	STUDENT NUMBER								LETTER
Figures									
Words									

MATHEMATICAL METHODS (CAS)

Written Examination Unit 1

SAMPLE

Reading time: 15 minutes
Section A writing time: 40 minutes
Section B writing time: 50 minutes

Note: At the end of the 40 minutes, Section A will be collected. Students may use a CAS calculator and annotated notes for the remainder of the examination only after Section A has been collected.

Multiple choice, short answer and extended answer questions

QUESTION AND ANSWER BOOK

Section	*Number of questions*	*Number of questions to be answered*	*Number of marks*
A	10 technology free questions	10	45
B	10 multiple choice questions	10	10
	3 extended questions	3	35

- Students are to write their names in the spaces provided on the front of Section A and on the front page of Section B.
- Section A is technology free and no reference material can be used.
- Section B: an approved CAS calculator and one bound reference text (which may be annotated) or lecture pad may be used.
- Unless otherwise stated, the diagrams in this booklet are not drawn to scale.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Student Name:____

Section A: Short answer and extended response questions. Technology free.

Specific instructions to students

- Answer **all** questions in the spaces provided.
- A decimal approximation will not be accepted if an **exact** answer is required to a question.
- In questions where more than one mark is available, appropriate working **must** be shown.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

QUESTION 1 **4 marks**

Write a matrix expression for the following transformations. Hence find the image of the point $(5, -7)$ under these transformations.

Dilation of -2 in the direction of the x axis, followed by reflection in the y axis, followed by a translation of 3 in the x axis direction and -2 in the y axis direction.

$\begin{bmatrix} 3 \\ -2 \end{bmatrix} + \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \times \begin{bmatrix} -2 & 0 \\ 0 & 1 \end{bmatrix}$ or $\begin{bmatrix} 3 \\ -2 \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$

Image is $\begin{bmatrix} 3 \\ -2 \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} \times \begin{bmatrix} 5 \\ -7 \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \end{bmatrix} + \begin{bmatrix} 10 \\ -7 \end{bmatrix} = \begin{bmatrix} 13 \\ -9 \end{bmatrix}$

QUESTION 2 **2 marks**

Write the factors of $a^3 + b^3$. Hence factorise $(x + 1)^3 + x^3$.

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

$$\begin{aligned}(x + 1)^3 + x^3 &= [(x + 1) + x][(x + 1)^2 - x(x + 1) + x^2] \\ &= [2x + 1][x^2 + 2x + 1 - x^2 - x + x^2] \\ &= (2x + 1)(x^2 + x + 1)\end{aligned}$$

QUESTION 3 **Total 3 marks**

a Without actually dividing, show that $(x - 2)$ is a factor of $P(x) = ax^3 - (2a + 1)x^2 + 3x - 2$. **1 mark**

$P(2) = 8a - 8a - 4 + 6 - 2 = 0$. Hence $(x - 2)$ is a factor.

b When $P(x)$ is divided by $x + 3$ the remainder is 70. Find the value of a. Write the particular polynomial of $P(x)$ for this value of a. **2 marks**

$$\begin{aligned}P(-3) &= -27a - 9(2a + 1) - 9 - 2 = 70 \\ -27a - 18a - 9 - 9 - 2 &= 70 \\ -45a &= 90 \\ a &= -2 \\ P(x) &= -2x^3 + 3x^2 + 3x - 2\end{aligned}$$

QUESTION 4 **Total 3 marks**

a If $z = x\sqrt{y}$ and $y = \dfrac{x}{x + 1}$, show that $z = \dfrac{x\sqrt{x^2 + x}}{x + 1}$. **2 marks**

$$\begin{aligned}z &= x\sqrt{\frac{x}{x + 1}} \\ &= x\sqrt{\frac{x}{x + 1}} \times \frac{\sqrt{x + 1}}{\sqrt{x + 1}} \\ &= \frac{x\sqrt{x}\,\sqrt{x + 1}}{x + 1} \\ &= \frac{x\sqrt{x^2 + x}}{x + 1}\end{aligned}$$

b Find z when $x = 1$. **1 mark**

$$z = \frac{1\sqrt{1+1}}{2} = \frac{\sqrt{2}}{2}$$

QUESTION 5 **2 marks**

Solve the simultaneous equations $2x - y + 7 = 0$ and $2y - 8 = 3x$ for x and y.

$$\begin{array}{l} 2x - y = -7 \\ -3x + 2y = 8 \end{array} \Rightarrow \begin{array}{l} 4x - 2y = -14 \\ -3x + 2y = 8 \end{array} \Rightarrow \begin{array}{l} x = -6 \\ y = -5 \end{array}$$

QUESTION 6 **Total 7 marks**

a Express $-x^2 + 5x + 3$ in the form $a(x + b)^2 + c$. **3 marks**

$$-\left[x^2 - 5x + \frac{25}{4} - 3 - \frac{25}{4}\right]$$
$$-\left[\left(x - \frac{5}{2}\right)^2 - \frac{37}{4}\right]$$
$$-\left(x - \frac{5}{2}\right)^2 + \frac{37}{4}$$

b Hence state the maximum value of $-x^2 + 5x + 3$ and the x value at which it occurs. **2 marks**

Maximum value is $\frac{37}{4}$ when $x = \frac{5}{2}$.

c Solve $-x^2 + 5x + 3 = 0$. **2 marks**

$$\begin{aligned} -\left(x - \frac{5}{2}\right)^2 + \frac{37}{4} &= 0 \\ \left(x - \frac{5}{2}\right)^2 &= \frac{37}{4} \\ x - \frac{5}{2} &= \pm\frac{\sqrt{37}}{2} \\ x &= \frac{5}{2} \pm \frac{\sqrt{37}}{2} \end{aligned}$$

QUESTION 7 **Total 4 marks**

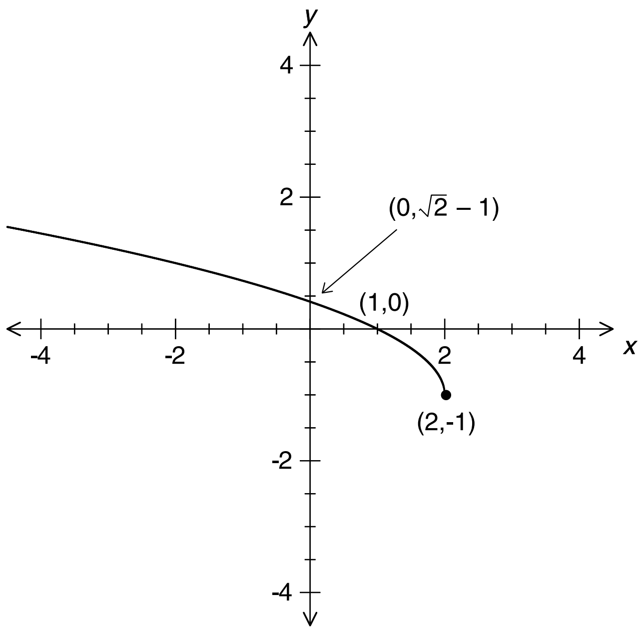
A graph has the equation $y = \sqrt{2 - x} - 1$.

a Find the values of the x and y intercepts. **2 marks**

x intercept ($y = 0$):
$$\begin{aligned} \sqrt{2 - x} - 1 &= 0 \\ \sqrt{2 - x} &= 1 \\ 2 - x &= 1 \\ -x &= -1 \\ x &= 1 \end{aligned}$$

y intercept ($x = 0$): $y = \sqrt{2} - 1$

b Sketch the graph of $y = \sqrt{2-x} - 1$ on the axes provided. **2 marks**



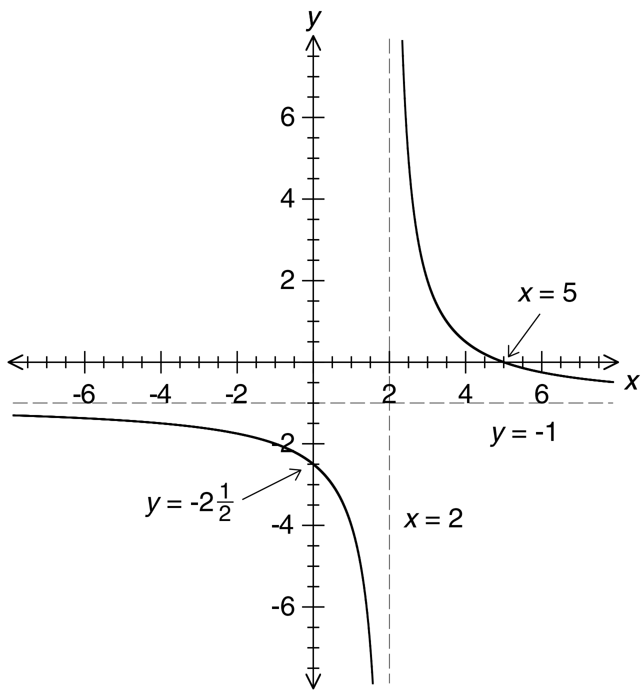
QUESTION 8

Total: 6 marks

a State the equations of any asymptotes for the graph of $y = \dfrac{3}{x-2} - 1$. **2 marks**

Vertical asymptote: $x = 2$; horizontal asymptote: $y = -1$.

b Sketch the graph of $y = \dfrac{3}{x-2} - 1$, locating asymptotes and any x and y intercepts. **3 marks**



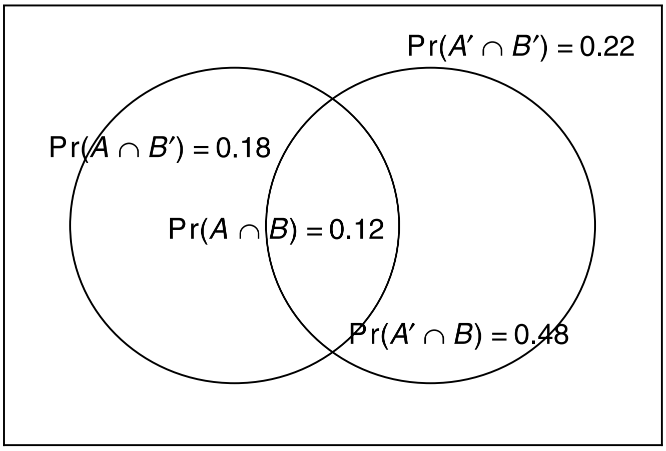
c State the domain of $y = \dfrac{3}{x-2} - 1$. **1 mark**

Domain: $x \in R \setminus \{2\}$

QUESTION 9 **Total 7 marks**

$\Pr(A) = 0.3$, $\Pr(B') = 0.4$ and $\Pr(A \cap B) = 0.12$.

a Construct a Venn diagram to represent this information. **2 marks**



b Hence, or otherwise, find the following:

i $\Pr(B)$ **1 mark**

$\Pr(B) = 1 - \Pr(B') = 1 - 0.4 = 0.6 = 0.78$

ii $\Pr(A \cup B)$ **2 marks**

$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B) = 0.3 + 0.6 - 0.12$

c Are A and B mutually exclusive? Give a reason. **1 mark**

No, since $\Pr(A \cup B) \neq \Pr(A) + \Pr(B)$.

d Are A and B independent events? Give a reason. **1 mark**

No, since $\Pr(A) \times \Pr(B) = 0.18 \neq \Pr(A \cap B)$.

QUESTION 10 **Total 7 marks**

The function f has the rule $f(x) = \sqrt{x + 1}$.

a Write f in function notation. **1 mark**

$f : [-1, \infty) \to \mathbf{R}$, $f(x) = \sqrt{x + 1}$

b Find the rule for f^{-1}, the inverse of f. Also state the domain of f^{-1}. **3 marks**

$x = \sqrt{y + 1}$

$y + 1 = x^2$

$y = x^2 - 1$

Rule is $f^{-1}(x) = x^2 - 1$. Domain f^{-1} = range $f = \mathbf{R}^+ \cup \{0\}$.

c Find the exact coordinates of any point of intersection between the function and its inverse. **3 marks**

At the point of intersection:

$$\begin{aligned} x^2 - 1 &= x \\ x^2 - x - 1 &= 0 \\ x &= \frac{1 \pm \sqrt{1+4}}{2}. \\ &= \frac{1 \pm \sqrt{5}}{2} \end{aligned}$$

Since $x \in \mathbf{R}^+ \cup \{0\}$, $x = \dfrac{1+\sqrt{5}}{2}$

Hence the point of intersection is $\left(\dfrac{1+\sqrt{5}}{2}, \dfrac{1+\sqrt{5}}{2}\right)$.

Student Name:____

Section B: Multiple-choice and extended response questions.
CAS technology assumed.

Specific instructions to students

- A correct answer scores 1, and an incorrect answer scores 0.
- Marks are not deducted for incorrect answers.
- No marks are given if more than one answer is given.
- Choose the alternative which most correctly answers the question and mark your choice on the multiple choice answer section at the bottom of the page.

1 A B C D E — USE PENCIL ONLY

- Use pencil only.

QUESTION 11
The midpoint of the line joining $(x, 4)$ and $(-8, y)$ is $(-1, 8)$. The sum of x and y is:

A 18

B 6

C 26

D 10

E 1

QUESTION 12
A cyclist completes 10 laps of a race circuit. Seven of the laps are at an average speed of 35 km/h and the remaining laps at an average speed of 15 km/h. If the total time of the 10 laps is 48 minutes, what is the length of each circuit?

A 400 m

B 800 m

C 1 km

D 2 km

E 3 km

QUESTION 13
The matrix representation for a dilation of -2 from the x axis, followed by a reflection in the x axis, is:

A $\begin{bmatrix} -2 & 0 \\ 0 & 1 \end{bmatrix}$

B $\begin{bmatrix} 2 & 0 \\ 0 & -1 \end{bmatrix}$

C $\begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix}$

D $\begin{bmatrix} -1 & 0 \\ 0 & \frac{1}{2} \end{bmatrix}$

E $\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$

ONE ANSWER PER LINE — USE PENCIL ONLY

11 A B C D E
12 A B C D E
13 A B C D E

QUESTION 14

The graph of $f(x) = (x - k)^2 + k$ touches, without crossing, the x axis when k is:

A 0

B 1

C -1

D $0, \frac{1}{2}$

E $0, -\frac{1}{2}$

QUESTION 15

The end points of the diameter of a circle are (2, 3) and (−4, 3). The equation of the circle is:

A $(x - 1)^2 + (y + 3)^2 = 9$

B $(x - 1)^2 + (y + 3)^2 = 81$

C $(x + 1)^2 + (y - 3)^2 = 81$

D $(x + 1)^2 + (y - 3)^2 = 9$

E $(x + 1)^2 + (y - 3)^2 = 3$

QUESTION 16

Which of the following is not a one-one function?

A $f(x) = \frac{1}{x^2}, x > 0$

B $f(x) = \sqrt{1 - x}$

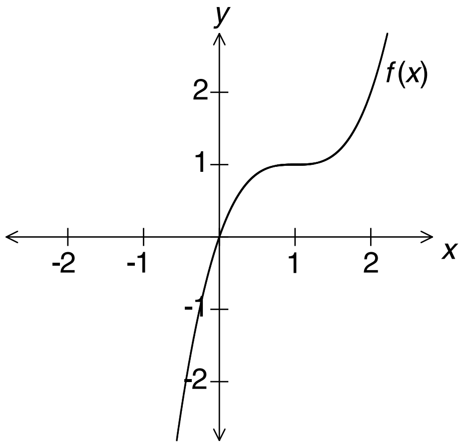
C $f(x) = 1 - x^2$

D $f(x) = \sqrt{4 - x^2}, 0 \le x \le 2$

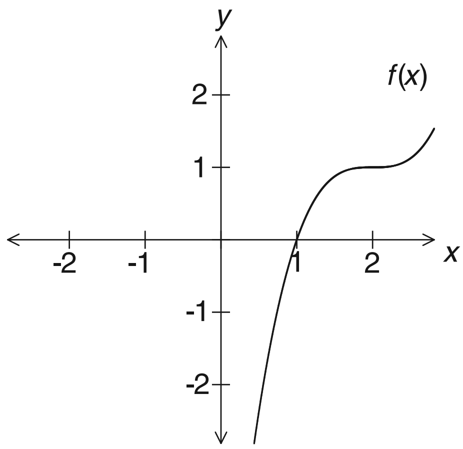
E $f(x) = \frac{1}{x}$

QUESTION 17

The graph of $y = f(x)$ is shown. Which graph could be $y = -f(x + 1) + 2$?



A



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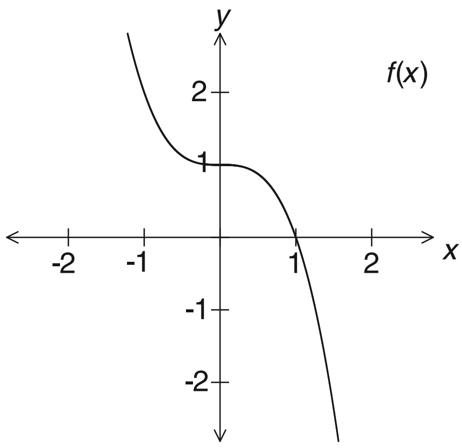
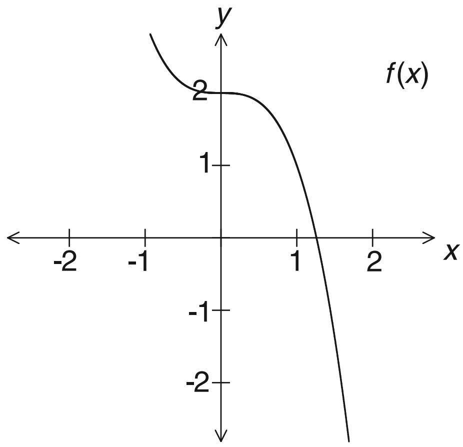
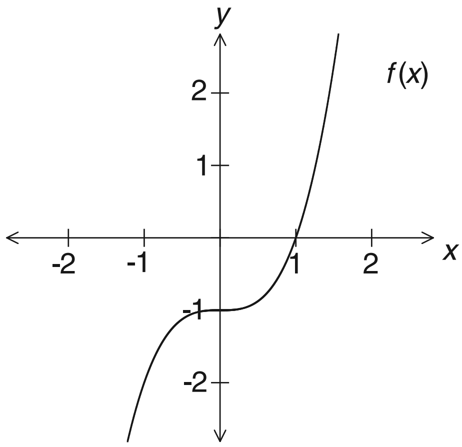


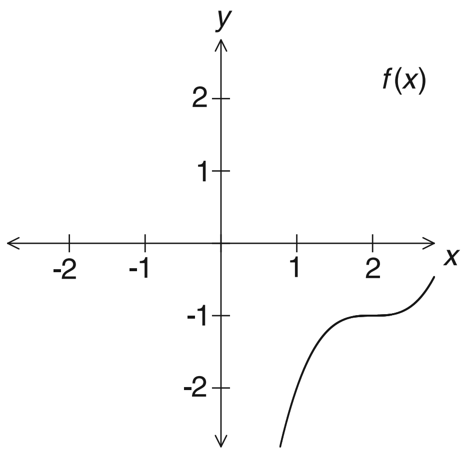
C



D



E



QUESTION 18

Each of 20 students enters one swimming event. Eight choose freestyle, six choose breaststroke, two select butterfly and the rest choose backstroke. A student is selected at random. What is the probability the student chose backstroke?

A $\frac{8}{20}$

B $\frac{1}{4}$

C $\frac{2}{5}$

D $\frac{1}{10}$

E $\frac{1}{5}$

ONE ANSWER PER LINE **USE PENCIL ONLY**

14	A	B	C	D	E	17	A	B	C	D	E
15	A	B	C	D	E	18	A	B	C	D	E
16	A	B	C	D	E						

QUESTION 19

The equation of the parabola shown is:

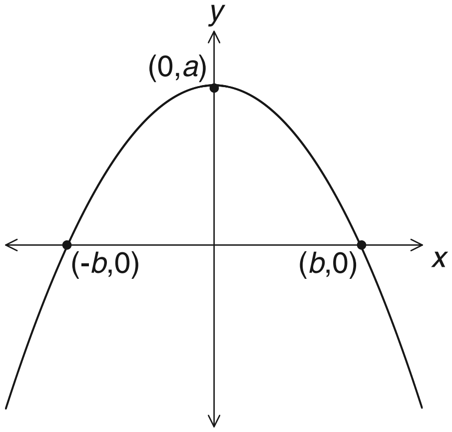
A $y = bx^2 + a$

B $a - bx^2$

C $y = a - \frac{1}{b}x^2$

D $y = a\left(\left(\frac{x}{b}\right)^2 - 1\right)$

E $y = a\left(1 - \left(\frac{x}{b}\right)^2\right)$



QUESTION 20

The image of $y = \sqrt{x}$ under a dilation of 2 from the x axis followed by a reflection in the y axis followed by a translation of 1 unit in the direction of the x axis, has a y intercept of:

A 2

B $\sqrt{2}$

C 1

D -1

E no real solution

ONE ANSWER PER LINE

USE PENCIL ONLY

19 A B C D E

20 A B C D E

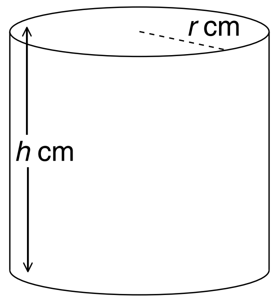
Section B: Extended response questions. CAS technology assumed.

Specific instructions to students

- Answer **all** questions in the spaces provided.
- In questions where more than one mark is available, appropriate working **must** be shown.

QUESTION 21 **Total 11 marks**

A cylindrical water tank has a capacity of 2000 litres.

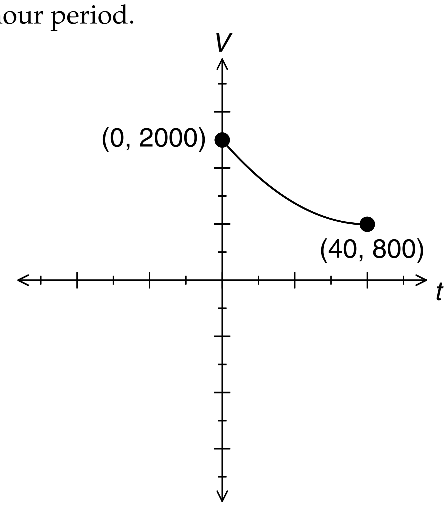


a If the height of the tank is 200 cm, find the radius of the tank, in terms of π. ($\text{Volume}_{\text{cylinder}} = \pi r^2 h$.) **2 marks**

$$2000 \times 1000 = \pi \times r^2 \times 200$$
$$r^2 = \frac{2000 \times 1000}{200 \times \pi}$$
$$r = \frac{100}{\sqrt{\pi}}, \text{ since } r > 0$$

The tank leaks water from a hole in its side. The volume of water remaining in the tank, in litres, over a 40-hour period, is given by $V = \frac{3}{4}(t - 40)^2 + 800$, where t is the time in hours from when the tank began to leak.

b Sketch the graph of $V(t)$ over the 40-hour period. **3 marks**



c What is the percentage of water left in the tank after 20 minutes? **1 mark**

$$V(20) = \frac{3}{4}(20 - 40)^2 + 800 = 1100 \text{ litres.}$$

$$\text{Percentage} = \frac{1100}{2000} \times 100 = 55\%.$$

d If the leak is stopped after 40 hours, what is the height of water left in the tank after the leak is stopped? **3 marks**

$V(40) = \frac{3}{4}(40 - 40)^2 + 800 = 800$ litres left in the tank.

Using $V = \pi r^2 h$ where $r^2 = \frac{10000}{\pi}$ and $V = 800 \times 1000$ cm^3

$h = \frac{V}{\pi r^2} = \frac{800 \times 1000 \times \pi}{\pi \times 10000} = 80$ cm

or use CAS to solve

$800 \times 1000 = \pi \times \frac{10000}{\pi} \times h.$

e Find the average rate of change for the height of water in the tank over the 40 hours. **2 marks**

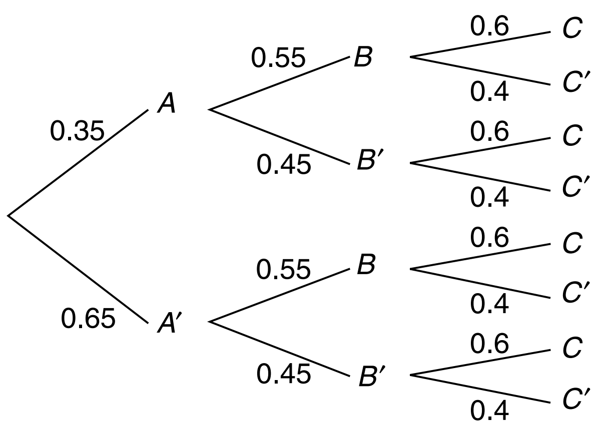
Average rate of change $= \frac{80 - 200}{40 - 0} = -\frac{120}{40} = -30.$

The height of water in the tank drops at a rate of 30 cm per hour.

QUESTION 22

a A psychologist conducts a screening experiment that consists of three tasks, A, B and C. The probability of success at task A is 0.35, at task B is 0.55 and task C is 0.6. **Total 12 marks**

i Draw a tree diagram to represent this information. **2 marks**



ii What is the probability that a person succeeds at all three tasks? **1 mark**

$0.35 \times 0.55 \times 0.6 = 0.1155$

iii If 350 people take the screening experiment, how many would be expected to succeed in all three tasks? **1 mark**

$350 \times 0.1155 = 40.425.$

$\therefore$ 40 people.

To be accepted into a special training program, a person must pass test A and at least one of the other two tests.

iv What is the probability that a person is accepted into the training course? **2 marks**

Passes the three tests = 0.1155

Passes tests A and B, fails $C = 0.35 \times 0.55 \times 0.4 = 0.077$

Passes tests A and C, fails $B = 0.35 \times 0.45 \times 0.6 = 0.0945$

Total probability $= 0.1155 + 0.077 + 0.0945 = 0.287$

b In a certain population, 47% are males. It is also known that 8% of the population are males with a certain gene abnormality and that 12% of the full population have this gene abnormality. M represents a male and G represents the gene abnormality.

i Set up a Karnaugh square to represent this data. **3 marks**

	M	M'	
G	0.08	0.04	0.12
G'	0.39	0.49	0.88
	0.47	0.53	

ii A person is selected at random. What is the probability the person is female and has the abnormal gene? **1 mark**

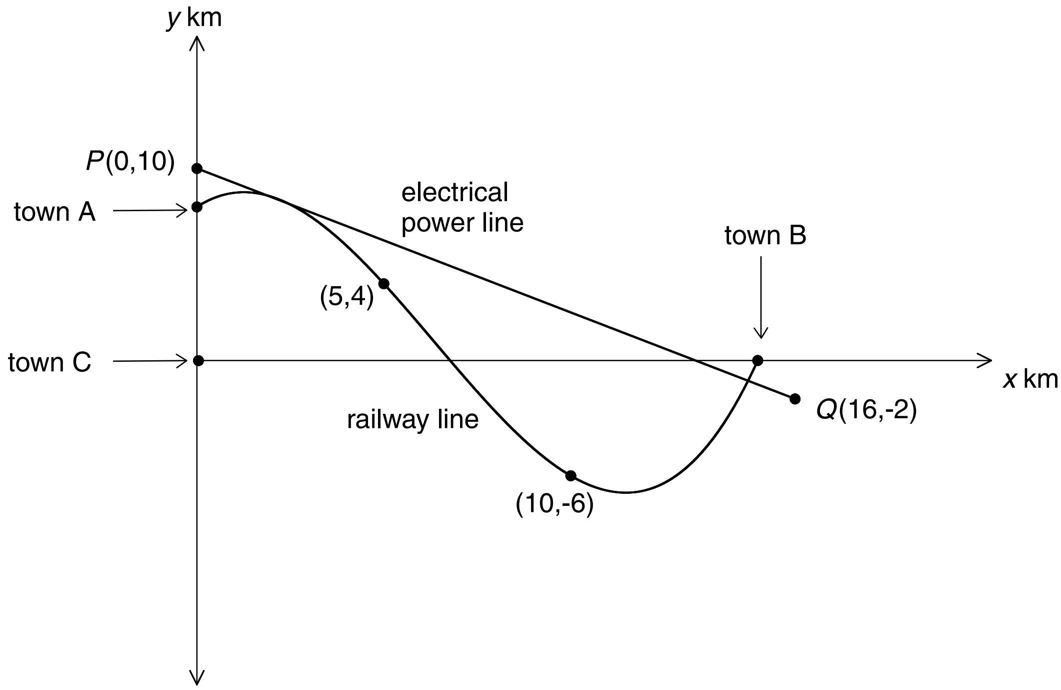
From the Karnaugh square, 0.04.

iii What is the probability the person is male or has the abnormal gene? **2 marks**

From the diagram there are the following possibilities: The person is male and has the gene + is male and does not have the gene + is female and has the gene = 0.08 + 0.39 + 0.04 = 0.51 or 1 − female and does not have the gene = 1 − 0.49 = 0.51

QUESTION 23 **Total 12 marks**

A high-voltage electrical power line runs in a straight line between the points $P(0, 10)$ and $Q(16, -2)$ in the neighbourhood of two towns, A and B, which have coordinates (0, 8) and (15, 0) respectively. All measurements are in kilometres.



a Find the equation of the straight line joining the points P and Q. **2 marks**

Gradient $= \dfrac{-2 - 10}{16 - 0} = -\dfrac{12}{16} = -\dfrac{3}{4}$.

Equation is $y = -\dfrac{3}{4}x + 10$.

A scenic railway line is to be built between the two towns and will also pass through the points (5, 4) and (10, −6). The route of the railway line can be modelled by the polynomial $y = ax^3 + bx^2 + cx + d$.

b Write a matrix equation to find the values of a, b, c and d. **2 marks**

$$\begin{bmatrix} 0 & 0 & 0 & 1 \\ 5^3 & 5^2 & 5 & 1 \\ 10^3 & 10^2 & 10 & 1 \\ 15^3 & 15^2 & 15 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 8 \\ 4 \\ -6 \\ 0 \end{bmatrix}$$

c Hence, or otherwise, find the exact values of a, b, c and d. **1 mark**

Using CAS: $a = \frac{11}{375}$, $b = -\frac{14}{25}$, $c = \frac{19}{15}$, $d = 8$.

A straight road runs from town B to town C, which is at the point (0, 0).

d Determine the distance along this road between the railway crossings, correct to the nearest metre. **2 marks**

Since the road lies along the x axis, the railway crossing will occur at the x intercepts of the polynomial. From the graph, the x intercepts are 6.7747 and 15.

Distance between railway crossings = 15 − 6.7747 = 8.225 km.

e How many times must the railway line pass under the electricity power lines? **1 mark**

From the graphs: 3 times. Twice near town A and once at town B.

In order to reduce the number of times the power line and the railway line are close to each other it is decided to change the route of the railway line from the point (5, 4) to (5, k) , where k is a real number.

f Write a cubic equation for the path of the railway line in terms of k. **1 mark**

CAS: Solve

$$\begin{bmatrix} 0 & 0 & 0 & 1 \\ 5^3 & 5^2 & 5 & 1 \\ 10^3 & 10^2 & 10 & 1 \\ 15^3 & 15^2 & 15 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 8 \\ k \\ -6 \\ 0 \end{bmatrix}$$

$$y = \frac{3k + 10}{750}x^3 + \frac{-5k - 8}{50}x^2 + \frac{9k - 17}{15}x + 8$$

g Find a value of k so that the coefficient of x^3 is zero. Hence write this new equation for the path of the railway line. Give the coordinates of the point (5, k) which lie on this new line. **3 marks**

$3k + 10 = 0$

$3k = -10$

$k = -\frac{10}{3}$

The new equation is $y = \frac{13}{75}x^2 - \frac{47}{15}x + 8$. The coordinates are $\left(5, -3\frac{1}{3}\right)$.

Mathematical Methods (CAS) Formulas

Mensuration

area of trapezium:	$\frac{1}{2}(a+b)h$
curved surface area of a cylinder:	$2\pi rh$
volume of a cylinder:	$\pi r^2 h$
volume of a cone:	$\frac{1}{3}\pi r^2 \mathrm{h}$
volume of a pyramid:	$\frac{1}{3}Ah$
volume of a sphere:	$\frac{4}{3}\pi r^3$
area of a triangle:	$\frac{1}{2}bc \sin A$

Matrix algebra

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$A^{-1} = \frac{1}{\triangle}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$\triangle = ad - bc$$

Calculus

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$$\int x^n \, dx = \frac{1}{n+1}x^{n+1} + c, n \neq -1$$

Probability

$$\Pr(A) = 1 - \Pr(A')$$

$$\Pr(A \mid B) = \frac{\Pr(A \cap B)}{\Pr(B)}$$

$$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$$