

Student Name: _____

MATHEMATICAL METHODS

Units 3 & 4 – Written examination 1



2007 Trial Examination

Reading Time: 15 minutes

Writing Time: 1 hour

QUESTION AND ANSWER BOOK

Structure of Book

<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
11	11	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers.
- Students are NOT permitted to bring into the examination room: notes of any kind, blank sheets of paper and/or white out liquid/tape, a calculator.

Materials supplied

- Question and answer book of 12 pages.
- Working space is provided throughout the book.

Instructions

- Print your name in the space provided on the top of this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

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Instructions

Answer **all** questions in the spaces provided.

A decimal approximation will not be accepted if an **exact** answer is required to a question.

In questions where more than one mark is available, appropriate working must be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1

Let $f(x) = 2x^2 - 1$ and $g(x) = x - 5$.

a. Write the rule for $f(g(x))$.

1 mark

b. State the domain of $f(g(x))$.

1 mark

TURN OVER

Question 2

For the function $f : \left(-\frac{1}{2}, \infty\right) \rightarrow \mathbb{R}$ where $f(x) = \log_e(2x+1)$

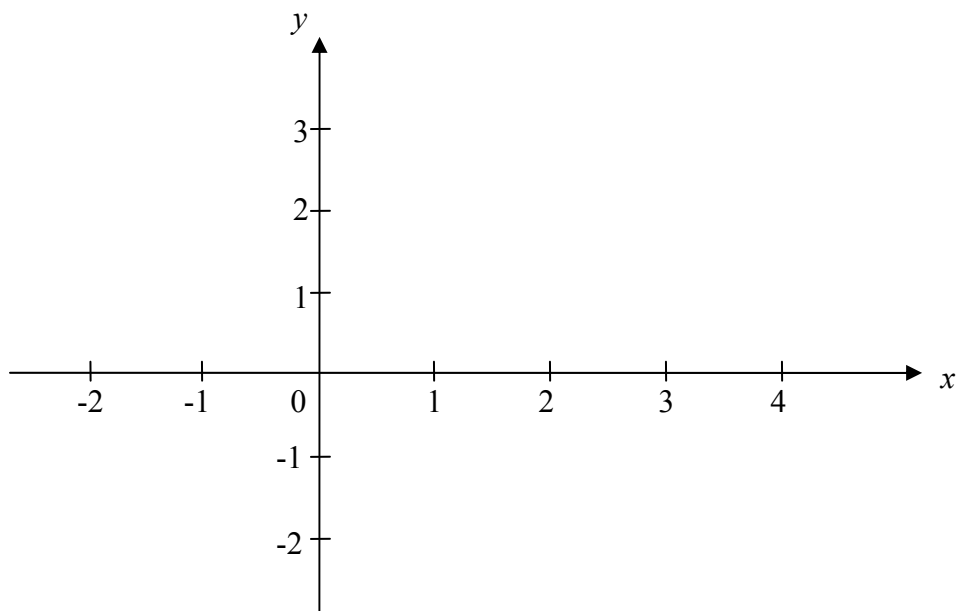
- a. Find the rule for the inverse function, f^{-1} .

2 marks

- b. Find the range of the inverse function, f^{-1} .

1 mark

- c. Sketch the graph of the $f(x) = \log_e(2x+1)$ and its inverse function, f^{-1} on the axes below.



2 marks

Question 3

a. Let $f(x) = e^{x^2 - 3x}$. Find $f'(x)$.

1 mark

b. Find the exact value of $f'(x)$ when $x = 1$.

1 mark

Question 4

Let $y = x \cos 2x$. Evaluate $\frac{dy}{dx}$ when $x = \frac{\pi}{6}$.

2 marks

TURN OVER

Question 5

The graph of the function $y = \frac{1}{x}$ undergoes the following transformations

- a dilation by a factor of $\frac{1}{2}$ from the y -axis
- a reflection in the y -axis
- a translation of +3 units parallel to the x -axis
- a translation of +1 units parallel to the y -axis

a. Write down the equation of the transformed function.

2 marks

b. Hence state the domain and range of the transformed function.

1 mark

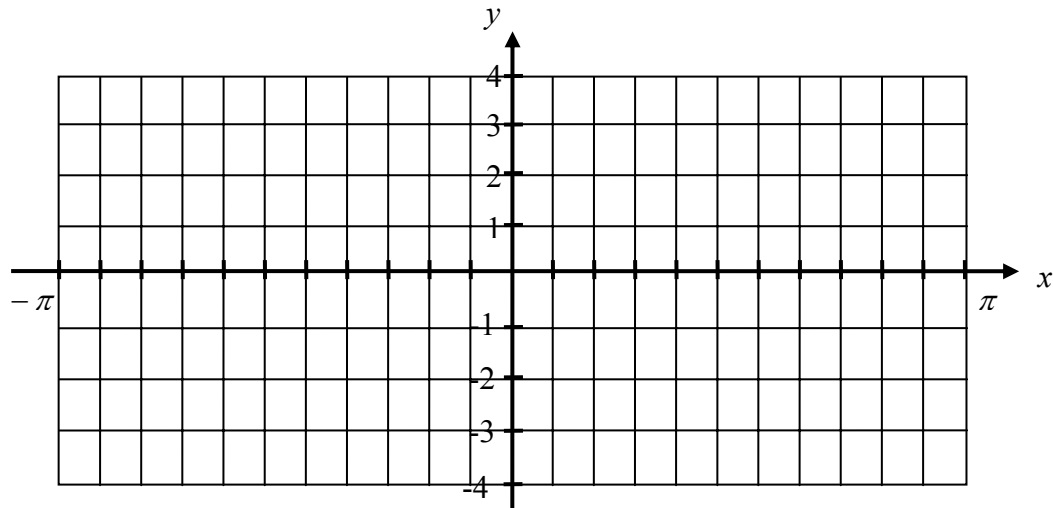
Question 6

For the functions $f : [-\pi, \pi] \rightarrow R$, $f(x) = 3 \sin\left(2\left(x - \frac{\pi}{4}\right)\right)$ and $g : [-\pi, \pi] \rightarrow R$, $g(x) = \frac{3}{2}x$

- a. find the x -coordinates of the points of intersection between the two functions.

2 marks

- b. sketch the graph of both functions f and g on the same set of axes below. Label axes intercepts and the intersection points with their coordinates. Label endpoints of the two graphs with their coordinates.

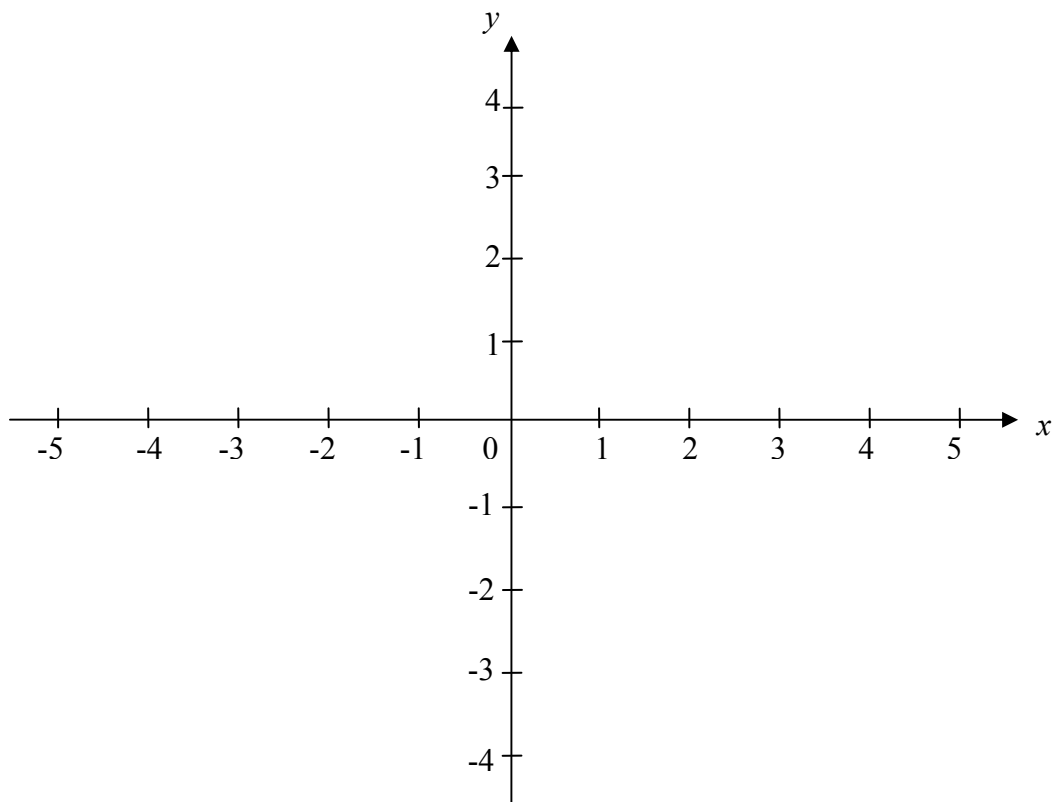


3 marks

TURN OVER

Question 7

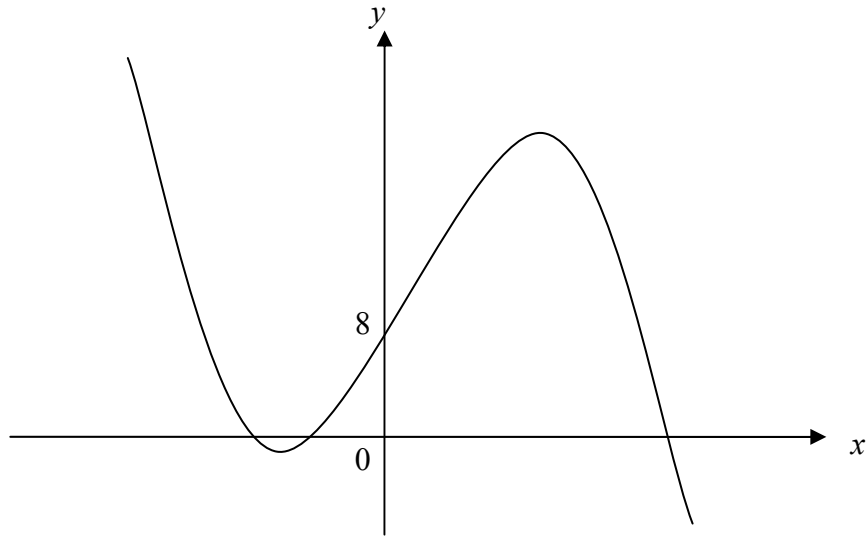
Sketch the graph of $f : [-2, 4] \rightarrow \mathbb{R}$ where $f(x) = 4 - 2|1 - x|$ on the axes below. Clearly label axes intercepts and the vertex (turning points) with their co-ordinates and end-points.



3 marks

Question 8

The graph of the curve with equation $y = -x^3 - 3x^2 + 6x + 8$ is shown below



- a.** Find the value of b and c and then write the equation in the form $y = -(x+1)(x^2 + bx + c)$.

2 marks

- b.** Hence, find the exact value of the x -intercepts.

1 mark

TURN OVER

- c. Use calculus to find the exact total area enclosed by the graph of the function with rule $y = -x^3 - 3x^2 + 6x + 8$ and the x -axis.

3 marks

Question 9

A probability density function is given by

$$f(x) = \begin{cases} k(a+x) & -a \leq x \leq a \\ 0 & x > a \text{ or } x < -a \end{cases}$$

- a. Show that $k = \frac{1}{2a^2}$.

3 marks

- b. Find the value of a given an expected value of 1.

3 marks

TURN OVER

Question 10

If it is raining on a particular day the chance that it will rain the next day is 0.7. If it is not raining on a particular day, the chance that it will rain on the following day is 0.4.

Calculate the probability that, if it is raining on Monday, it will also be raining on Thursday of the same week.

2 marks

Question 11

Let X be a normally distributed random variable with mean of 50 and variance of 100. **Use the result that $\Pr(z < 2) = 0.98$, correct to two decimal places,** to find

a. The probability that X is greater than 70.

1 mark

b. The probability that $X < 30$ given that $X < 70$.

3 marks

END OF QUESTION AND ANSWER BOOK