

SECTION 1

1	2	3	4	5	6	7	8	9	10	11
C	A	E	B	D	D	E	D	B	D	A

12	13	14	15	16	17	18	19	20	21	22
B	E	A	B	A	A	C	A	A	D	C

Q1 $f(x) = \tan\left(\frac{x}{2}\right)$ is defined over D . C

Q2 $b = 2\log_2\left(\frac{a}{2}\right)$, $\frac{b}{2} = \log_2\left(\frac{a}{2}\right)$, $\frac{a}{2} = 2^{\frac{b}{2}}$,
 $a = 2^{\frac{b}{2}} \times 2 = 2^{\frac{b}{2}+1} = 2^{\frac{1}{2}(b+2)} = (e^{\log_e 2})^{\frac{1}{2}(b+2)} = e^{\frac{1}{2}(b+2)\log_e 2}$. A

Q3 $e^{2x} - 2e^x + k = 0$, $(e^x)^2 - 2(e^x) + k = 0$,
 $e^x = \frac{2 \pm \sqrt{4-4k}}{2} = 1 \pm \sqrt{1-k}$.

Two solutions exist when $1 - \sqrt{1-k} > 0$ and $1 - k > 0$.
 $\therefore 1 > \sqrt{1-k}$ and $k < 1$, i.e. $1 > 1-k$ and $k < 1$.
Hence $0 < k < 1$. E

Q4 $f(x) = 2\cos(3x)$, $\frac{1}{4}f\left(\frac{\pi}{6} - \frac{1}{3}x\right) = \frac{1}{4}\left(2\cos\left(3\left(\frac{\pi}{6} - \frac{1}{3}x\right)\right)\right)$,
 $= \frac{1}{2}\cos\left(\frac{\pi}{2} - x\right) = \frac{1}{2}\sin x$. B

Q5 The graph is part of a circle centred at $(0,0)$ and with a radius of 1 unit.
 $x^2 + y^2 = 1$, $y = \sqrt{1-x^2}$, $\therefore a = 1$ and $b = 1$. D

Q6 Transform e^x to $|a - be^{-x}| - c$ graphically, or choose positive a , b and c values, and use graphics calculator to sketch graph. D

Q7 $(3\sqrt{x} + x)(3\sqrt{x} - x) = 9x - x^2$,
 $(1 - x\sqrt{2})(2 + x\sqrt{2}) = 2 - x\sqrt{2} - 2x^2$,
 $\sqrt[3]{x^3 - 3x^2 + 3x - 1} = \sqrt[3]{(x-1)^3} = x-1$,
 $\frac{x^{\frac{3}{2}} - (2x)^{\frac{5}{2}}}{x^{-\frac{3}{2}}} = x^3 - 2^{\frac{5}{2}}x^4$. E

Q8 D

Q9 The range of g is $(-1,0]$, $\therefore$ the range of $f \circ g$ is $(0,1]$. B

Q10 $f\left(\frac{x}{y}\right) = 1 - \sqrt{\frac{x}{y}} = 1 - \frac{\sqrt{x}}{\sqrt{y}} = \frac{\sqrt{y} - \sqrt{x}}{\sqrt{y}}$.
 $\frac{f(x) - f(y)}{1 - f(y)} = \frac{1 - \sqrt{x} - (1 - \sqrt{y})}{1 - (1 - \sqrt{y})} = \frac{\sqrt{y} - \sqrt{x}}{\sqrt{y}}$. D

Q11 A

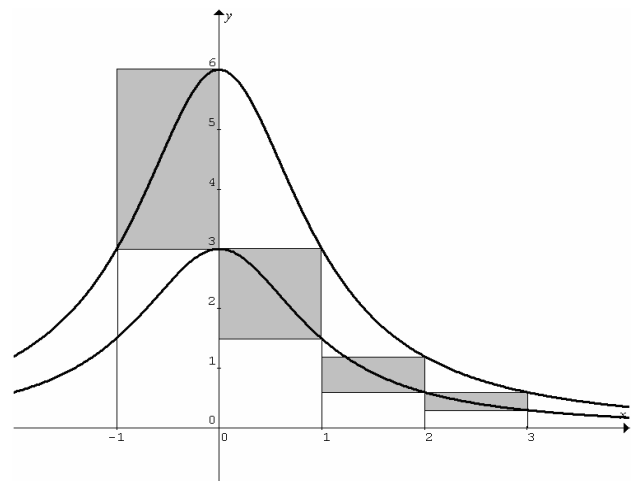
Q12 $P(x) = \frac{f(\log_e x)}{g(\sqrt{x})}$, use the quotient rule and the chain rule,
 $P'(x) = \frac{(g(\sqrt{x}))\left(\frac{1}{x}\right)(f'(\log_e x)) - (f(\log_e x))\left(\frac{1}{2\sqrt{x}}\right)(g'(\sqrt{x}))}{[g(\sqrt{x})]^2}$
 $= \frac{2\sqrt{x}g(\sqrt{x})f'(\log_e x) - xf(\log_e x)g'(\sqrt{x})}{2x\sqrt{x}[g(\sqrt{x})]^2}$. B

Q13 $y = 2x^3 - 3ax^2 + 5$, $\frac{dy}{dx} = 6x^2 - 6ax = 6x(x-a)$. $\therefore$ it is a stationary point at $x = a$. Equation of the normal at $x = a$ is $x = a$. E

Q14 Use graphics calculator, at $x = \frac{3}{4}$, $\frac{dy}{dx} = 2.373$. A

Q15 $f(x) = \sqrt{1-x}$, $f'(x) = \frac{-1}{2\sqrt{1-x}}$. Let $x = -3$ and $h = -0.1$.
 $f(x+h) \approx f(x) + hf'(x)$
 $\therefore f(-3.1) \approx f(-3) + (-0.1)f'(-3)$
 $f(-3.1) \approx \sqrt{1-(-3)} + (-0.1)\left(\frac{-1}{2\sqrt{1-(-3)}}\right) = \frac{81}{40}$. B

Q16 $3 \times 1 + 1.6 \times 1 + 0.5 \times 1 + 0.3 \times 1 \approx 5.4$ A



Q17 $f(x) = (x+2)g(x)$ is a continuous decreasing function,
 $\therefore$ it has only one x -intercept at $x = -2$.

$$\begin{aligned}\text{Required area} &= \int_a^{-2} f(x) dx - \int_{-2}^b f(x) dx = [F(x)]_a^{-2} - [F(x)]_{-2}^b \\ &= 2F(-2) - F(a) - F(b). \quad \text{A}\end{aligned}$$

Q18 $\frac{20}{36} = \frac{5}{9}$. C

(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

Q19

$$\begin{aligned}\bar{X} &= (-1)\left(2 \times \frac{1}{30}\right) + 1\left(2 \times \frac{1}{10}\right) + 3\left(2 \times \frac{1}{6}\right) + 5\left(2 \times \frac{2}{15}\right) + 7\left(2 \times \frac{1}{15}\right) \\ &= \frac{51}{15}. \quad \text{A}\end{aligned}$$

Q20

	$X > 0.36$	$X < 0.36$	
$X > 0.64$	0.19	0	0.19
$X < 0.64$	0.33	0.48	0.81
	0.52	0.48	1

$$\Pr(X > 0.36 | X < 0.64) = \frac{\Pr(0.36 < X < 0.64)}{\Pr(X < 0.64)} = \frac{0.33}{0.81} = \frac{11}{27}. \quad \text{A}$$

Q21 $\mu = \frac{1.25 + 1.35}{2} = 1.30$ and $\sigma = \frac{1.35 - 1.30}{2} = 0.025$.

$$\Pr(1.28 < X < 1.38) = \text{normalcdf}(1.28, 1.38, 1.30, 0.025) \approx 0.79.$$

D

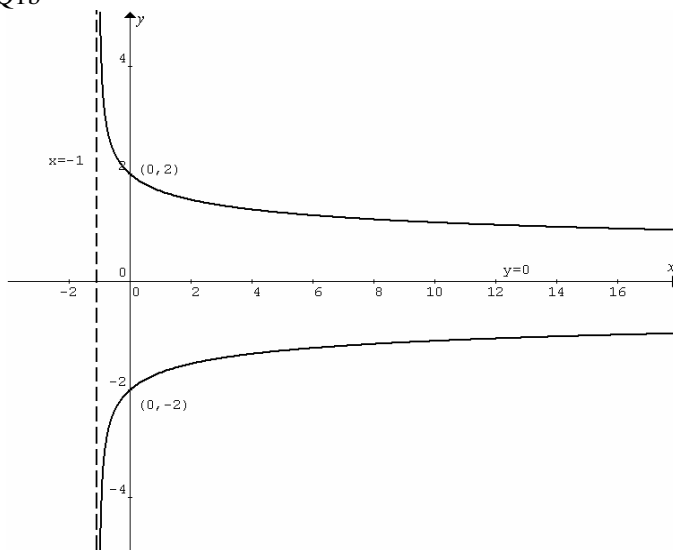
Q22 $\int_0^M \frac{1}{\sqrt{3} \cos^2 t} dt = 0.5$, $\int_0^M \frac{1}{\sqrt{3}} \sec^2 t dt = 0.5$,

$$\therefore \left[\frac{1}{\sqrt{3}} \tan t \right]_0^M = 0.5, \quad \frac{1}{\sqrt{3}} \tan M = 0.5, \quad M = 0.714. \quad \text{C}$$

SECTION 2

Q1a $x = \frac{16}{y^4} - 1$, $x + 1 = \frac{16}{y^4}$, $y^4 = \frac{16}{x+1}$, $y = \pm \left(\frac{2^4}{x+1} \right)^{\frac{1}{4}}$,
 $y = \pm \frac{2}{(x+1)^{\frac{1}{4}}}.$

Q1b



Q1c $L(x) = \frac{2}{(x+1)^{\frac{1}{4}}} - \frac{-2}{(x+1)^{\frac{1}{4}}} = \frac{4}{(x+1)^{\frac{1}{4}}}.$

Q1di $\frac{dL}{dt} = \frac{dL}{dx} \times \frac{dx}{dt} = -\frac{1}{4} \times 4(x+1)^{-\frac{5}{4}} \times 2 = -\frac{2}{(x+1)^{\frac{5}{4}}}.$

Rate of decrease of $L = \frac{2}{(x+1)^{\frac{5}{4}}}.$

Q1dii When $t = 7.5$, $x = vt = 2 \times 7.5 = 15$.

Rate of decrease of $L = \frac{2}{(15+1)^{\frac{5}{4}}} = \frac{1}{16}.$

Q1e $\Delta A = 2 \times \int_0^{15} \frac{2}{(x+1)^{\frac{1}{4}}} dx = 2 \times \left[\frac{8(x+1)^{\frac{3}{4}}}{3} \right]_0^{15} = \frac{112}{3}.$

Average rate of increase of the area $= \frac{\Delta A}{\Delta t} = \frac{\frac{112}{3}}{7.5} = \frac{224}{45}.$

Q2a $2\pi[1 - (1-h)^2] = 2\pi(1 - (1-h))(1 + (1-h)) = 2\pi h(2-h).$

Q2b $\frac{2}{3} - (1-h) + \frac{(1-h)^3}{3} = \frac{2}{3} - 1 + h + \frac{1-3h+3h^2-h^3}{3}$
 $= h^2 - \frac{h^3}{3}.$

Q2c When $h=1$, $\max V = 2\pi\left(\frac{2}{3}\right) = \frac{4\pi}{3} \text{ m}^3$.

Q2d $V = \frac{4\pi}{3} \text{ m}^3 = \frac{4\pi}{3} \times 10^6 \text{ cm}^3 = \frac{4\pi}{3} \times 10^3 \text{ litres}$.

Time required $= \frac{\frac{4\pi}{3} \times 10^3}{2} = \frac{2\pi}{3} \times 10^3 \text{ seconds}$.

Q2e 2 litres per second $= 2 \times 10^{-3} \text{ m}^3 \text{ per second}$.

$V = 2\pi\left(h^2 - \frac{h^3}{3}\right)$, $\frac{dV}{dt} = \frac{dV}{dh} \times \frac{dh}{dt}$,

$\frac{dV}{dt} = 2\pi(2h - h^2) \frac{dh}{dt}$, $\therefore \frac{dh}{dt} = \frac{1}{2\pi(2h - h^2)} \times \frac{dV}{dt}$.

When $h = 0.5$, $\frac{dh}{dt} = \frac{1}{2\pi(1 - 0.25)} \times (-2 \times 10^{-3}) = -1.82 \times 10^{-4}$

Rate of decrease $= 1.82 \times 10^{-4} \text{ ms}^{-1}$.

Q2f $A = 2\pi(2h - h^2)$, $\frac{dA}{dt} = \frac{dA}{dh} \times \frac{dh}{dt} = 2\pi(2 - 2h) \frac{dh}{dt}$.

When $h = 0.5$, $\frac{dA}{dt} = 2\pi \times (-1.82 \times 10^{-4}) = -1.14 \times 10^{-3}$.

Rate of decrease $= 1.14 \times 10^{-3} \text{ m}^2 \text{ s}^{-1}$.

Q2gi When $h = 0.5$,

volume of water $= 2\pi\left(0.5^2 - \frac{0.5^3}{3}\right) = 1.309 \text{ m}^3$.

Volume of water plus pebbles $= 1.309 + 0.831 = 2.140 \text{ m}^3$.

$\therefore 2.140 = 2\pi\left(h^2 - \frac{h^3}{3}\right)$. Using graphics calculator, $h = 0.661 \text{ m}$.

Q2gii Yes. $\frac{dh}{dt} = \frac{dV}{dt} \times \frac{1}{2\pi(2h - h^2)}$.

Since $\frac{dV}{dt}$ is constant, $\frac{dh}{dt} \propto \frac{1}{2h - h^2}$.

Q3a For $y = c - a \cos(bx)$, $\frac{T}{2} = 6$, $\therefore T = 12 = \frac{2\pi}{b}$, $\therefore b = \frac{\pi}{6}$.

Its amplitude is 2, $\therefore a = 2$. $y = -2 \cos\left(\frac{\pi}{6}x\right)$ is translated upwards by 3, $\therefore c = 3$.

For semi-circle $(x-h)^2 + (y-k)^2 = 1.5^2$, $x \in [6.5, 8]$, radius is 1.5. It is the translation of the semi-circle $x^2 + y^2 = 1.5^2$, $x \in [-1.5, 0]$ to the right by 8 units and upwards by 2.5 units. $\therefore h = 8$ and $k = 2.5$.

Q3b $P(8, 4)$

Q3ci The semi-circle in the third wave crest is the translation of the semi-circle $x^2 + y^2 = 1.5^2$, $x \in [-1.5, 0]$ to the right by 24 units and upwards by 2.5 units, $\therefore$ its equation is $(x-24)^2 + (y-2.5)^2 = 1.5^2$, $x \in [22.5, 24]$.

Q3cii The cosine curve in the fourth wave crest is the

translation of $y = 3 - 2 \cos\left(\frac{\pi}{6}x\right)$, $x \in [0, 8]$, to the right by 24 units, $\therefore$ its equation is $y = 3 - 2 \cos\left(\frac{\pi}{6}(x-24)\right)$, $x \in [24, 32]$.

Q3di Same area as the first wave crest,

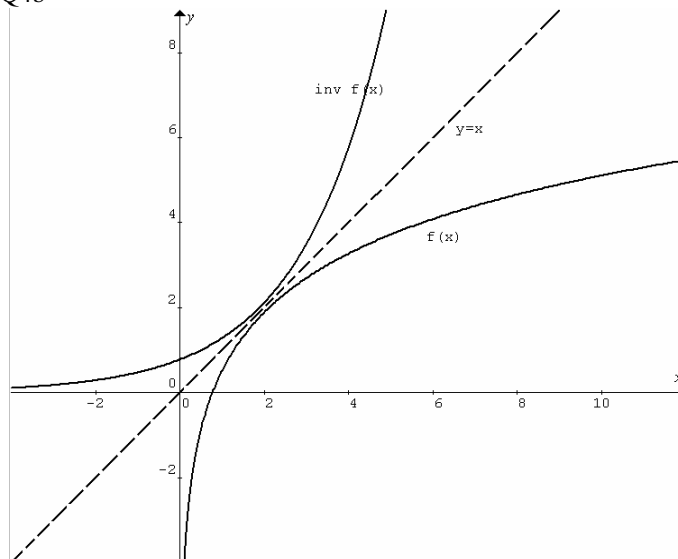
$A = \int_0^8 \left(3 - 2 \cos\left(\frac{\pi}{6}x\right) - (-2)\right) dx - \frac{1}{2}\pi(1.5)^2$
 $= \int_0^8 \left(5 - 2 \cos\left(\frac{\pi}{6}x\right)\right) dx - \frac{9\pi}{8} = \left[5x - \frac{12 \sin\left(\frac{\pi}{6}x\right)}{\pi}\right]_0^8 - \frac{9\pi}{8}$
 $= 40 + \frac{6\sqrt{3}}{\pi} - \frac{9\pi}{8} \text{ m}^2$.

Q3dii $V = A \times l = \left(40 + \frac{6\sqrt{3}}{\pi} - \frac{9\pi}{8}\right) \times 10 = 398 \text{ m}^3$.

Q4a Inverse equation: $x = a \log_e y + \frac{1}{2}$, $\log_e y = \frac{x - \frac{1}{2}}{a}$,

$y = e^{\frac{x - \frac{1}{2}}{a}} = e^{\frac{2x-1}{2a}}$, $\therefore f^{-1}(x) = e^{\frac{2x-1}{2a}}$.

Q4b



Q4ci $y = a \log_e x + \frac{1}{2}$. At the contact point, $\frac{dy}{dx} = \frac{a}{x} = 1$ and $y = x$, $\therefore y = x = a$, $\therefore a = a \log_e a + \frac{1}{2}$, $\therefore a - a \log_e a - \frac{1}{2} = 0$. Using graphics calculator, $a = 2.156$.

Q4cii $(2.156, 2.156)$.

Q5a $1000 \times \Pr(X > 375) = 1000 \times \int_{375}^{\infty} e^{-\pi(x-375.5)^2} dx = 895$. Note:

let ∞ be 380, use graphics calculator to evaluate the definite integral.

Q5b $1000 \times \Pr(X > 375.3) = 1000 \times \int_{375.3}^{\infty} e^{-\pi(x-375.5)^2} dx = 692$.

Translate the pdf and all x -values to the left by 0.3,

$\therefore 1000 \times \Pr(X > 375) = 1000 \times \int_{375}^{\infty} e^{-\pi(x-375.2)^2} dx = 692$.

$\therefore k = 375.2$.

Q5c Binomial, $n = 5$, $p = 1 - 0.692 = 0.308$,

$\Pr(X = 4) = \text{binompdf}(5, 0.308, 4) = 0.031$.

Q5d Since the machines are identical, it makes no difference to the probability which machine the cans are selected from.

$\Pr(X = 8) = \text{binompdf}(10, 0.308, 8) = 0.002$.

Q5e Given that 2 cans were under and 1 can was over 375 ml,
 $\therefore$ the probability that 6 of the remaining 7 cans are under 375 ml
 $= \text{binompdf}(7, 0.308, 6) = 0.004$.

Q5fi Symmetric bell shape.

Q5fii

If Y is normally distributed, then $\sigma_Y = \frac{376.85 - 375.5}{2} = 0.675$.

$\Pr(\mu - \sigma < Y < \mu + \sigma)$
 $= \frac{\text{normalcdf}(374.825, 376.175, 375.5, 0.3989) + \text{normalcdf}(374.825, 376.175, 375.5, 0.7978)}{2}$
 $= 0.756 \neq 0.683$.

$\Pr(\mu - 3\sigma < Y < \mu + 3\sigma)$
 $= \frac{\text{normalcdf}(373.475, 377.525, 375.5, 0.3989) + \text{normalcdf}(373.475, 377.525, 375.5, 0.7978)}{2}$
 $= 0.994 \approx 0.997$.

$\therefore Y$ is **not** exactly normally distributed.

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