# Test 2

**Section A: Technology free. 47 marks**
**Section B: CAS technology assumed. 43 marks**
**Suggested time: 90 minutes**

## Section A: Short answer and extended response questions. Technology free.

**Specific instructions to students**

- Answer **all** questions in the spaces provided.
- A decimal approximation will not be accepted if an **exact** answer is required to a question.
- In questions where more than one mark is available, appropriate working **must** be shown.

### QUESTION 1 — Total 4 marks

**a** Find the equation of the line passing through the points $A(-2, 4)$ and $B(5, 1)$. Write the equation in the form $y = mx + c$. **2 marks**

$$m = \frac{4 - 1}{-2 - 5}$$
$$= \frac{3}{-7}$$

The equation is:

$$y - 4 = \frac{3}{-7}(x + 2)$$
$$y = \frac{3}{-7}x - \frac{6}{7} + 4$$
$$y = -\frac{3}{7}x + \frac{22}{7}$$

**b** Find the midpoint of the line joining the two points $A(-2, 4)$ and $B(5, 1)$. **1 mark**

$$\left(\frac{-2 + 5}{2}, \frac{4 + 1}{2}\right)$$
$$\left(\frac{3}{2}, \frac{5}{2}\right)$$

**c** Find the *exact* distance between the points $A(-2, 4)$ and $B(5, 1)$. **1 mark**

Distance:

$$\sqrt{(-2 - 5)^2 + (4 - 1)^2}$$
$$\sqrt{49 + 9}$$
$$\sqrt{58}$$

### QUESTION 2 — Total 4 marks

Find the equation of the straight line which passes through the point $(1, -2)$ and is:

**a** parallel to the line with equation $2x - 3y = 6$. **2 marks**

$$3y = 2x - 6$$
$$y = \frac{2}{3}x - 2$$

The gradient of the parallel line is $\frac{2}{3}$.

The equation of the parallel line through $(1, -2)$ is

$$y = \frac{2}{3}x + c.$$
$$-2 = \frac{2}{3} + c$$
$$c = -2\frac{2}{3}$$

The parallel line is $y = \frac{2}{3}x - 2\frac{2}{3}$.

**b** perpendicular to the line $2x - 3y = 6$. **2 marks**

The gradient of the perpendicular line is $\frac{-3}{2}$.

The equation of the perpendicular line through $(1, -2)$ is

$$-2 = \frac{-3}{2} \times 1 + c$$
$$c = -\frac{1}{2}$$

The perpendicular line is $y = -\frac{3}{2}x - \frac{1}{2}$.

### QUESTION 3 — Total 3 marks

State the maximal domain for each of the following.

**a** $y = \sqrt{x + 2}$ **1 mark**

$x + 2 \geq 0$

$x \leq -2$ or $x \in (-\infty, -2]$

**b** $y = \sqrt{4 - x^2}$ **2 marks**

$4 - x^2 \geq 0$

$x^2 \geq 4$

$-2 \leq x \leq 2$ or $x \in [-2, 2]$

**QUESTION 4** **Total 5 marks**

**a** Find the $x$ and $y$ intercepts for the graph of $f(x) = 4 - (x + 1)^2, x \geq -1$. **3 marks**

$y$ intercept

$$f(0) = -(1)^2 + 4$$
$$= 3$$

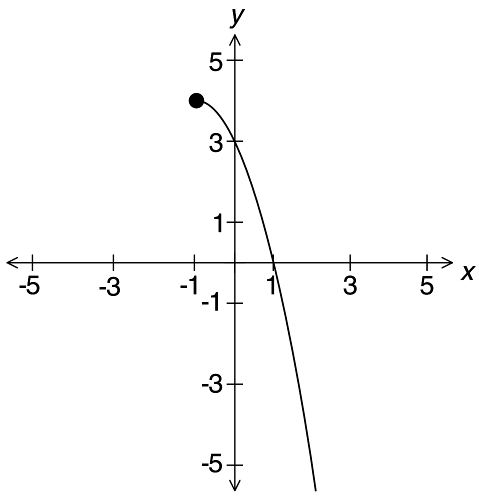
$x$ intercept

$$f(x) = 0$$
$$(x + 1)^2 = 4$$
$$x + 1 = \pm 2$$
$$x = 1, -3$$

(Since $x \geq -1$)

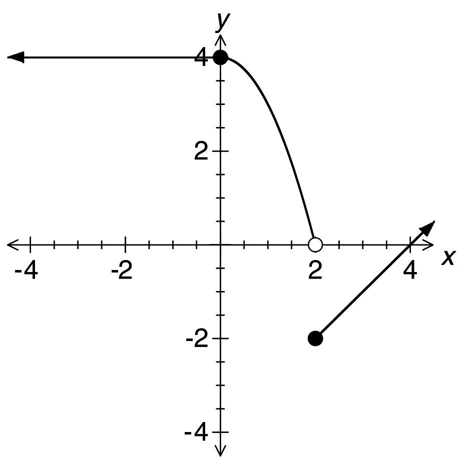
$x = 1$ is the $x$ intercept.

**b** Sketch the graph of the function. **2 marks**



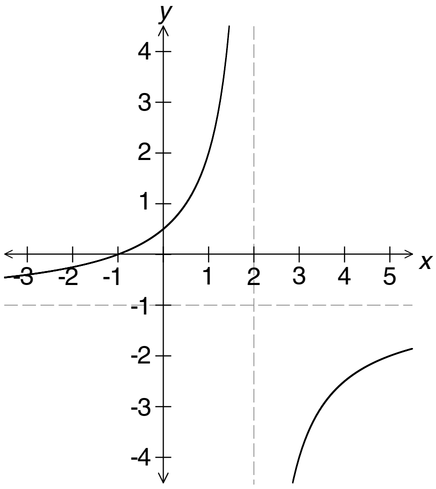

**QUESTION 5** **3 marks**

Sketch the graph of $f(x) = \begin{cases} 4, & x \in (-\infty, 0] \\ 4 - x^2, & x \in (0, 2) \\ x - 4, & x \in [2, \infty) \end{cases}$




**QUESTION 6** **Total 5 marks**

The graph of the function with rule $y = \dfrac{a}{x + b} + c$ is shown below.




**a** Find the values of $a$, $b$ and $c$. **3 marks**

Since the vertical asymptote is $x = 2$, $b = -2$, and since the horizontal asymptote is $y = -1$, $c = -1$, the $x$ intercept is $(-1, 0)$, hence $0 = \dfrac{a}{-1 - 2} - 1$.

$$a = -3$$

**b** State the domain and range of the function. **2 marks**

Domain: $x \in \mathbf{R}\backslash\{2\}$. Range: $y \in \mathbf{R}\backslash\{-1\}$.

**QUESTION 7** **Total 5 marks**

A quadratic function is defined by $f: [-3, 5] \rightarrow R$, $f(x) = (5 - x)(3 + x)$.

**a** Find the values of the $x$ and $y$ intercepts. **2 marks**

$y$ intercept: $f(0) = (5 - 0)(3 + 0) = 15$

$x$ intercepts: $(5 - x)(3 + x) = 0$

$$x = -3, 5$$

**b** Find the maximum value of $f$ and the $x$ value at which it occurs. **2 marks**

By symmetry, the $x$ value of the turning point is 1.

$f(1) = (5 - 1)(3 + 1) = 16$

$\therefore$ The maximum value is 16 when $x = 1$.

**c** State the range of $f$. **1 mark**

Range: $f(x) \in [0, 16]$

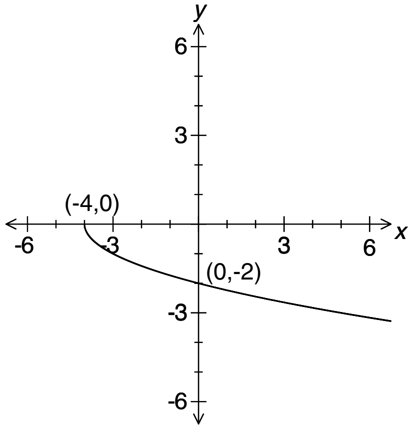
**QUESTION 8** **Total 5 marks**

The graph of $y = -\sqrt{x + 4}$ is an image of $y = \sqrt{x}$ under a set of transformations.

**a** State the transformations in the order they were applied. **2 marks**

Reflection in the $x$ axis followed by a translation of $-4$ in the $x$ direction.

**b** Sketch the graph of $y = -\sqrt{x+4}$, labelling any intercepts with the axes. **3 marks**




## QUESTION 9 — Total 7 marks

The equation of a circle is given by $(x-1)^2 + (y+3)^2 = 10$.

**a** State the coordinates of the centre and the exact length of the radius. **2 marks**

Centre $(1, -3)$, radius $\sqrt{10}$ units.

**b** Find the exact values of the $x$ and $y$ intercepts. **2 marks**

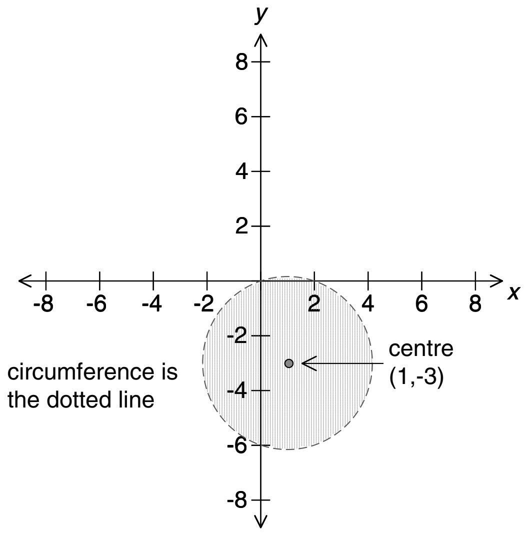
$x$ intercepts:

$$(x-1)^2 + 9 = 10$$
$$(x-1)^2 = 1$$
$$x - 1 = \pm 1$$
$$x = 0, 2$$

$y$ intercepts:

$$1 + (y+3)^2 = 10$$
$$(y+3)^2 = 9$$
$$y + 3 = \pm 3$$
$$y = 0, -6$$

**c** Sketch the graph of the inequality $(x-1)^2 + (y+3)^2 < 10$. **3 marks**




## QUESTION 10 — Total 9 marks

The graph of $y = \frac{a}{(x-2)^2} - 3$ has a $y$ intercept of 1.

**a** Show that $a = 16$. **1 mark**

The curve passes through (0, 1).

$$1 = \frac{a}{4} - 3$$
$$\frac{a}{4} = 4$$
$$a = 16$$

**b** State the equations of any asymptotes. **2 marks**

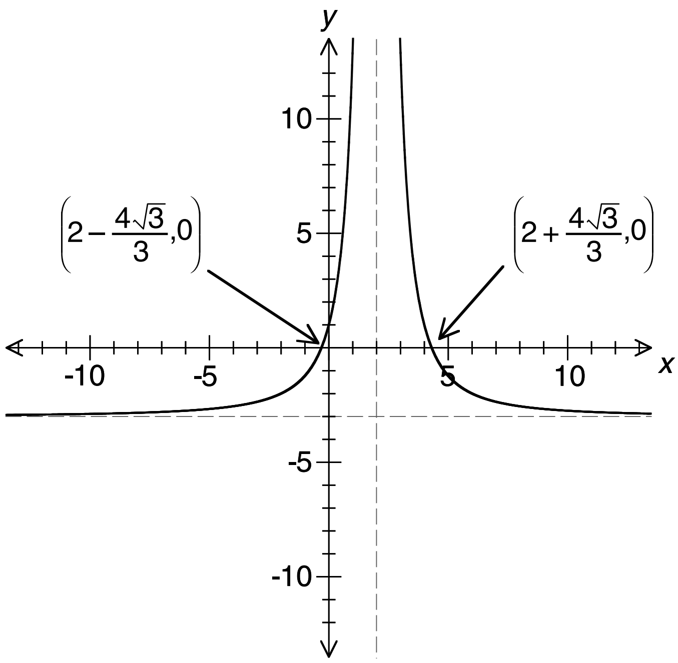
Vertical asymptote: $x = 2$

Horizontal asymptote: $y = -3$

**c** Find the exact value of any $x$ intercepts. **2 marks**

$$\frac{16}{(x-2)^2} - 3 = 0$$
$$\frac{(x-2)^2}{16} = \frac{1}{3}$$
$$(x-2)^2 = \frac{16}{3}$$
$$x - 2 = \pm\frac{4}{\sqrt{3}} \quad \text{or} \quad \left(\pm\frac{4\sqrt{3}}{3}\right)$$
$$x = 2 \pm \frac{4}{\sqrt{3}} \quad \text{or} \quad x = 2 \pm \frac{4\sqrt{3}}{3}$$

**d** Sketch the graph of $y = \frac{16}{(x-2)^2} - 3$ on the axes provided. **4 marks**



## Section B: Multiple-choice questions. CAS technology assumed.

**Specific instructions to students**

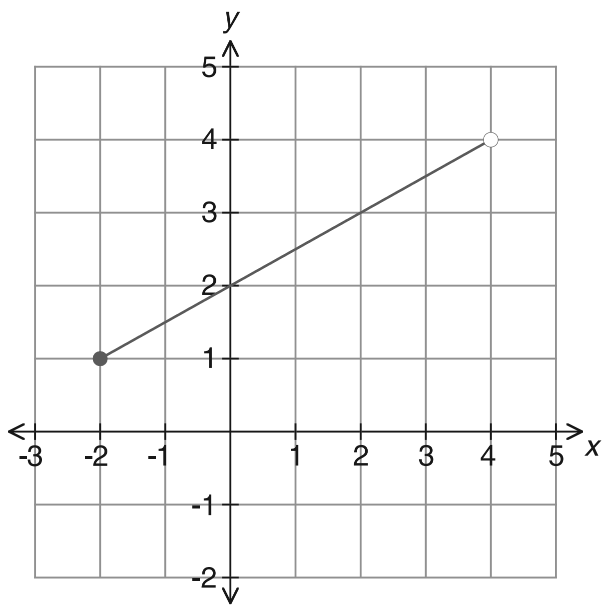
- A correct answer scores 1, and an incorrect answer scores 0.
- Marks are not deducted for incorrect answers.
- No marks are given if more than one answer is given.
- Choose the alternative which most correctly answers the question and mark your choice on the multiple-choice answer section at the bottom of each page, as shown in the example below.

1 A B C D E **USE PENCIL ONLY**

- Use pencil only.

**QUESTION 11**

Which group of characteristics best describes the graph shown in the diagram?




**A** $\{(x, y) : x - 2y + 2 = 0\}$.

**B** The domain is $\{x : x \in [-2, 4)\}$. The range is $\{y : y \in [1, 4)\}$. The gradient is $\frac{1}{2}$.

**C** The domain is $\{x : x \in (-2, 4]\}$. The range is $\{y : y \in (1, 4]\}$. The gradient is $\frac{1}{2}$.

**D** $g : R \to R, g(x) = \frac{1}{2}x + 2$.

**E** $h : [-2, 4) \to [1, 4), h(x) = \frac{1}{2}x + 2$.

**QUESTION 12**

For the function $f : \mathbf{R} \to \mathbf{R}, f(x) = x^2 - 3x + 2$, the range is:

**A** R

**B** $R^+ \cup \{0\}$

**C** $\{(x, y) : x \geq -2\}$

**D** $\{(x, y) : x \geq 2\}$

**E** $[-\frac{1}{4}, \infty)$

**QUESTION 13**

Let $f(x) = 2x(x + 2)$. Then $f(x - 1) - f(x)$ is:

**A** $-4x - 2$

**B** 1

**C** 2

**D** $-1$

**E** $4x^2 - 4x - 2$

**QUESTION 14**

The maximal domain for $f(x) = \sqrt{x + 1}$ is best given by:

**A** $R^+ \cup \{0\}$

**B** $R^+$

**C** $(-\infty, -1]$

**D** $[-1, \infty)$

**E** $[1, \infty)$

**QUESTION 15**

Which of the following is not a one-to-one function?

**A** $f(x) = 5 - x$

**B** $f(x) = \sqrt{5 - x^2}$

**C** $f(x) = \sqrt{5 - x}$

**D** $f(x) = \frac{1}{5x}, x > 0$

**E** $f(x) = 5 - x^2, x \geq 0$

**ONE ANSWER PER LINE** **USE PENCIL ONLY**

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 11 | A | B | C | D | E | 14 | A | B | C | D | E |
| 12 | A | B | C | D | E | 15 | A | B | C | D | E |
| 13 | A | B | C | D | E | | | | | | |

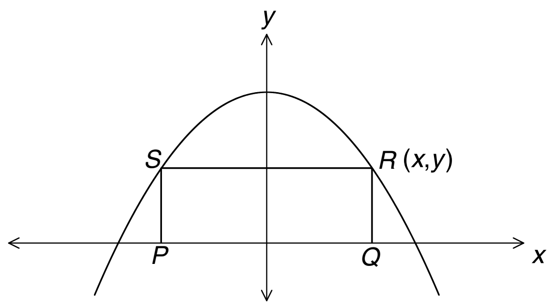
## Section B: Extended response questions. CAS technology assumed.

**Specific instructions to students**

- Answer **all** questions in the spaces provided.
- In questions where more than one mark is available, appropriate working **must** be shown.

**QUESTION 16** **Total 7 marks**

A rectangle $PQRS$ is inscribed between the $x$ axis and the graph of $y = 4 - x^2$. The vertices $P$ and $Q$ are on the $x$ axis and the vertices $R$ and $S$ are on the graph, as shown in the diagram. The coordinates of $R$ are $(x, y)$.




**a** Find the distance $PQ$ in terms of $x$. **1 mark**

By symmetry, $PQ$ is $2x$.

**b** Find the distance $QR$ in terms of $x$. **1 mark**

$QR = y = 4 - x^2$

**c** Find the area, $A$, of the rectangle $PQRS$ in terms of $x$. **1 mark**

$$A = PQ \times QR$$
$$= 2x(4 - x^2)$$

**d** What are the maximum and minimum values $x$ can take? **2 marks**

The $x$ intercepts are $\pm 2$. $x$ lies between 0 and 2.

**e** Find a value for $x$ for the maximum area of the rectangle. What is the maximum area? (Give answers correct to four decimal places.) **2 marks**

CAS: Use SOLVE, or work from the graph of $A$.
$x = 1.1547$, $A = 6.1584$.

**QUESTION 17** **Total 10 marks**

**a** Solve the simultaneous linear equations $mx + 3y = 6$ and $2x + (m - 2)y = m - 1$ for $x$ and $y$. Give the answer in terms of $m$. **1 mark**

CAS: Use SOLVE : $x = \dfrac{3(m-3)}{m^2 - 2m - 6}$

$$y = \frac{m^2 - m - 12}{m^2 - 2m - 6}$$

**b** Find the numerical solutions when $m = 1$. **2 marks**

$$x = \frac{3x - 2}{1 - 2 - 6}$$
$$= \frac{6}{7}$$
$$y = \frac{-3 \times 4}{1 - 2 - 6}$$
$$= \frac{12}{7}$$

or CAS.

**c** Show that when $m = 1 \pm \sqrt{7}$, there are no solutions to the simultaneous equations. **2 marks**

No solutions will occur when $m^2 - 2m - 6 = 0$
$m = 1 \pm \sqrt{7}$.

Or, there are no solutions when the gradients are the same, such as

$$-\frac{m}{3} = -\frac{2}{m-2}$$
$$m(m-2) = 6$$
$$m^2 - 2m - 6 = 0$$

**d** Write the particular pair of simultaneous linear equations for $m = 1 + \sqrt{7}$. Hence, give a reason why there is no solution. **2 marks**

Simultaneous equations $(1 + \sqrt{7})x + 3y = 6$ and $2x + (\sqrt{7} - 1)\,y = \sqrt{7}$, by CAS substitution.

The graphs are parallel.

**e** Show that the gradients of these two lines are equal. **3 marks**

Equation 1

$$y = \frac{(1 + \sqrt{7})}{-3}x + 2$$

Gradient of equation 1

$$\frac{(1 + \sqrt{7})}{-3}$$
$$= -\frac{\sqrt{7} + 1}{3}$$

Equation 2

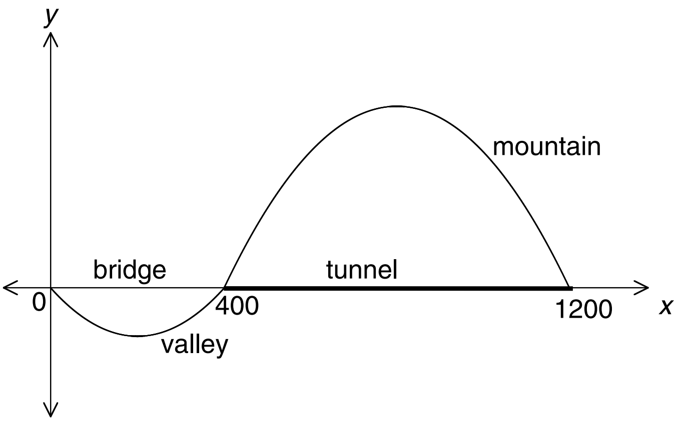
$$y = \frac{-2}{\sqrt{7} - 1}x + \frac{\sqrt{7}}{\sqrt{7} - 1}$$

Gradient of equation 2

$$\frac{-2}{\sqrt{7} - 1} \times \frac{\sqrt{7} + 1}{\sqrt{7} + 1}$$
$$= \frac{-2(\sqrt{7} + 1)}{7 - 1}$$
$$= -\frac{\sqrt{7} + 1}{3}$$

**QUESTION 18** **Total 7 marks**

Part of a new railway line is to pass across a new bridge over a valley then through a new tunnel in a mountain. The diagram shows a cross-section of the route.




The outline of the cross-section of the valley and the mountain can be modelled by the hybrid function

$$y = \begin{cases} \frac{3}{2000}x(x - 400), & 0 \leq x \leq 400 \\ -\frac{3}{4000}(x - 800)^2 + 120, & 400 < x \leq 1200 \end{cases}$$

where $y$ is the height above the bridge and $x$ is the horizontal distance from the start of the bridge. All measurements are in metres.

**a** What is the height, in metres, of the mountaintop above the bridge? **2 marks**

Since the height of the mountain is found between 400 m and 1200 m horizontally from the start of the bridge, $y = -\frac{3}{4000}(x - 800)^2 + 120$ is the required formula. The graph is a parabola with maximum value of 120 m, when $x = 800$. The height of the mountain is 120 m.

**b** What is the distance from the bridge to the bottom of the valley floor? **2 marks**

$y = \frac{3}{2000}x(x - 400)$ is the required quadratic formula for the depth of the valley. By symmetry, the lowest point is when $x = 200$.

$y = \frac{3}{2000} \times 200(200 - 400)$

$= -60$

The valley floor is 60 m below the bridge.

**c** During the wet season, the valley is subjected to flash flooding. As a safety precaution, it is decided to raise the level of the bridge and the tunnel by 20 metres. What are the revised lengths for each of the bridge and the tunnel? (Correct to the nearest metre.)
Note: the bridge will still start at $x = 0$. **3 marks**

Using CAS:

The bridge: From the graph, find the point of intersection between the equation for the mountain and $y = 20$. The length of the bridge is 435 m.

The tunnel: The second point of intersection is 1165 m. The length of the tunnel is 730 m.

**QUESTION 19** **Total 11 marks**

A function $f$ has the rule $f(x) = 2 - 2x$, where $-2 < x \leq 3$.

**a** Write $f$ in function notation. **1 mark**

$f : (-2, 3] \rightarrow \mathbf{R}$, $f(x) = 2 - 2x$

**b** Find $f(0)$ and $\{x : f(x) = 0\}$. **2 marks**

$f(0) = 2 - 2 \times 0$

$= 2$

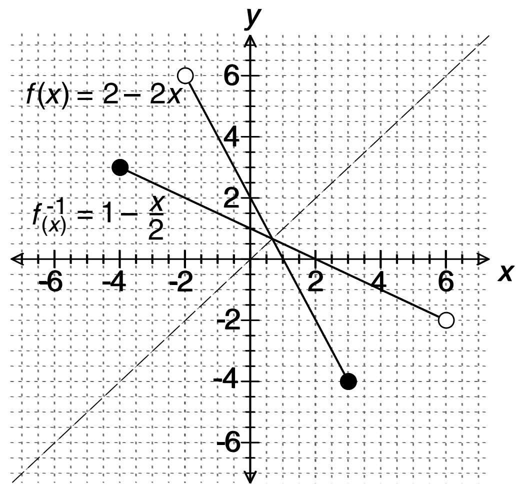
$f(x) = 0$

$2 - 2x = 0$

$2x = 2$

$x = 1$

**c** Sketch the graph of $f$ on the axes provided. **2 marks**




**d** State the domain and range of $f^{-1}$, the inverse of $f$. **2 marks**

The domain of the inverse is $[-4, 6)$. The range of the inverse is $(-2, 3]$.

**e** Find the rule of $f^{-1}$. **1 mark**

Rule of inverse:

$$x = 2 - 2y$$
$$2y = 2 - x$$
$$y = 1 - \frac{x}{2}$$
$$f^{-1}(x) = 1 - \frac{x}{2}$$

**f** Find the point of intersection between $f$ and $f^{-1}$. **1 mark**

The point of intersection is given by:

$x = 2 - 2x$

$3x = 2$

$x = \frac{2}{3}$

$\therefore$ The point of intersection is $\left(\frac{2}{3}, \frac{2}{3}\right)$.

**g** Sketch the graph of $f^{-1}$ on the same set of axes (part **c**), as the graph of $f$. **2 marks**

See the graph in part **c** above.