Test 1

Section A: Technology free. 39 marks
Section B: CAS technology assumed. 26 marks
Suggested time: 80 minutes

Section A: Short answer and extended response questions. Technology free.

Specific instructions to students

- Answer **all** questions in the spaces provided.
- A decimal approximation will not be accepted if an **exact** answer is required to a question.
- In questions where more than one mark is available, appropriate working **must** be shown.

QUESTION 1 — 2 marks

Solve $\frac{3x-1}{4} - \frac{2x+3}{5} \geq x$ for x.

$5(3x-1) - 4(2x+3) \geq 20x$
$15x - 5 - 8x - 12 \geq 20x$
$-13x \geq 17$
$x \leq -\frac{17}{13}$

QUESTION 2 — Total 4 marks

a Transpose $V = \frac{2R}{R-r}$ to make r the subject. **2 marks**

$V(R-r) = 2R$
$VR - Vr = 2R$
$Vr = VR - 2R$
$r = \frac{VR-2R}{V}$ or $r = R\left(1 - \frac{2}{V}\right)$

b Find r when $V = 4$ and $R = \sqrt{2}$ in simplest surd form. **2 marks**

$r = \sqrt{2}\left(1 - \frac{2}{4}\right)$
$= \sqrt{2}\left(1 - \frac{1}{2}\right)$
$= \frac{\sqrt{2}}{2}$

QUESTION 3 — Total 7 marks

Factorise the following:

a $x^4 - 5x^2 - 6$

i Over the rational numbers. **2 marks**

$(x^2)^2 - 5(x^2) - 6$
$(x^2 - 6)(x^2 + 1)$

ii Over the real numbers. **1 mark**

$(x - \sqrt{6})(x + \sqrt{6})(x^2 + 1)$

b $2x^3 + 54$ **2 marks**

$2(x^3 + 27)$
$2(x + 3)(x^2 - 3x + 9)$

c $4 - (2x + 1)^2$ **2 marks**

$[2 - (2x + 1)][2 + (2x + 1)]$
$[2 - 2x - 1][2 + 2x + 1]$
$(1 - 2x)(2x + 3)$

QUESTION 4 — 2 marks

Solve $x^2(x^2 - 8x - 9) = 0$ for x.

$x^2(x - 9)(x + 1) = 0$
$x = 0, 9, -1$

QUESTION 5 — 2 marks

Use the quadratic formula to solve $2x(x - 2) = 1$ for x, in simplest form.

$2x^2 - 4x - 1 = 0$
$x = \frac{4 \pm \sqrt{16 - 4 \times 2 \times -1}}{2 \times 2}$,
where $a = 2$, $b = -4$, $c = -1$
$= \frac{4 \pm \sqrt{24}}{4}$
$= \frac{2(2 \pm \sqrt{6})}{4}$
$= \frac{2 \pm \sqrt{6}}{2}$ or $1 \pm \frac{\sqrt{6}}{2}$

QUESTION 6 — Total 4 marks

a Show that $P(x) = 2x^3 + x^2 - 5x + 2$ is exactly divisible by $x + 2$. **1 mark**

$P(-2) = 2 \times (-2)^3 + (-2)^2 - 5 \times (-2) + 2$
$= -16 + 4 + 10 + 2$
$= 0$
By the factor theorem, $x + 2$ is a factor of $P(x)$.

b Hence, find the linear factors of $P(x)$. **3 marks**

$$\begin{array}{r} 2x^2 - 3x + 1 \\ x + 2 \overline{)\,2x^3 + x^2 - 5x + 2} \\ \underline{2x^3 + 4x^2} \\ -3x^2 - 5x \\ \underline{-3x^2 - 6x} \\ x + 2 \\ \underline{x + 2} \\ 0 \end{array}$$

$(x + 2)(2x^2 - 3x + 1)$

$(x + 2)(2x - 1)(x - 1)$

or

$2x^2(x + 2) - 3x(x + 2) + 1(x + 2)$

$(x + 2)(2x^2 - 3x + 1)$

$(x + 2)(2x - 1)(x - 1)$

QUESTION 7 **2 marks**

If $f(x) = -2x^3 + 3x + 7$, find $f(a-1)$, expressed in expanded form.

$$\begin{aligned} f(a-1) &= -2(a-1)^3 + 3(a-1) + 7 \\ &= -2(a^3 - 3a^2 + 3a - 1) + 3a - 3 + 7 \\ &= -2a^3 + 6a^2 - 3a + 6 \end{aligned}$$

QUESTION 8 **Total 5 marks**

The point (5, 2) is dilated by 2 units from the y axis, followed by a reflection in the line $y = x$, followed by a translation of 1 unit to the left and 3 units up.

a Write a matrix for each transformation. **3 marks**

Dilation: $\begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$ Reflection: $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ Translation: $\begin{bmatrix} -1 \\ 3 \end{bmatrix}$

b Find the coordinates of the image point of (5, 2) under the above transformations. **2 marks**

$$\begin{aligned} \begin{bmatrix} x' \\ y' \end{bmatrix} &= \begin{bmatrix} -1 \\ 3 \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 5 \\ 2 \end{bmatrix} \\ &= \begin{bmatrix} -1 \\ 3 \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 10 \\ 2 \end{bmatrix} \\ &= \begin{bmatrix} -1 \\ 3 \end{bmatrix} + \begin{bmatrix} 2 \\ 10 \end{bmatrix} \\ &= \begin{bmatrix} 1 \\ 13 \end{bmatrix} \end{aligned}$$

QUESTION 9 **Total 7 marks**

For the function $f(x) = \dfrac{1}{x-1} + 2$:

a write the equation of any asymptotes. **2 marks**

$x = 1$ and $y = 2$

b write $f(x)$ in the form $\dfrac{ax + b}{x - 1}$. **1 mark**

$$\begin{aligned} f(x) &= \frac{1}{x-1} + \frac{2(x-1)}{x-1} \\ &= \frac{1 + 2x - 2}{x-1} \\ &= \frac{2x - 1}{x-1} \end{aligned}$$

c Evaluate any x or y intercepts of $f(x)$. **2 marks**

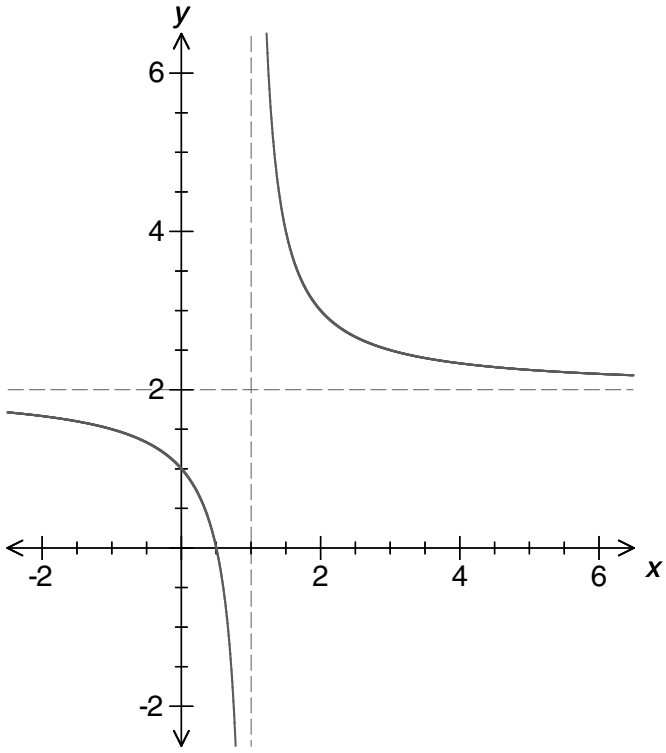
y intercept: $y = \dfrac{-1}{-1}$

$= 1$

x intercept: $2x - 1 = 0$

$x = \dfrac{1}{2}$

d Sketch the graph of $f(x)$. **2 marks**



QUESTION 10 **4 marks**

Find the equation of the polynomial that applies to the following table.

x	0	1	2	3	4	5	6
y	−1	2	11	38	95	194	347

3, 9, 27, 57, 99, 153

6, 18, 30, 42, 54

12, 12, 12, 12

Thus, a cubic polynomial:

$y = ax^3 + bx^2 + cx + d$

$6a = 12$

$a = 2$

(0, −1) implies $d = -1$

$y = 2x^3 + bx^2 + cx - 1$

(1, 2): $2 = 2 + b + c - 1$

$b + c = 1$ equation 1

(2, 11): $11 = 16 + 4b + 2c - 1$

$4b + 2c = -4$

$2b + c = -2$ equation 2

$b = -3$

$c = 4$

∴ The polynomial is $2x^3 - 3x^2 + 4x - 1$.

Section B: Multiple-choice questions. CAS technology assumed.

Specific instructions to students

- A correct answer scores 1, and an incorrect answer scores 0.
- Marks are not deducted for incorrect answers.
- No marks are given if more than one answer is given.
- Choose the alternative which most correctly answers the question and mark your choice on the multiple-choice answer section at the bottom of each page, as shown in the example below.

1 [A] [B] [C] [■] [E] **USE PENCIL ONLY**

- Use pencil only.

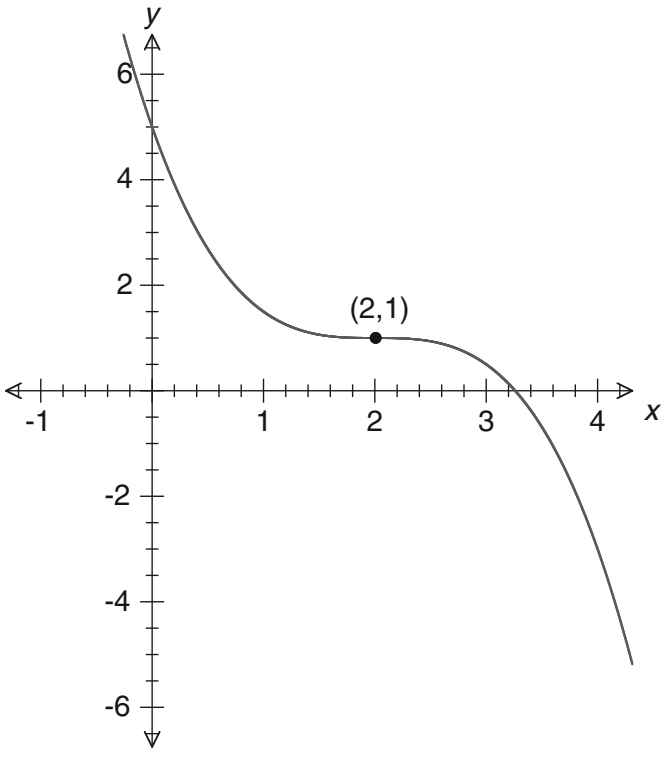
QUESTION 11

The graph of $f(x) = -(x - A)^2 + B$ has which of the following characteristics?

	Turning point	y-intercept
A	$(-A, B)$	$(0, A - B)$
B	(A, B)	$(0, A)$
C	$(-A, -B)$	$(0, B - A)$
D	(A, B)	$(0, B - A)$
E	(A, B)	$(0, B)$

QUESTION 12

The graph of $y = a(x + B)^3 + C$ is shown in the diagram.

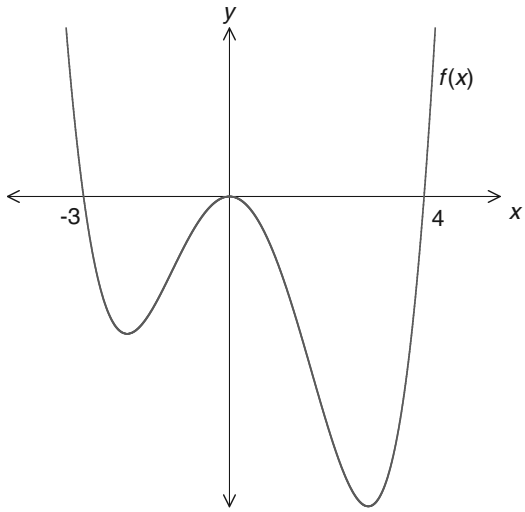


The values of a, B and C are:

	a	B	C
A	$\frac{1}{2}$	-2	1
B	$-\frac{1}{2}$	-2	5
C	$\frac{1}{2}$	2	5
D	$-\frac{1}{2}$	2	1
E	$-\frac{1}{2}$	-2	1

QUESTION 13

The graph of $f(x)$ is shown in the diagram.



The values of x for which $f(x) \geq 0$ are:

A $x < -3$ and $x > 4$

B $x \leq -3$ and $x \geq 4$

C $x = -3$ and 4

D $x \leq -3$ and $x = 0$ and $x \geq 4$

E $-3 \leq x \leq 4$

QUESTION 14

If $2(x - 3)^3 - 1 = 0$, then x is equal to:

A $3 - \frac{1}{\sqrt[3]{2}}$

B $3 + \frac{1}{\sqrt[3]{2}}$

C $3 \pm \sqrt[3]{\frac{1}{2}}$

D $3 - \sqrt[3]{2}$

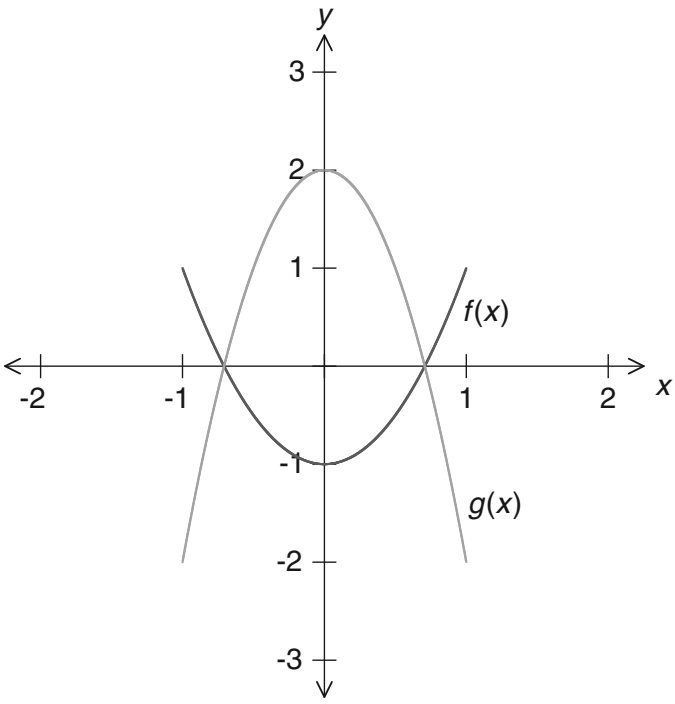
E $3 + \sqrt[3]{2}$

ONE ANSWER PER LINE **USE PENCIL ONLY**

11 [A] [B] [C] [D] [■] [E]
12 [A] [B] [C] [D] [■] [E]
13 [A] [B] [C] [D] [■] [E]
14 [A] [■] [C] [D] [E]

QUESTION 15

The diagram shows the graph of two parabolas, $f(x)$ and $g(x)$.



The graph of $y = f(x)$ is transformed into the graph of $y = g(x)$ by:

A a dilation by a factor of 2 units from the x axis and a reflection in the x axis

B a dilation by a factor of 2 units from the y axis and a reflection in the y axis

C a dilation by a factor of 2 units from the x axis and a reflection in the y axis

D a dilation by a factor of $\frac{1}{2}$ unit from the x axis and a reflection in the x axis

E a dilation by a factor of $\frac{1}{2}$ unit from the y axis and a reflection in the x axis

ONE ANSWER PER LINE **USE PENCIL ONLY**

15 A B C D E

Section B: Extended response questions. CAS technology assumed.

Specific instructions to students

- Answer **all** questions in the spaces provided.
- In questions where more than one mark is available, appropriate working **must** be shown.

QUESTION 16 **Total 6 marks**

A car hire company offers two hiring options, Standard and Premium. The Standard option costs \$35 per day plus \$0.40 per km travelled. The Premium option costs \$60 per day for unlimited travel.

While on holiday, Mary hires a car for d days and drives for a total of k kilometres.

a Write expressions for the total cost, \$C, for the Standard and Premium options, in terms of d and k, for Mary's holiday. **2 marks**

Standard: $C = 35d + 0.4k$

Premium: $C = 60d$

b Mary's holiday is for 6 days. Which plan is the more economic if she plans to travel 500 kms? **1 mark**

Standard: $C = 35 \times 7 + 0.4 \times 500$

$= \$445$

Premium: $C = 60 \times 7$

$= \$420$

$\therefore$ Premium is the more economic.

c Mary decides to extend her holiday by an extra 3 days. Find the minimum kilometres Mary can travel so that the Premium option is cheaper than the Standard option. **3 marks**

Mary travels for a total of 10 days.

Premium: $C = 60 \times 10$

$= \$600$

For the Premium to be cheaper, the Standard must cost more than \$600.

Standard: $600 < 35 \times 10 + 0.4k$

$0.4k > 600 - 350$

$k > 625$ km

Alternatively, solve using CAS.

The Premium option is cheaper than the standard option if Mary travels more than 625 km.

QUESTION 17 **Total 9 marks**

A family of graphs is represented by $f(x) = 2(x + 4)^2 + (3 - k)$, where k is a real number.

a Find the y intercept, in terms of k. **1 mark**

y intercept: $f(0) = 2(0 + 4)^2 + (3 - k)$

$= 35 - k$

Or use CAS.

b Find the x intercepts, in terms of k. Write the answer in the form $a \pm \frac{\sqrt{b(k-3)}}{2}$, where a and b are natural numbers. **2 marks**

$2(x + 4)^2 + (3 - k) = 0$

$(x + 4)^2 = \frac{k - 3}{2}$

x intercept: $x + 4 = \pm\sqrt{\frac{k-3}{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$

$= \pm\frac{\sqrt{2(k-3)}}{2}$

$x = -4 \pm \frac{\sqrt{2(k-3)}}{2}$ or

$\frac{-8 \pm \sqrt{2(k-3)}}{2}$

Or use CAS for working.

c Evaluate the x and y intercepts when $k = 5$. **2 marks**

y intercept: $35 - 5 = 30$

$x = -4 \pm \frac{\sqrt{2(5-3)}}{2}$

x intercept: $= -4 \pm \frac{\sqrt{4}}{2}$

$= -3, -5$

Or use CAS.

d Find the values of the x and y intercepts and the minimum value of $f(x)$, when $k = 8$. Give the value of any x intercepts to four decimal places. **3 marks**

y intercept is 27

x intercepts are -5.5811 and -2.4189

minimum is -5

e For what values of k will $f(x)$ have two x intercepts? **1 mark**

When $f(x)$ has two x intercepts, $k - 3 > 0$

$k > 3$

QUESTION 18 **Total 6 marks**

A cubic graph passes through the points $(1, 2)$, $(-1, 3)$, $(4, 2)$ and $(0, 1)$.

a Using the general cubic equation, $f(x) = ax^3 + bx^2 + cx + d$, write equations in terms of a, b, c and d that can be used to find the equation of the cubic graph that passes through these four points. **2 marks**

$(1, 2)$: $a + b + c + d = 2$

$(-1, 3)$: $-a + b - c + d = 3$

$(4, 2)$: $64a + 16b + 4c + d = 2$

$(0, 1)$: $d = 1$

b Hence, or otherwise, find exact values for a, b, c and d. Write the equation of the cubic function that passes through these four points. **1 mark**

CAS: $a = -\frac{7}{20}$, $b = \frac{3}{2}$, $c = -\frac{3}{20}$, $d = 1$

$f(x) = -\frac{7}{20}x^3 + \frac{3}{2}x^2 - \frac{3}{20}x + 1$

c Find the values of any x intercepts and the coordinates of any stationary points. Give the answers correct to three decimal places. **3 marks**

CAS: The stationary points are $(0.051, 0.996)$, $(2.806, 4.657)$.

The x intercept is 4.339.