

MATHEMATICAL METHODS

Units 3 & 4 – Written examination 1



2007 Trial Examination

SOLUTIONS

Question 1

a.

$$\begin{aligned} f(g(x)) &= f(x-5) \\ &= 2(x-5)^2 - 1 \end{aligned}$$

A1

b.

$$\text{dom } f(g(x)) = R$$

A1

Question 2

a.

$$\begin{aligned} f(x) &= \log_e(2x+1) \\ \text{inverse } x &= \log_e(2y+1) \end{aligned}$$

M1

$$e^x = 2y + 1$$

$$2y = e^x - 1$$

$$y = \frac{1}{2}(e^x - 1)$$

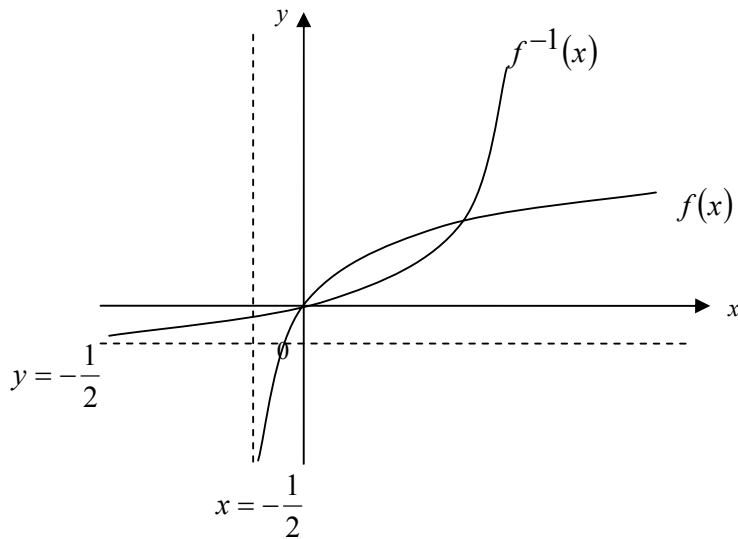
$$f^{-1}(x) = \frac{1}{2}(e^x - 1)$$

A1

b. $\text{range } f^{-1} = \text{dom } f = \left(-\frac{1}{2}, \infty\right)$

A1

c.

M1 correct shape of graph of $f(x)$ with asymptotes labelled.M1 correct shape of graph of $f^{-1}(x)$ with asymptotes labelled**Question 3**

a. $f'(x) = (2x - 3)e^{x^2-3}$

A1

b.

$$\begin{aligned}
 \text{when } x = 1 \quad f'(1) &= (2 - 3)e^{1-3} \\
 &= -e^{-2} \\
 &= -\frac{1}{e^2}
 \end{aligned}$$

A1

Question 4

$$\frac{dy}{dx} = \cos 2x - 2x \sin 2x$$

A1

$$\begin{aligned}
 \text{when } x = \frac{\pi}{6} \quad \frac{dy}{dx} &= \cos 2\left(\frac{\pi}{6}\right) - 2\left(\frac{\pi}{6}\right) \sin 2\left(\frac{\pi}{6}\right) \\
 &= \cos \frac{\pi}{3} - \frac{\pi}{3} \sin \frac{\pi}{3} \\
 &= \frac{1}{2} - \frac{\pi}{3} \frac{\sqrt{3}}{2} \\
 &= \frac{1}{2} - \frac{\pi\sqrt{3}}{6}
 \end{aligned}$$

A1

Question 5

a. $y = -\frac{1}{2(x-3)} + 1$

M1 for 2 correct transformations

M1 for 2 correct transformations

b. Dom = $\mathbb{R} \setminus \{3\}$
Range = $\mathbb{R} \setminus \{1\}$

A1 must have both domain and range correct

Question 6

a. $f(x) = g(x)$

$$\frac{3}{2} = 3 \sin\left(2\left(x - \frac{\pi}{4}\right)\right)$$

$$\frac{1}{2} = \sin\left(2\left(x - \frac{\pi}{4}\right)\right)$$

M1

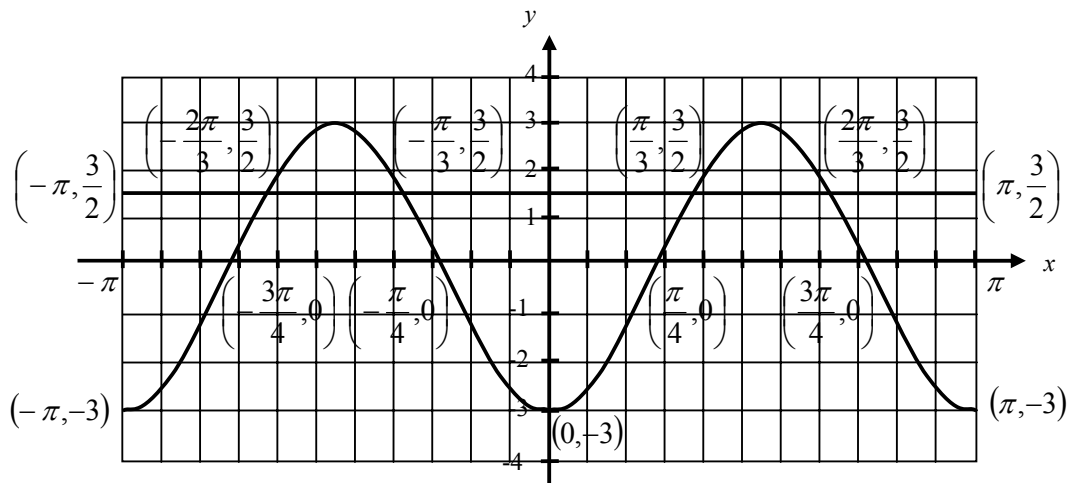
$$2\left(x - \frac{\pi}{4}\right) = \frac{\pi}{6}, \frac{5\pi}{6}, -\frac{7\pi}{6}, -\frac{11\pi}{6}$$

$$x - \frac{\pi}{4} = \frac{\pi}{12}, \frac{5\pi}{12}, -\frac{7\pi}{12}, -\frac{11\pi}{12}$$

$$x = \frac{\pi}{3}, \frac{2\pi}{3}, -\frac{\pi}{3}, -\frac{2\pi}{3}$$

A1

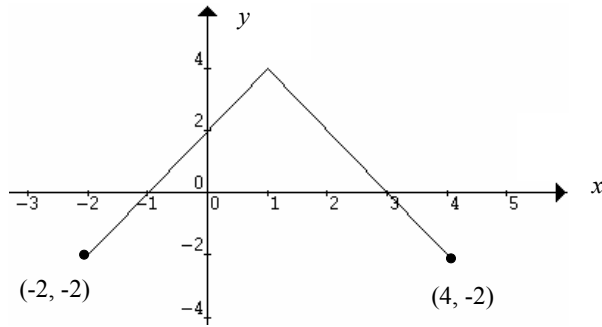
b.



M1 correct shape of graphs

M1 intercepts labelled

M1 end points labelled

Question 7

M1 correct shape of graph

M1 stationary point and all intercepts labelled

M1 both end points labelled

Question 8**a.**

$$\begin{array}{r}
 -x^2 - 2x + 8 \\
 \overline{) -x^3 - 3x^2 + 6x + 8} \\
 \underline{-x^3 - x^2} \\
 2x^2 + 6x \\
 \underline{-2x^2 - 2x} \\
 8x + 8 \\
 \underline{8x + 8} \\
 0
 \end{array}$$

$$b = 2, c = -8$$

$$y = -(x+1)(x^2 + 2x - 8)$$

M1

M1

b.

$$-(x+1)(x-2)(x+4) = 0$$

$$x = -1, x = 2, x = -4$$

A1

c.

$$A = 2 \int_{-1}^2 (-x^3 - 3x^2 + 6x + 8) dx$$

M1

$$= 2 \left[-\frac{x^4}{4} - x^3 + 3x^2 + 8x \right]_{-1}^2$$

$$= 2 \left[(-4 - 8 + 12 + 16) - \left(-\frac{1}{4} + 1 + 3 - 8 \right) \right]$$

M1

$$= 2 \left[16 - \left(4\frac{1}{4} \right) \right]$$

$$= 40\frac{1}{2} \text{ units squared}$$

A1

Question 9**a.**

$$\int_{-a}^a k(a+x)dx = 1 \quad \text{M1}$$

$$k \int_{-a}^a (a+x)dx = 1$$

$$\left[ax + \frac{x^2}{2} \right]_{-a}^a = \frac{1}{k} \quad \text{M1}$$

$$\left(a^2 + \frac{a^2}{2} \right) - \left(-a^2 + \frac{a^2}{2} \right) = \frac{1}{k}$$

$$2a^2 = \frac{1}{k} \quad \text{A1}$$

$$k = \frac{1}{2a^2}$$

b.

$$E(X) = \int_{-a}^a xf(x)dx$$

$$1 = \int_{-a}^a x \frac{1}{2a^2}(a+x)dx$$

$$1 = \frac{1}{2a^2} \int_{-a}^a x(a+x)dx \quad \text{M1}$$

$$2a^2 = \int_{-a}^a x(a+x)dx$$

$$2a^2 = \int_{-a}^a (ax + x^2)dx$$

$$2a^2 = \left[\frac{ax^2}{2} + \frac{x^3}{3} \right]_{-a}^a$$

$$2a^2 = \left(\frac{a^3}{2} + \frac{a^3}{3} \right) - \left(\frac{a^3}{2} - \frac{a^3}{3} \right) \quad \text{M1}$$

$$2a^2 = \frac{2a^3}{3}$$

$$6a^2 = 2a^3$$

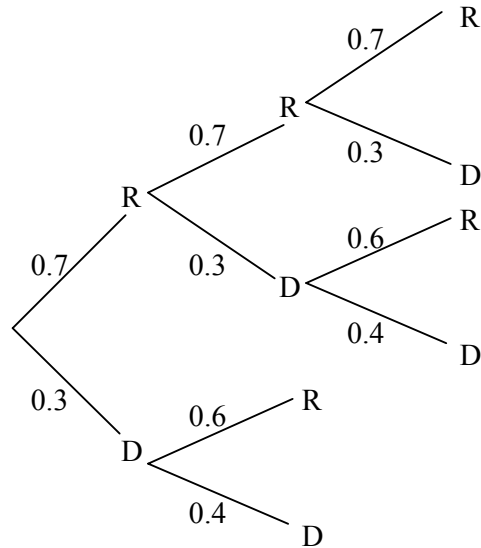
$$3a^2 = a^3$$

$$a^2(a-3) = 0$$

$$a = 0 \text{ (reject) or } a = 3$$

$$\therefore a = 3$$

A1

Question 10

$$\begin{aligned}
 \Pr(\text{if it is raining on Monday, it will be raining on Thursday}) &= (0.7 \times 0.7 \times 0.7) + (0.7 \times 0.3 \times 0.6) \text{ M1} \\
 &= 0.343 + 0.126 \\
 &= 0.469 \qquad \qquad \qquad \text{A1}
 \end{aligned}$$

Question 11**a.**

$$\begin{aligned}
 \Pr(X > 70) &= \Pr\left(z > \frac{70 - 50}{10}\right) \\
 &= \Pr(z > 2) \\
 &= 1 - \Pr(z < 2) \\
 &= 1 - 0.98 \\
 &= 0.02 \qquad \qquad \qquad \text{A1}
 \end{aligned}$$

b.

$$\Pr(X < 30 | X < 70) = \frac{\Pr(X < 30 \cap X < 70)}{X < 70} \qquad \qquad \qquad \text{M1}$$

$$\begin{aligned}
 &= \frac{\Pr(X < 30)}{\Pr(X < 70)} \\
 &= \frac{\Pr(z < -2)}{\Pr(z < 2)} \qquad \qquad \qquad \text{M1}
 \end{aligned}$$

$$\begin{aligned}
 &= \frac{\Pr(z > 2)}{0.98} \\
 &= \frac{1 - \Pr(z < 2)}{0.98}
 \end{aligned}$$

$$\begin{aligned}
 &= \frac{0.02}{0.98} \\
 &= \frac{1}{49} \qquad \qquad \qquad \text{A1}
 \end{aligned}$$