1}(2x) = -\frac{1}{2} \log_e(2x+1),$

$$f(x) = e^{2x} - 1, f(-f^{-1}(2x)) = e^{2\left(-\frac{1}{2} \log_e(2x+1)\right)} - 1$$

$$= e^{-\log_e(2x+1)} - 1 = (2x+1)^{-1} - 1 = \frac{1}{2x+1} - 1 = \frac{-2x}{2x+1}.$$

Please inform mathline@itute.com re conceptual, mathematical and/or typing errors