



THE SCHOOL FOR EXCELLENCE (TSFX)

UNIT 4 MATHEMATICAL METHODS 2007

WRITTEN EXAMINATION 1

Reading Time: 15 minutes

Writing time: 1 hour

QUESTION AND ANSWER BOOKLET

Structure of Booklet

Number of questions	Number of questions to be answered	Number of marks
10	10	40

Students are permitted to bring into the examination rooms: pens, pencils, highlighters, erasers, sharpeners, rulers.

Students are **NOT** permitted to bring into the examination room: notes of any kind, a calculator, blank sheets of paper and/or white out liquid/tape.

Students are **NOT** permitted to bring mobile phones and/or any electronic communication devices into the examination room.

All written responses must be in English.

COMPLIMENTS OF THE SCHOOL FOR EXCELLENCE

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Instructions

- Answer **all** questions in the spaces provided.
- A decimal approximation will not be accepted if an **exact** answer is required to a question.
- In questions where more than 1 mark is available, appropriate working must be shown.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

QUESTION 1

- a. If $f(x) = (x^2 + 2x + 2)e^{-x}$ show that $f'(x) = -\frac{x^2}{e^x}$.

2 marks

- b. Use calculus to find the coordinates of the stationary point(s).

1 mark

- c. Use the sign of the first derivative to determine the nature of the stationary point(s).

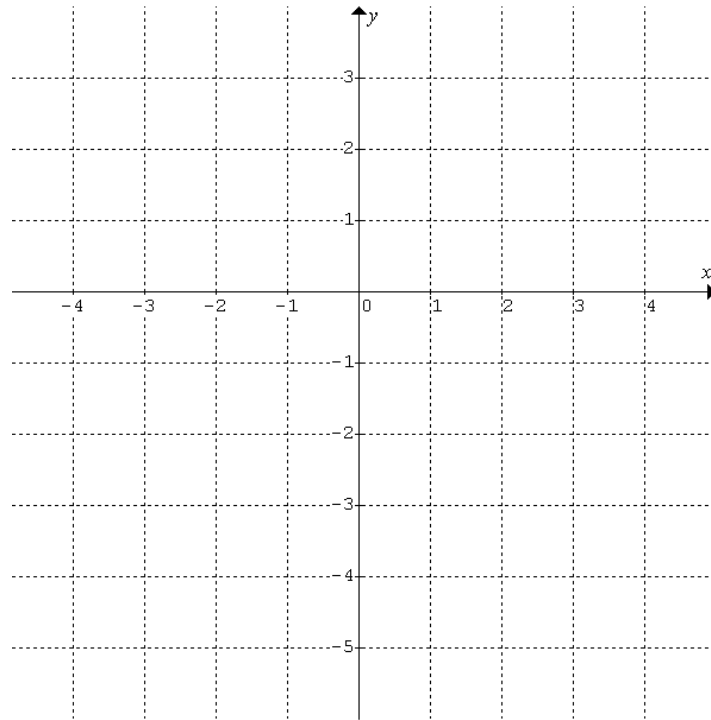
1 mark

Total = 4 Marks

QUESTION 2

Given $f(x) = 3 - |(x - 2)^3 + 1|$:

- a. Sketch the graph of $f(x)$ on the set of axes below.



2 marks

- b. Find $f'(x)$ over the relevant intervals.

2 marks

- c. Hence find (if it exists):

(i) $f'\left(\frac{1}{2}\right)$

1 mark

(ii) $f'(1)$

1 mark

Total = 6 Marks

QUESTION 3

Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be a continuous function with the following properties:

$$f(-2) = f(1) = 0$$

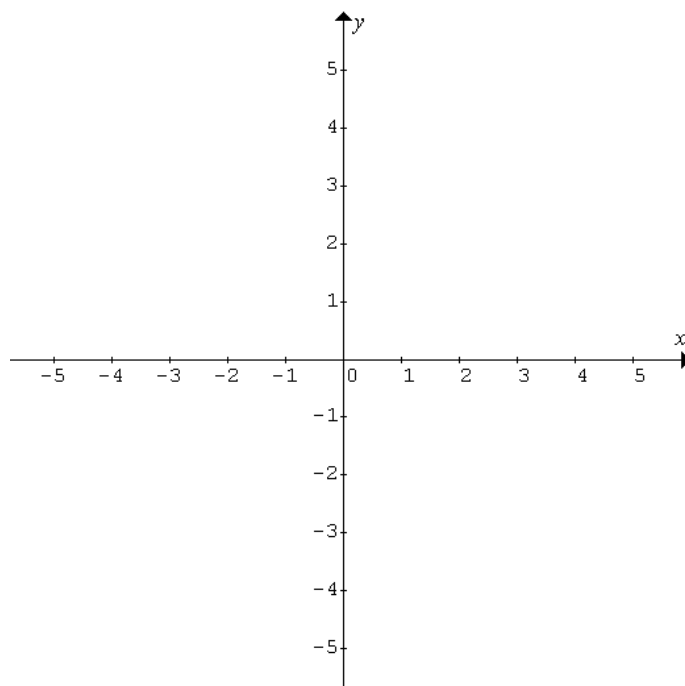
$$f(0) = 4$$

$$f'(-2) = f'(0) = 0$$

$$f'(x) < 0 \text{ for } x < -2 \text{ and } x > 0$$

$$f'(x) > 0 \text{ for } -2 < x < 0$$

- a. Sketch a possible graph of f on the set of axes below.



2 marks

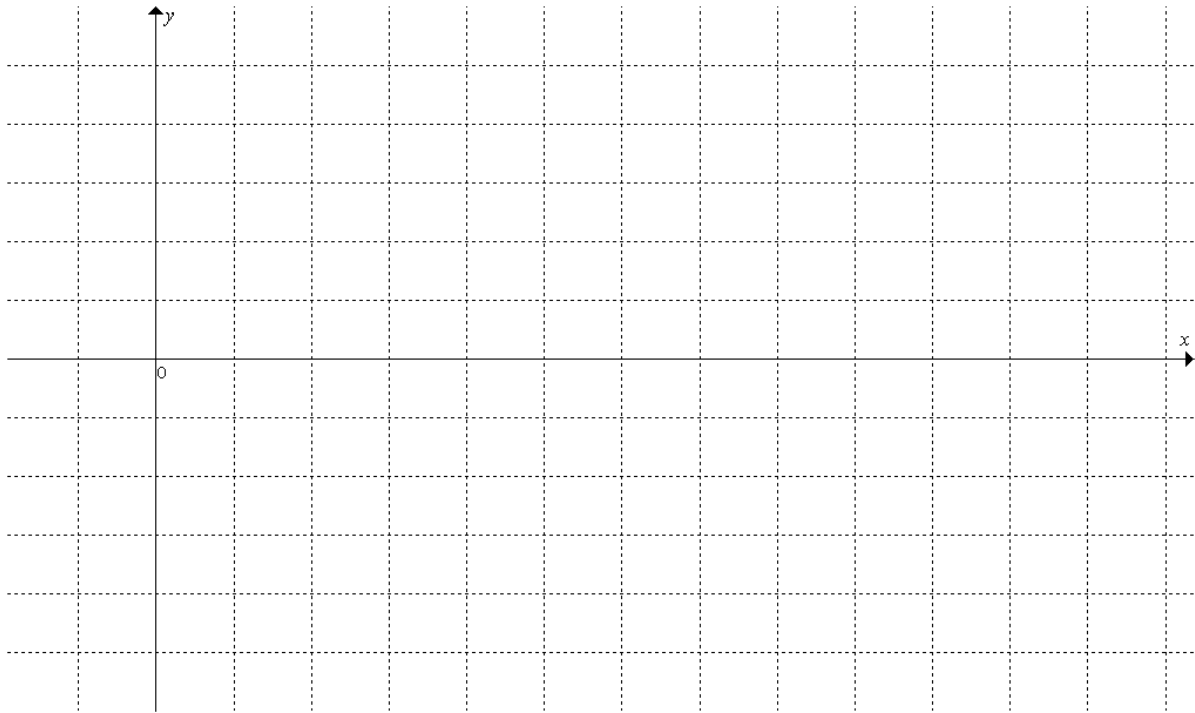
- b. Write the equation describing f in the form $a(x+b)^2(x+c)$ where a, b, c represent real number constants.

2 marks

Total = 4 Marks

QUESTION 4

Sketch the graph of $y = 5\sin 2\left(x - \frac{\pi}{6}\right)$, for $0 \leq x \leq 2\pi$, on the set of axes below, labelling the coordinates of the endpoints and all axes intercepts.



Total = 4 Marks

QUESTION 5

- a. Given $\int_{-1}^1 (x^4 - 2x^3 + 1 + a) dx = \frac{2}{5}$, show that $a = -1$.

2 marks

- b. Find the exact area enclosed by the graph of $f(x) = x^4 - 2x^3 + 1 + a$, the x-axis and the lines $x = -1$ and $x = 1$.

2 marks

Total = 4 Marks

QUESTION 6

Chris either walks or takes a tram to school. If Chris walks to school on a particular morning, the probability that he takes the tram the next morning is 0.3. If he takes the tram to school on a particular morning, the probability that he walks to school the next morning is 0.2. Suppose Chris takes the tram to school on Wednesday morning.

- a. Draw a tree diagram to illustrate how Chris may travel to school on Friday morning.

2 marks

- b. Hence find the probability that Chris walks to school on Friday morning.

1 mark

Total = 3 Marks

QUESTION 7

Let X be a normally distributed random variable with a mean of 22 and a standard deviation of 7. Let Z be the standard normal random variable. Given $\Pr(Z < 2) = 0.98$, find the following correct to two decimal places:

- a. The probability that X is greater than 36.

1 mark

- b. The probability that X is less than 8 given that X is less than 22.

2 marks

Total = 3 Marks

QUESTION 8

A circular patch of oil spreads over a road in such a way that its area increases at the rate of $15 \text{ cm}^2/\text{minute}$. Find the exact rate at which the radius of the oil patch is increasing when its radius is 5 cm .

This image shows a blank sheet of white paper with horizontal ruling lines. The lines are evenly spaced and extend across the width of the page. There are no margins, text, or other markings on the paper.

Total = 3 Marks

QUESTION 9

Consider the function $f : (1, \infty) \rightarrow \mathbb{R}$ where $f(x) = \frac{1}{2} \log_e(x-1)$.

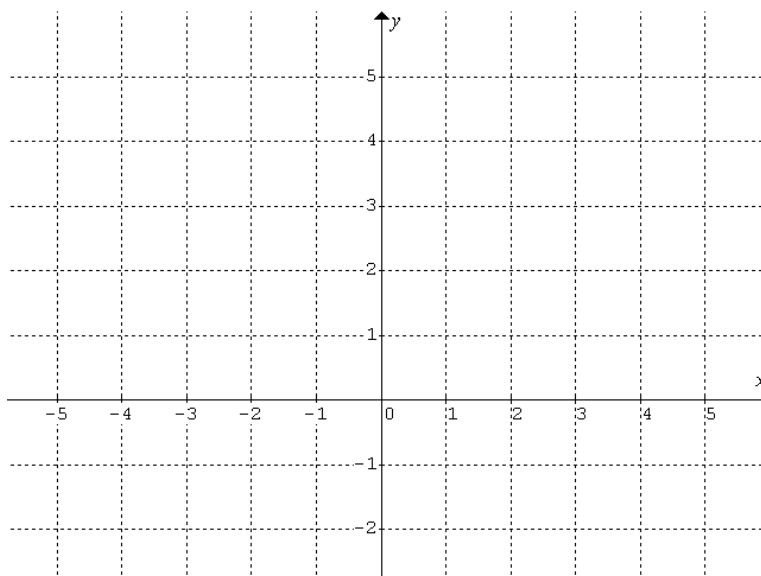
- a. (i) Find the equation of the inverse function f^{-1} .

2 marks

- (ii) State the domain and range of f^{-1} .

1 mark

- b. Sketch the graph of both f and f^{-1} on the set of axes below.



2 marks

Total = 5 Marks

QUESTION 10

A continuous random variable X has a probability distribution function given by

$$f(x) = \begin{cases} k(a-x), & -a \leq x \leq a \\ 0 & \text{otherwise} \end{cases}$$

where a and k are positive constants:

- a. Show that $k = \frac{1}{2a^2}$.

2 marks

- b. Find the value of a for which the expected value of X is equal to -2 .

2 marks

Total = 5 Marks

End of Paper