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Mathematical Methods

2008

Trial Examination 1

Instructions

Answer **all** questions. Do **not** use calculators.

A decimal approximation will not be accepted if an **exact** answer is required to a question.

In questions where more than one mark is available, appropriate working must be shown.

Unless otherwise indicated, the diagrams in this exam are **not** drawn to scale.

Question 1 Let $f(x) = \sqrt{x} + \frac{x}{2}$ and $g(x) = 2f(x+1)$.

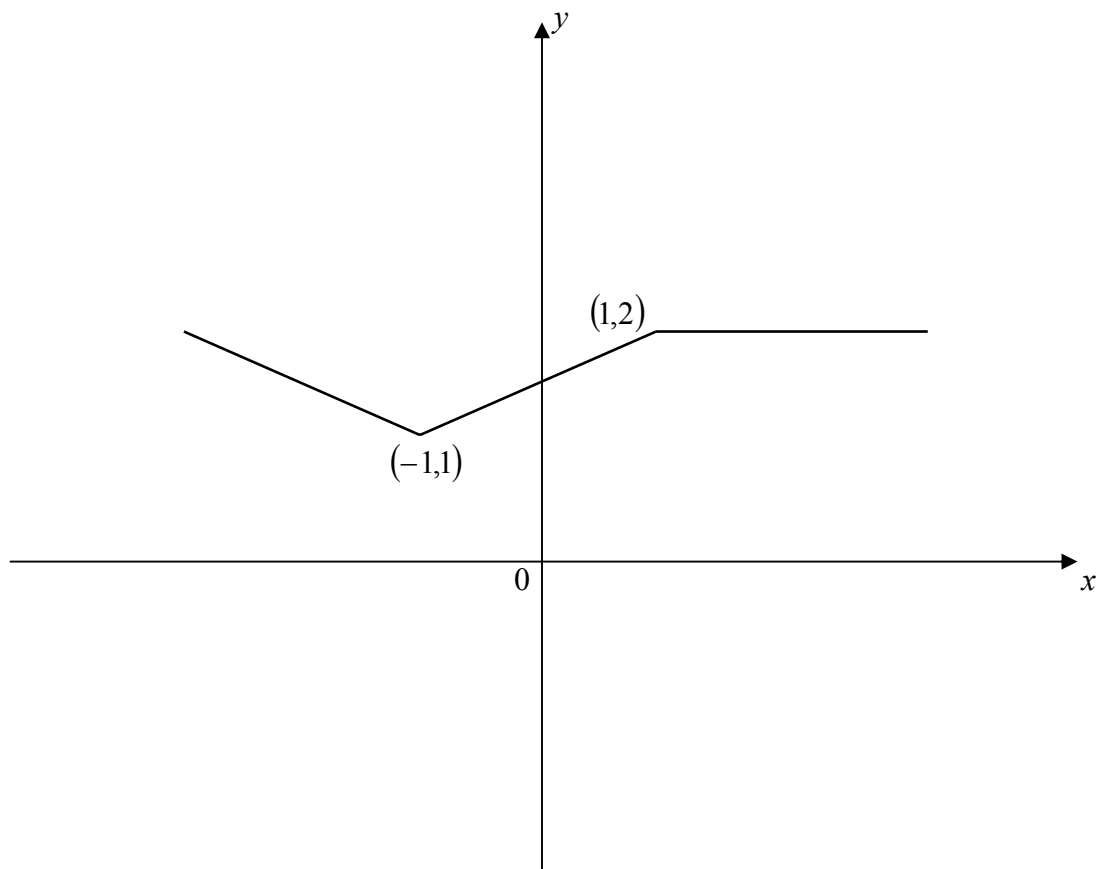
a. Find $g(x)$. 1 mark

b. Solve the equation $g(x) = 0$ for x . 2 marks

Question 2 Let $y = 1 + 3 \log_e \left(\frac{2x-b}{a} \right)$. If $y = -2$ when $x = b$, find a in terms of b . 2 marks

Question 3 Given $f(x) = \frac{\log_e(ax)}{ax}$, find $f'(a^{-1})$ in terms of a . 3 marks

Question 4 The diagram shows the graph of $y = f'(x)$ with domain R .

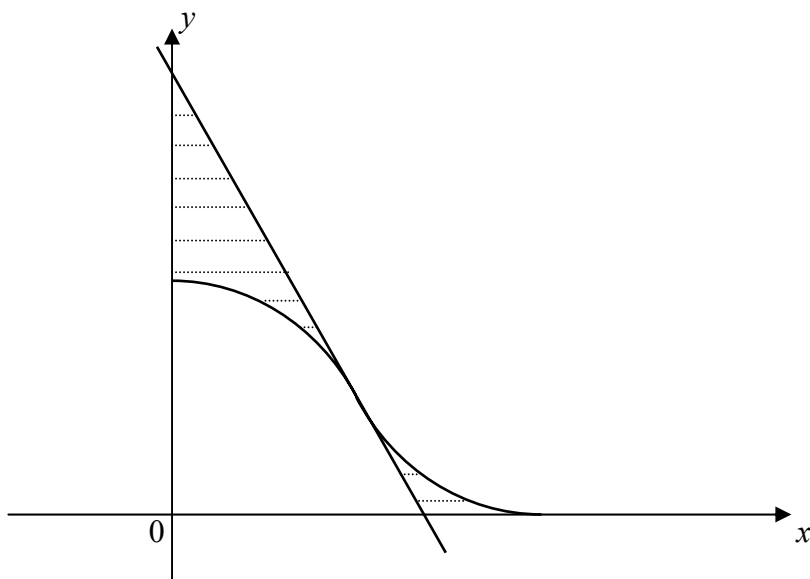


a. For the graph shown above, find the equation of $y = f(x)$ for $-1 \leq x < 1$, given $f(-1) = -1$. 3 marks

b. For the graph shown above, sketch on the same set of axes the graph of $y = f(x)$, given that $y = f(x)$ is continuous and $f(-1) = 0$.

3 marks

Question 5 The graph of $y = 1 + \cos \frac{x}{2}$ for $x \in [0, 2\pi]$ is shown. The tangent to the graph at the point of inflection is also shown.



a. Find the equation of the tangent to the graph $y = 1 + \cos \frac{x}{2}$ for $x \in [0, 2\pi]$. 2 marks

b. Find the exact area of the shaded region. 3 marks

Question 6 Given $g(x) = a \sin x + b \cos x$, $g\left(\frac{\pi}{4}\right) = 2\sqrt{2}$ and $g\left(-\frac{\pi}{6}\right) = -1$, find the values of a and b .

3 marks

Question 7 Given $f'(x) = \frac{1}{1-6x+9x^2}$, evaluate $[f(x)]_{-\frac{1}{3}}^0$.

3 marks

Question 8 Consider $g : (-\infty, -3] \rightarrow \mathbb{R}$, $g(x) = 1 - \frac{1}{2}|x+2|$.

a. Find the rule of g^{-1} .

3 marks

b. State the domain of g^{-1} .

1 mark

Question 9 Use Euler's method of linear approximation to find the value of $\sqrt{\tan 1} - \sqrt{\tan \frac{\pi}{4}}$. Leave the approximation in exact form.

3 marks

Question 10 The following table shows the probability distribution of random variable X with one missing entry.

x	0	1	2	3	4	5	6
$\Pr(X = x)$	$\left(\frac{2}{3}\right)^6$	$6\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)^5$	$15\left(\frac{1}{3}\right)^2\left(\frac{2}{3}\right)^4$		$15\left(\frac{1}{3}\right)^4\left(\frac{2}{3}\right)^2$	$6\left(\frac{1}{3}\right)^5\left(\frac{2}{3}\right)$	$\left(\frac{1}{3}\right)^6$

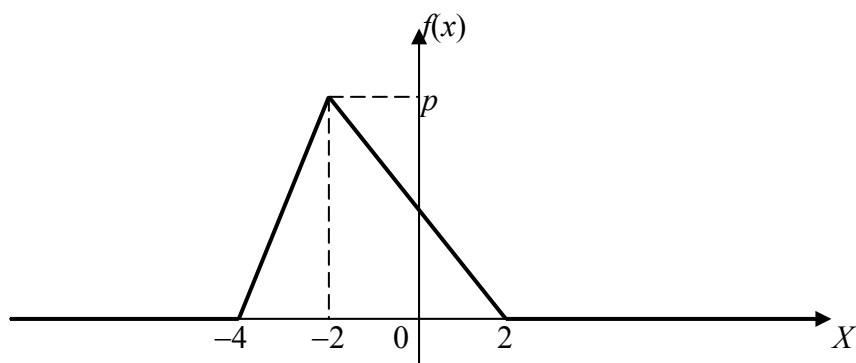
a. Complete the table.

1 mark

b. Calculate the exact values of $E(X)$ and $Var(X)$.

2 marks

Question 11 The probability density function of random variable X is shown below.



a. Find the exact value of p . 1 mark

b. Find the exact value of $\Pr(X \leq 0)$. 1 mark

Question 12

a. Consider the standard normal distribution. Find $\Pr(Z < 1)$. 1 mark

b. The scores on an examination are normally distributed with $\mu = 72$ and $\sigma^2 = 36$. Find the score x such that $\Pr(X \geq x) = \Pr(Z < 1)$. 2 marks

End of exam 1