

Student Name: _____

MATHEMATICAL METHODS

Units 3 & 4 – Written examination 2



2007 Trial Examination

Reading Time: 15 minutes

Writing Time: 2 hours

QUESTION AND ANSWER BOOK

Structure of book

<i>Section</i>	<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
1	22	22	22
2	4	4	58
			Total 80

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, and rulers, a protractor, set-squares, aids for curve sketching, one bound reference and an approved **graphics** calculator and if desired, one scientific calculator.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied

- Question and answer book of 26 pages.

Instructions

- Print your name in the space provided on the top of this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

SECTION 1

Instructions for Section 1

Answer **all** questions.

Choose the response that is **correct** or **best answers** the question.

A correct answer scores 1, an incorrect answer scores 0.

No mark will be given if more than one answer is completed for any question.

Marks will **not** be deducted for incorrect answers

Question 1

The range of the function $f : [-1, 5) \rightarrow R, f(x) = 3 - x$ is

- A. $[-2, 4]$
- B. $(-2, 4)$
- C. R
- D. $(-2, 4]$
- E. $(-2, \infty)$

Question 2

If $f(x) = \cos x$ and $g(x) = \sqrt{x}$, then the domain of $f(g(x))$ is

- A. $(-\infty, \infty)$
- B. $[0, \infty)$
- C. R
- D. $(0, \infty)$
- E. $[-1, 1]$

Question 3

The function $f: [1, \infty) \rightarrow R, f(x) = 3e^{x-1} + 2$ has an inverse function $f^{-1}(x)$.

The function $f^{-1}(x)$ is given by

- A. $f^{-1} : [5, \infty) \rightarrow R, f^{-1}(x) = 3e^{x-1} + 2$
- B. $f^{-1} : [1, \infty) \rightarrow R, f^{-1}(x) = \log_e \left(\frac{x-2}{3} \right)$
- C. $f^{-1} : [5, \infty) \rightarrow R, f^{-1}(x) = 1 + \log_e \left(\frac{x-2}{3} \right)$
- D. $f^{-1} : [1, \infty) \rightarrow R, f^{-1}(x) = 1 + \log_e \left(\frac{x-2}{3} \right)$
- E. $f^{-1} : R \rightarrow R, f^{-1}(x) = \frac{x+2}{3} - e^{-1}$

SECTION 1- continued

Question 4

The graph of $y = -\log_2(3-x) + 2$ is obtained from the graph of $y = \log_2(x)$ by

- A. a reflection in the x and y -axis, a translation by 3 units right and a translation of 2 units up.
- B. a reflection in the y -axis, a translation by 3 units right and a translation of 2 units up.
- C. a reflection in the x -axis, a translation by 3 units right and a translation of 2 units up.
- D. a reflection in the y -axis, a translation by 3 units left and a translation of 2 down.
- E. a dilation factor of 2, a translation by 3 units left and a translation of 2 units up.

Question 5

For the function $f : R \rightarrow R$, where $f(x) = a \sin(bx) + c$, where a , b and c are positive constants, the period, amplitude and range are respectively

- A. $\frac{2\pi}{a}$, a , $[a+c, -a+c]$
- B. $\frac{2\pi}{b}$, a , $[-a+c, a+c]$
- C. $\frac{2\pi}{b}$, a , $[-a, a]$
- D. $\frac{2\pi}{a}$, $-a$, $[a, b]$
- E. $\frac{2\pi}{x}$, c , $[-a, a]$

Question 6

The sum of the solutions of the equation $2\cos\left(2x + \frac{\pi}{6}\right) = \sqrt{3}$ in the interval $[0, \pi]$ is equal to

- A. $\frac{\pi}{6}$
- B. 2π
- C. $\frac{11\pi}{6}$
- D. π
- E. 4π

Question 7

For the graph of $y = |3\cos 2x|$ over the domain $[0, 2\pi]$ then the range is

- A. $[-3, 3]$
- B. R
- C. $[0, 3]$
- D. $(0, 3]$
- E. $(0, 3)$

SECTION 1- continued
TURN OVER

Question 8

If $2 \log_e 3x - 1 = \log_e a$ where $a > 0$, then x is equal to

- A. $\frac{a}{9}$
- B. $\frac{\sqrt{a}}{3}$
- C. $\frac{ae}{3}$
- D. $\frac{\sqrt{ae}}{3}$
- E. $\frac{a}{e}$

Question 9

The derivative of $y = \frac{\cos(2x)}{3e^x - x}$ is

- A. $\frac{2 \sin 2x(3e^x - x) - \cos 2x(3e^x - 1)}{3e^x - x}$
- B. $\frac{-2 \sin 2x(3e^x - x) - \cos 2x(3e^x - 1)}{9e^{2x} - 6xe^x + x}$
- C. $\frac{2 \sin 2x(3e^x - x) - \cos 2x(3e^x - 1)}{9e^{2x} - 6xe^x + x^2}$
- D. $\frac{-2 \sin 2x(3e^x - x) - \cos 2x(3e^x - 1)}{9e^{2x} - 6xe^x + x^2}$
- E. $\frac{2 \sin 2x(3e^x - x) - \cos 2x(3e^x - 1)}{(3x^x - x)^2}$

Question 10

A spherical balloon of Volume, $V \text{ cm}^3$ has a radius, $r \text{ cm}$ where $V = \frac{4}{3}\pi r^3$.

When $r = 7$ the rate of increase in volume of the balloon with respect to the radius is

- A. $49\pi \text{ cm}^3/\text{cm}$
- B. $196\pi \text{ cm}^3/\text{cm}$
- C. $169\pi \text{ cm}^2/\text{cm}$
- D. $\frac{196}{\pi} \text{ cm}^2/\text{cm}$
- E. $196\pi \text{ cm}^2/\text{cm}$

Question 11

If $f(x) = 2x^3 + ax^2 + bx$ has a stationary point at $(-2, -4)$ then the values of a and b respectively are

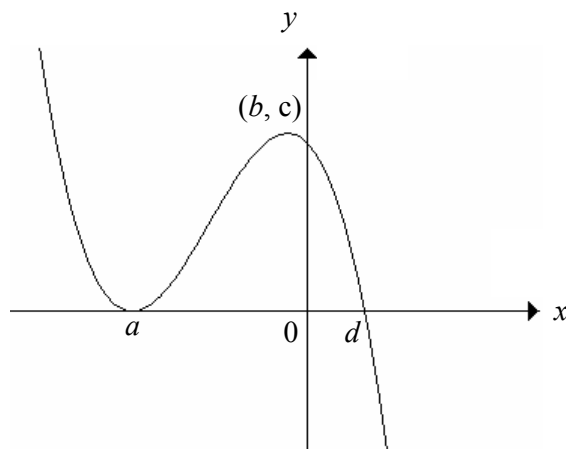
- A. 9 and 12
- B. 12 and 9
- C. -9 and -12
- D. 5 and 4
- E. 9 and -12

Question 12

An approximate value for $\frac{1}{\sqrt{9.5}}$ is

- A. $-\frac{1}{54} \times 0.5 + \frac{1}{3}$
- B. $\frac{1}{54} \times 0.5 + \frac{1}{3}$
- C. $-\frac{1}{54} \times 0.5 - \frac{1}{3}$
- D. $\frac{1}{54} \times 0.5 - \frac{1}{3}$
- E. $\frac{1}{3} - \frac{1}{54} \times 0.5$

SECTION 1- continued
TURN OVER

Question 13

For the graph of $y = f(x)$ shown above $f'(x)$ is negative when

- A. $x > -a$ or $x < a$
- B. $-a < x < a$
- C. $x < a$ or $x > b$
- D. $-a < x < b$ or $x > d$
- E. $-\infty < x < d$

Question 14

The hybrid function $f(x) = \begin{cases} 9 - x^2, & x \leq 3 \\ x - 3, & x > 3 \end{cases}$ is

- A. differentiable but not continuous at $x = 3$
- B. neither continuous nor differentiable at $x = 3$
- C. continuous and smooth at $x = 3$
- D. continuous and differentiable for all $x \in \mathbb{R}$
- E. continuous but not differentiable at $x = 3$

SECTION 1- continued

Question 15

A random variable X , has its frequency curve defined as

$$f(x) = \begin{cases} \frac{1}{2} e^{-\frac{1}{2}x} & , x > 0 \\ 0 & , \text{elsewhere} \end{cases}$$

The probability X is smaller than 2 is

- A. -0.3679
- B. 1.367
- C. -0.6321
- D. 0.5
- E. 0.6321

Question 16

A manufacturer who produces sheets of coloured paper believes there is a probability of 0.2 of any one sheet being defective. The coloured paper is packed 10 to a box. The probability that exactly half of the coloured paper per box is defective is

- A. 0.0026
- B. 0.9736
- C. 0.246
- D. 0.0264
- E. 0.264

Question 17

The discrete random variable X has the probability distribution given by

x	1	2	3	4
$\Pr(X=x)$	0.2	$3k$	$2a$	$3a$

If the mean is 2.6, then the values of a and k respectively are

- A. $a = 0.1$ and $k = 0.1$
- B. $a = 0.1$ and $k = 0.9$
- C. $a = 0.6$ and $k = 0.2$
- D. $a = 0.6$ and $k = 0.3$
- E. $a = 0.4$ and $k = 0.5$

SECTION 1- continued
TURN OVER

Question 18

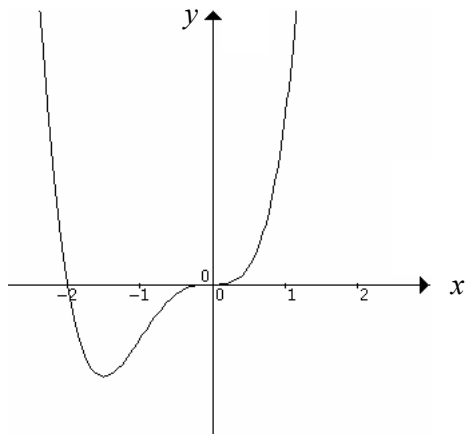
A normal random variable X has a mean of 16 and a standard deviation of 4, then the probability that X is greater than or equal to 20 is

- A. $\Pr(X \geq 1)$
- B. $1 - \Pr(z \geq 1)$
- C. $\Pr(z \geq 20)$
- D. $\Pr(z \geq 1)$
- E. $\Pr(z \geq -1)$

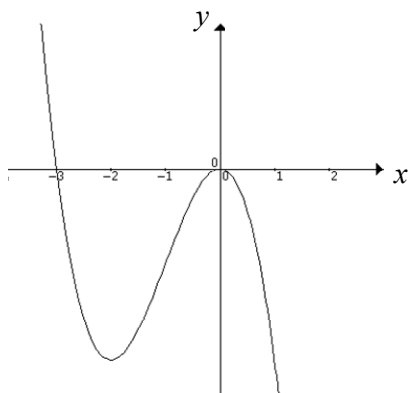
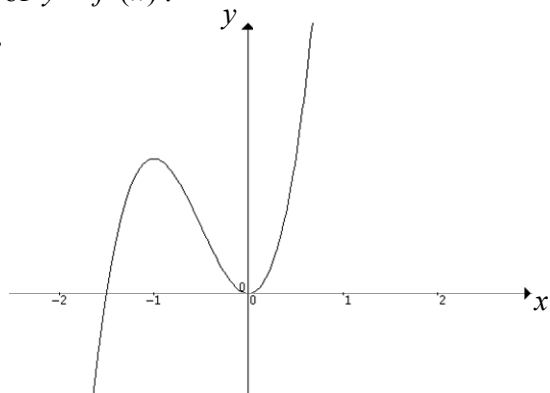
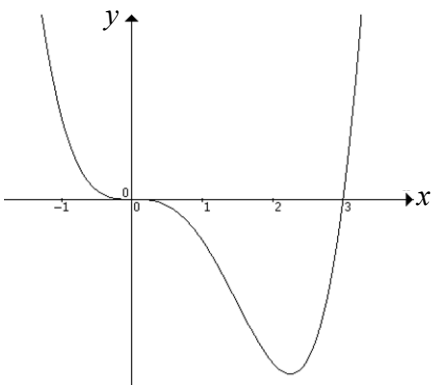
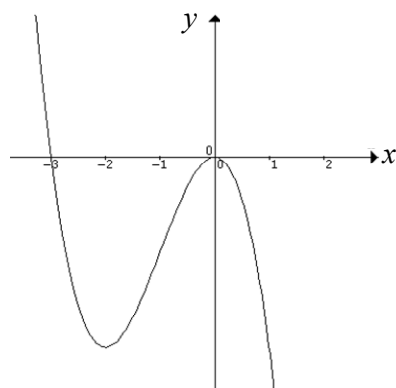
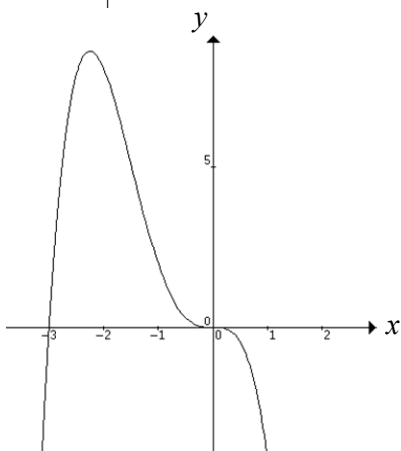
SECTION 1- continued

Question 19

The graph of the function f with $y = f(x)$ is shown below.



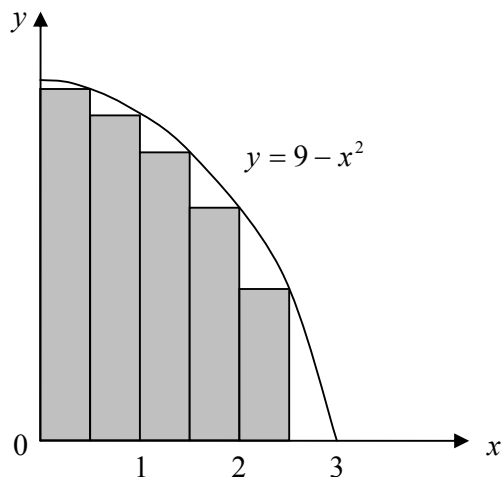
Which one of the following could be the graph of $y = f'(x)$?

A.**B.****C.****D.****E.**

SECTION 1- continued
TURN OVER

Question 20

The total area of the shaded rectangles can be used as an approximation for the area bounded by the curve $y = 9 - x^2$, and the x -axis.



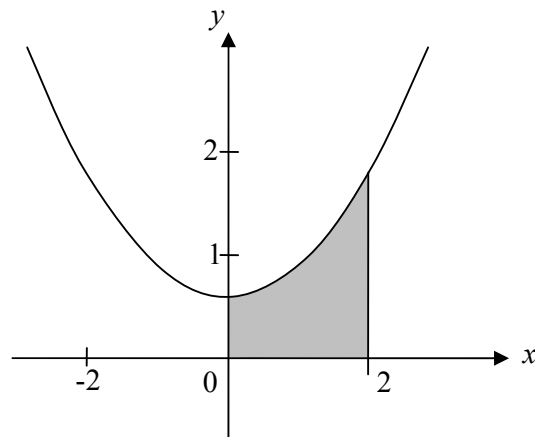
The area of the shaded region shown above is

- A. 15.625 units squared
- B. 16.625 units squared
- C. 17.295 units squared
- D. 31.25 units squared
- E. 18 units squared

SECTION 1- continued

Question 21

The graph with equation $y = m(x^2 + 2)$ where $m > 0$ is shown below.



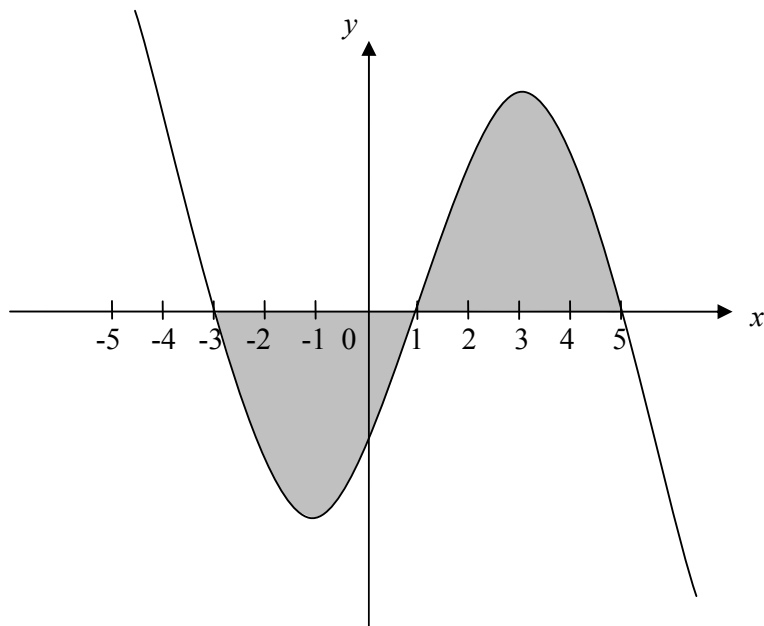
If the area of the shaded region is 2, then the value of m is

- A. 10
- B. 3
- C. $\frac{3}{10}$
- D. $\frac{10}{3}$
- E. $\frac{1}{10}$

SECTION 1- continued
TURNOVER

Question 22

Which of the following integrals represents the shaded area shown in the graph below?



- A. $\int_{-3}^1 f(x)dx + \int_1^5 f(x)dx$
- B. $-\int_{-3}^1 f(x)dx + \int_1^5 f(x)dx$
- C. $\left| \int_{-3}^5 f(x)dx \right|$
- D. $2 \int_{-3}^1 f(x)dx$
- E. $\int_{-3}^5 f(x)dx$

END OF SECTION 1

SECTION 2**Instructions for Section 2**

Answer **all** questions in the spaces provided.

A decimal approximation will not be accepted if an **exact** answer is required to a question.

In questions where more than one mark is available, appropriate working must be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1

Let f be the function $f : D \rightarrow R$, $f(x) = \log_e(5 - 2x)$, where D is the largest possible domain for which f is defined.

- a. Find the **exact** co-ordinates of the x and y intercepts of the graph of $y = f(x)$.

2 marks

- b. Find D the **largest** possible domain over which f is defined.

1 mark

- c. Use calculus to show that the rate of change is always negative.

2 marks

SECTION 2-Question 1- continued
TURN OVER

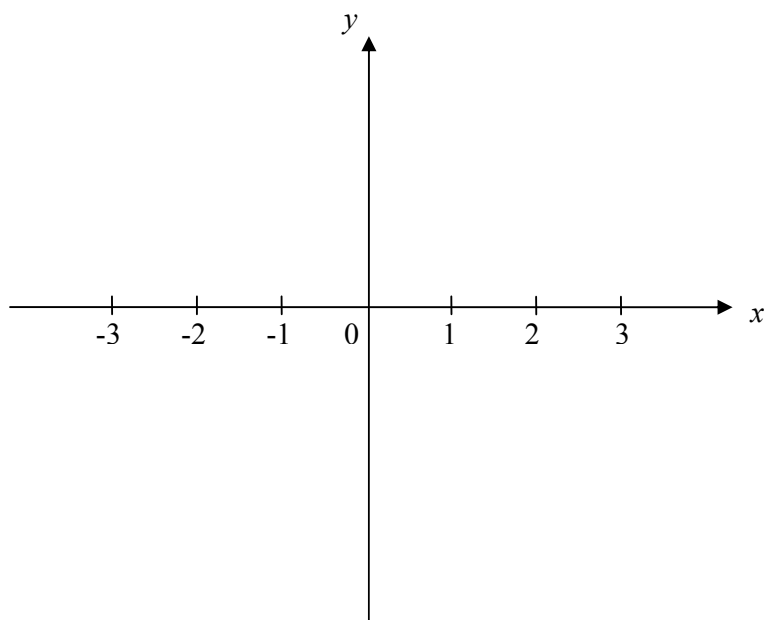
d.

- i.**
- Find
- $f^{-1}(x)$
- , the rule for
- f^{-1}
- .

- ii.**
- State the domain and range of
- f^{-1}
- .

2 + 1 = 3 marks

- e.**
- Sketch the graph of
- $f^{-1}(x)$
- . Clearly label any intercepts with the axes and vertical or horizontal asymptotes with its equation.



2 marks

SECTION 2-Question 1- continued

- f. Using calculus find the exact area bounded by the graph of $y = f^{-1}(x)$ the x -axis and y -axis.

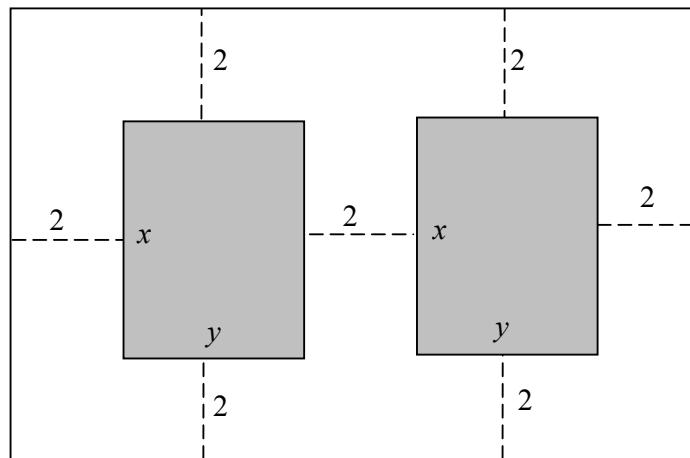
2 marks

Total 12 marks

SECTION 2-continued
TURN OVER

Question 2

The diagram below shows a design for a shop with two identical display windows situated on a wall. The two windows are equally spaced so that all of the dotted lines are 2 metres long. The total area of each window is to be 12 square metres.



- a. Show that $y = \frac{12}{x}$ where y is the width of a window.

2 marks

- b. Find the length of the wall, L in terms of x .

1 mark

SECTION 2-Question 2- continued

- c. Find the height of the wall, H in terms of x .

1 mark

- d. Show that the total area, A square metres of the wall excluding the windows is a function of the window height x metres is modeled by

$$A = 24 + 6x + \frac{96}{x}$$

3 marks

SECTION 2-Question 2- continued
TURN OVER

- e. Find $\frac{dA}{dx}$.

1 mark

- f. Find the minimum value of x for which the area, A is a minimum. Hence find the value of A .

3 marks

- g. Using calculus verify that the area is a minimum.

2 marks

SECTION 2-Question 2- continued

- h.** Sketch the graph of A against x .



2 marks

- i.** Find the width of the window, y , which will give a minimum area, A .

1 mark

Total 16 marks

SECTION 2-continued
TURN OVER

Question 3

On 'Lee's Celebrity Quiz' show, each team has to answer a total of 20 questions. There are 5 questions in each category and 4 categories are covered in each show.

The score X , represents the number of correctly answered questions out of a set of 5 questions in a particular category.

The probability of the celebrity team obtaining a particular score of X in the category of **Music** is shown in the table below:

x	0	1	2	3	4	5
$\Pr(X=x)$	0.06	0.04	0.3	0.1	0.2	0.3

- a. Find the probability that on the next Quiz Show the celebrity team will answer
- all the Music questions correctly.

- less than half of their Music questions correctly.

1 + 2 = 3 marks

SECTION 2-Question 3- continued

For the celebrity team, the probability of obtaining a particular value of x in the category of **Maths** is shown in the table below:

x	0	1	2	3	4	5
$\Pr(X=x)$	p	$3q$	q	p	p	p

- b. If $p = 4q$, find the probability that on the next Quiz Show the celebrity team will answer three of their Maths questions correctly.

3 marks

- c. Find the probability that the celebrity team scores a total of ten for their Maths and Music questions on the next Quiz Show.

1 mark

SECTION 2-Question 3- continued
TURN OVER

The Quiz Show has a regular team of celebrities who frequently appear on the show.
The probability of this regular team of celebrities appearing on the show is 0.85.
The teams are chosen at random.

- d. Find the probability that the regular celebrity team will compete on the next six shows.

2 marks

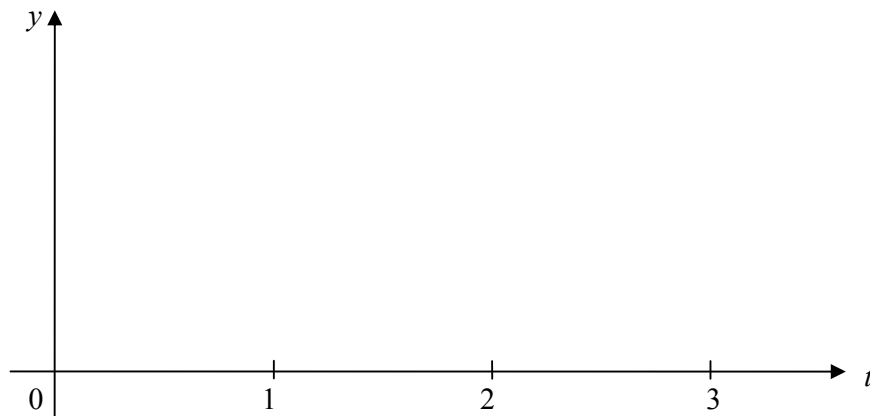
- e. For how many successive shows coming up will there be a better than even chance of having the regular celebrity team on the show?

2 marks

When Lee is on the Celebrity Quiz Show, the time, t hours, that he spends on the show is a continuous random variable with probability density function given by

$$f(t) = \begin{cases} t^3 - 9t^2 + 26t - 24 & , \quad 2 \leq t \leq 3 \\ 0 & , \quad \text{otherwise} \end{cases}$$

- f. Sketch the graph of $f(t)$ on the axes below. Label any stationary points with their coordinates, correct to two decimal places.



2 marks

SECTION 2-Question 3- continued

- g. What is the probability, correct to three decimal places, that Lee spends less than 150 minutes on the quiz show?

2 marks

- h. Using calculus, find the expected time, to the nearest minute, that he spends on the quiz show.

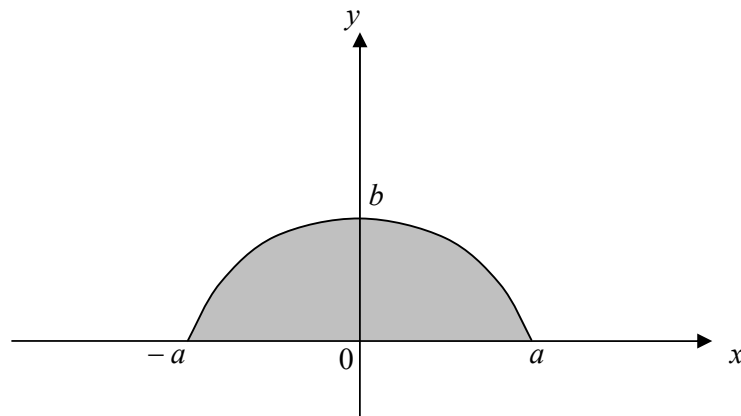
2 marks

Total 17 marks

SECTION 2-continued
TURN OVER

Question 4

The entrance to a shopping centre is given by the curve $y = 4 - \frac{1}{2}e^{\frac{x}{2}} - \frac{1}{2}e^{-\frac{x}{2}}$ as shown in the diagram below:



- a. The y -axis intercept of the graph represents the height of the entrance door is b . Show that $b = 3$.

1 mark

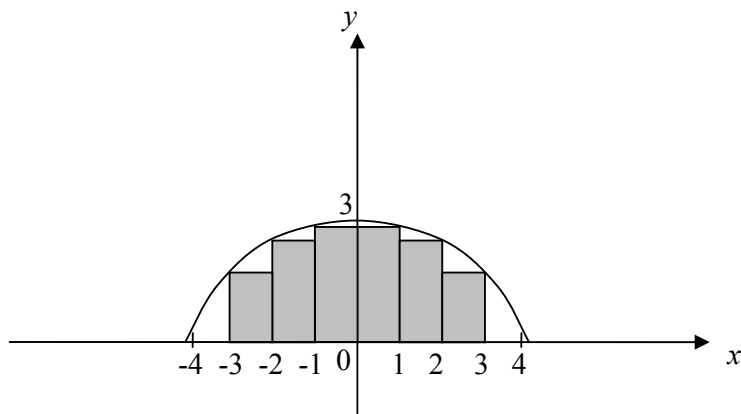
- b. Find the exact value of a . Show your working.

4 marks

SECTION 2-Question 4- continued

c. To approximate the area of the door, assume the value of $a = 4$.

- i. Use the curve of $y = 4 - \frac{1}{2}e^{\frac{x}{2}} - \frac{1}{2}e^{-\frac{x}{2}}$ to find the approximate area of the door correct to two decimal places, using rectangles of width 1 metre as shown in the diagram below:



- ii. The shopping centre uses this to approximation to estimate the cost of painting the entrance door. If the cost is \$35 per square metre, find the cost of painting the door.

2 + 1 = 3 marks

SECTION 2-Question 4- continued
TURN OVER

- d. The area of the entrance door to the shopping centre is approximated with a parabola by using the points $(0, 3)$, $(4, 0)$, $(-4, 0)$. Find the equation of this parabola.

3 marks

- e. Use calculus to find the area enclosed by the parabola and the x -axis, giving answer correct to two decimal places.

2 marks

Total 13 marks

END OF QUESTION AND ANSWER BOOK