2008 Mathematical Methods 3/4 Trial Exam 1 Solutions
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1a. $f(x)=\sqrt{x}+\dfrac{x}{2}$, $f(x+1)=\sqrt{x+1}+\dfrac{x+1}{2}$,

$\therefore\ g(x)=2f(x+1)=2\sqrt{x+1}+x+1$.

1b. $g(x)=0$. $\therefore\ 2\sqrt{x+1}+x+1=0$, $2\sqrt{x+1}=-(x+1)$,

$\therefore\ 4(x+1)=(x+1)^2$, $4(x+1)-(x+1)^2=0$,

$(x+1)[4-(x+1)]=0$, $(x+1)(3-x)=0$.

Only $x=-1$ satisfies $g(x)=0$.

2. $y=1+3\log_e\left(\dfrac{2x-b}{a}\right)$ and $y=-2$ when $x=b$.

$\therefore\ -2=1+3\log_e\left(\dfrac{b}{a}\right)$, $\therefore\ \log_e\left(\dfrac{b}{a}\right)=-1$, $\therefore\ \log_e\left(\dfrac{a}{b}\right)=1$.

Hence $\dfrac{a}{b}=e$, $\therefore\ a=be$.

3. $f(x)=\dfrac{\log_e(ax)}{ax}$,

$$f'(x)=\frac{(ax)\left(\frac{1}{x}\right)-(a)(\log_e(ax))}{(ax)^2}=\frac{1-\log_e(ax)}{ax^2}.$$

$$f'(a^{-1})=\frac{1-\log_e(1)}{a^{-1}}=\frac{1}{a^{-1}}=a.$$

4a. $y=f'(x)=\dfrac{1}{2}x+\dfrac{3}{2}$ for $-1\le x<1$.

$\therefore\ f(x)=\dfrac{1}{4}x^2+\dfrac{3}{2}x+c$.

Given $f(-1)=-1$, $\therefore\ \dfrac{1}{4}(-1)^2+\dfrac{3}{2}(-1)+c=-1$, $\therefore\ c=\dfrac{1}{4}$.

$\therefore\ f(x)=\dfrac{1}{4}x^2+\dfrac{3}{2}x+\dfrac{1}{4}$ for $-1\le x<1$.

Hence $y=\dfrac{1}{4}x^2+\dfrac{3}{2}x+\dfrac{1}{4}$ for $-1\le x<1$.

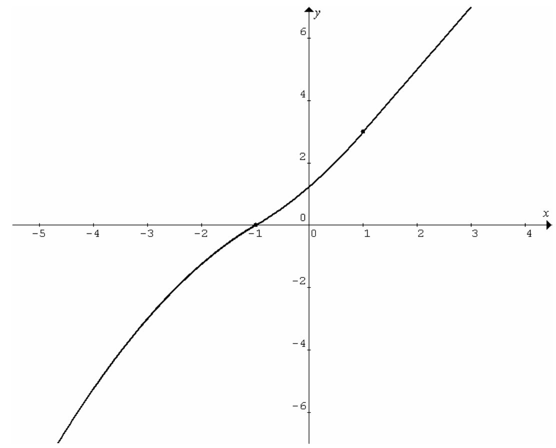
4b. For $-1\le x<1$, if $f(-1)=0$, then $\dfrac{1}{4}(-1)^2+\dfrac{3}{2}(-1)+c=0$,

$\therefore\ c=\dfrac{5}{4}$. Hence $y=\dfrac{1}{4}x^2+\dfrac{3}{2}x+\dfrac{5}{4}=\dfrac{1}{4}(x+1)(x+5)$.

Similarly, for $x\in(-\infty,-1)$,

$y=-\dfrac{1}{4}x^2+\dfrac{1}{2}x+\dfrac{3}{4}=-\dfrac{1}{4}(x+1)(x-3)$.

For $x\in[1,\infty)$, $y=2x+1$.



5a. $y=1+\cos\dfrac{x}{2}$, $x\in[0,2\pi]$.

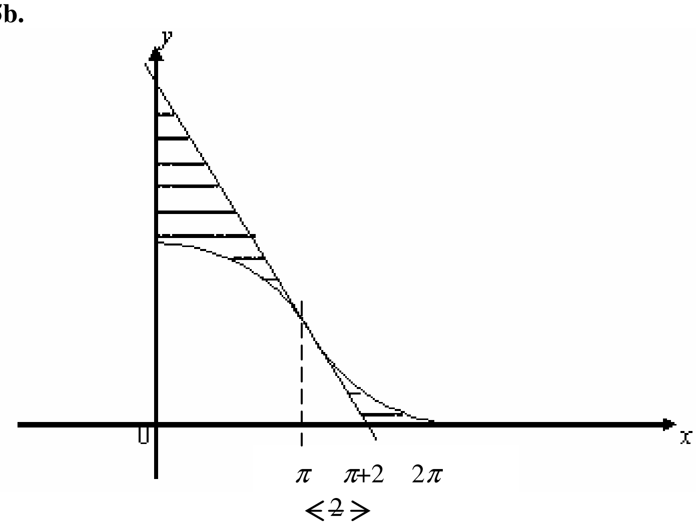
Coordinates of the inflection point are $(\pi,1)$.

$\dfrac{dy}{dx}=-\dfrac{1}{2}\sin\dfrac{x}{2}$,

$\therefore$ gradient of the tangent at $(\pi,1)=-\dfrac{1}{2}\sin\dfrac{\pi}{2}=-\dfrac{1}{2}$.

Equation of the tangent: $y-1=-\dfrac{1}{2}(x-\pi)$, $y=-\dfrac{1}{2}x+\left(1+\dfrac{\pi}{2}\right)$

5b.



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$$=\int_0^{\pi}\left\{\left(-\frac{1}{2}x+\left(1+\frac{\pi}{2}\right)\right)-\left(1+\cos\frac{x}{2}\right)\right\}dx+\int_{\pi}^{2\pi}\left(1+\cos\frac{x}{2}\right)dx-\frac{1}{2}(2)(1)$$

$$=\int_0^{\pi}\left(-\frac{1}{2}x+\frac{\pi}{2}-\cos\frac{x}{2}\right)dx+\int_{\pi}^{2\pi}\left(1+\cos\frac{x}{2}\right)dx-1$$

$$=\left[-\frac{x^2}{4}+\frac{\pi x}{2}-2\sin\frac{x}{2}\right]_0^{\pi}+\left[x+2\sin\frac{x}{2}\right]_{\pi}^{2\pi}-1$$

$$=\left(\frac{\pi^2}{4}-2\right)+(\pi-2)-1$$

$$=\frac{\pi^2}{4}+\pi-5.$$

6. $g(x)=a\sin x+b\cos x$, $g\left(\frac{\pi}{4}\right)=2\sqrt{2}$, $g\left(-\frac{\pi}{6}\right)=-1$.

$g\left(\frac{\pi}{4}\right)=a\sin\frac{\pi}{4}+b\cos\frac{\pi}{4}=2\sqrt{2}$, $\therefore a\left(\frac{1}{\sqrt{2}}\right)+b\left(\frac{1}{\sqrt{2}}\right)=2\sqrt{2}$,

$\therefore a+b=4$ ....(1)

$g\left(-\frac{\pi}{6}\right)=a\sin\left(-\frac{\pi}{6}\right)+b\cos\left(-\frac{\pi}{6}\right)=-1$,

$\therefore a\left(-\frac{1}{2}\right)+b\left(\frac{\sqrt{3}}{2}\right)=-1$, $\therefore a-\sqrt{3}b=2$ ....(2)

(1) − (2), $b+\sqrt{3}b=2$, $b\left(\sqrt{3}+1\right)=2$,

$\therefore b=\frac{2}{\sqrt{3}+1}=\sqrt{3}-1$ ....(3)

Substitute (3) in (1), $a=4-\left(\sqrt{3}-1\right)=5-\sqrt{3}$.

7. $f'(x)=\frac{1}{1-6x+9x^2}=\frac{1}{(1-3x)^2}$,

$f(x)=\int\frac{1}{(1-3x)^2}dx=\int(1-3x)^{-2}dx=\frac{(1-3x)^{-1}}{3}+c$

$=\frac{1}{3(1-3x)}+c$.

$\left[f(x)\right]_{-\frac{1}{3}}^{0}=\left[\frac{1}{3(1-3x)}+c\right]_{-\frac{1}{3}}^{0}=\frac{1}{3}-\frac{1}{6}=\frac{1}{6}$.

8a. $g:(-\infty,-3]\rightarrow R$, $g(x)=1-\frac{1}{2}|x+2|$.

g is an increasing function. Its range is $(-\infty,g(-3)]$,

i.e. $\left(-\infty,\frac{1}{2}\right]$.

Equation of $g(x)$:

$y=1-\frac{1}{2}|x+2|=1-\frac{1}{2}\left(-(x+2)\right)=\frac{1}{2}x+2$

Equation of $g^{-1}(x)$:

$x=\frac{1}{2}y+2$, $\therefore y=2(x-2)$.

$\therefore g^{-1}(x)=2(x-2)$.

8b. Domain of g^{-1} is the same as the range of g , i.e. $\left(-\infty,\frac{1}{2}\right]$.

9. Let $f(x)=\sqrt{\tan x}$,

$f'(x)=\frac{\sec^2 x}{2\sqrt{\tan x}}=\frac{1}{2\sqrt{\tan x}\left(\cos^2 x\right)}$,

$f'\left(\frac{\pi}{4}\right)=\frac{1}{2\sqrt{\tan\frac{\pi}{4}}\left(\cos\frac{\pi}{4}\right)^2}=1$.

$\frac{\sqrt{\tan 1}-\sqrt{\tan\frac{\pi}{4}}}{1-\frac{\pi}{4}}\approx f'\left(\frac{\pi}{4}\right)$, $\therefore\sqrt{\tan 1}-\sqrt{\tan\frac{\pi}{4}}\approx 1-\frac{\pi}{4}$.

10a. ${}^{6}C_{3}\left(\frac{1}{3}\right)^3\left(\frac{2}{3}\right)^3=20\left(\frac{1}{3}\right)^3\left(\frac{2}{3}\right)^3$.

10b. Random variable X has a binomial distribution,

$n=6$, $p=\frac{1}{3}$, $q=\frac{2}{3}$.

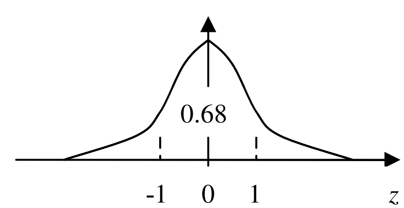
$E(X)=np=2$, $Var(X)=npq=\frac{4}{3}$.

11a. $\frac{1}{2}p\left(2-{}^{-}4\right)=1$, $\therefore 3p=1$, $p=\frac{1}{3}$.

11b. $f(0)=\frac{1}{2}p=\frac{1}{2}\left(\frac{1}{3}\right)=\frac{1}{6}$.

$\Pr(X\le 0)=1-\Pr(X>0)=1-\frac{1}{2}(2)\left(\frac{1}{6}\right)=\frac{5}{6}$.

12a.



$\Pr(Z<1)=1-\frac{1}{2}(1-0.68)=0.84$.

12b. $\mu=72$, $\sigma=6$.

$\Pr(Z<1)=\Pr(Z>-1)$, $\therefore\Pr(X\ge x)=\Pr(Z<1)=\Pr(Z>-1)$.

$\therefore\frac{x-72}{6}=-1$, $\therefore x=66$.

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