



Trial Examination 2007

VCE Mathematical Methods Units 3 & 4

Written Examination 1

Suggested Solutions

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Question 1

$$g(f(x)) = g((x-1)^2)$$

$$= \sqrt{(x-1)^2} + 1 \text{ or } |x-1| + 1$$

A1

Note: $|x-1| + 1 \neq x$

Question 2

- a. The turning point of the graph of $y = f(x)$ is $(-1, -1)$.

Therefore the maximal domain for f to be a one-to-one function is $(-\infty, -1)$.

Therefore $a = -1$.

A1

- b. Originally, we have $y = x^2 + 2x$.

For the inverse, $x = y^2 + 2y$.

M1

$$x = y^2 + 2y + 1 - 1$$

$$= (y+1)^2 - 1$$

M1

$$x+1 = (y+1)^2$$

$$y = -1 \pm \sqrt{x+1}$$

However, $\text{ran } f^{-1} = \text{dom } f = (-\infty, -1)$

$$f^{-1}(x) = -1 - \sqrt{x+1}$$

A1

Question 3

a. i. period = $\frac{2\pi}{\frac{\pi}{6}} = 12$

A1

ii. range : $[-1, 3]$

A1

b. $g(x) = 1 - 2\sin\left(\frac{\pi}{6}(x-2)\right)$

$g(x)$ has a minimum where $\sin\left(\frac{\pi}{6}(x-2)\right) = 1$

M1

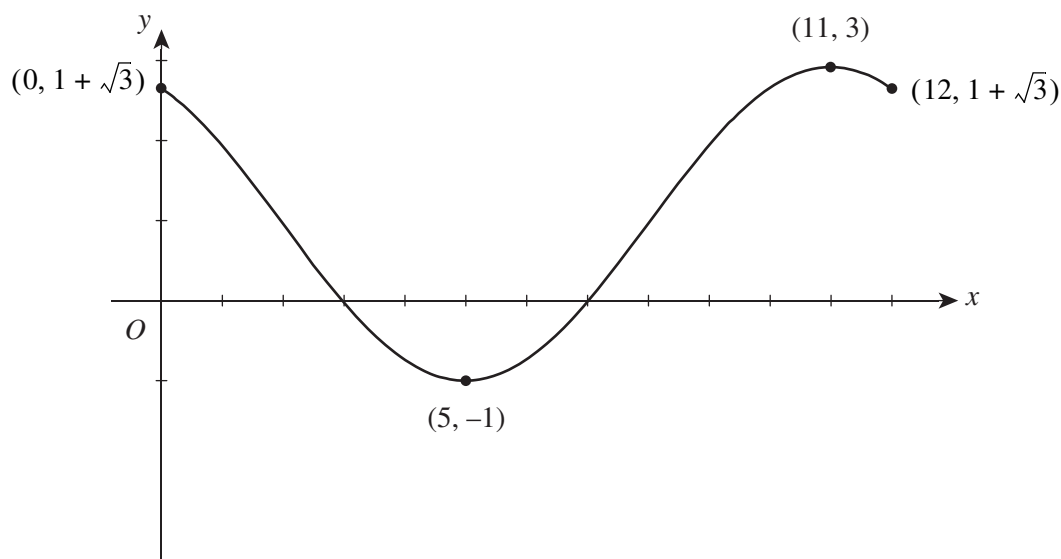
$$\therefore \frac{\pi}{6}(x-2) = \frac{\pi}{2}$$

$$x = 5$$

So the minimum occurs at $(5, -1)$.

A1

c.



Correct shape A1

Correct endpoints A1

Correct maximum and minimum A1

Question 4

$$f(x) = \log_e \left(\frac{x}{x-1} \right)$$

$$= \log_e \left(\frac{-x}{1-x} \right)$$

$$= \log_e(-x) - \log_e(1-x) \text{ (so that } f\left(-\frac{1}{2}\right) \text{ exists)}$$

$$f'(x) = \frac{-1}{-x} - \frac{-1}{1-x}$$

$$= \frac{1}{x} - \frac{1}{x-1}$$

$$= \frac{x-1-x}{x(x-1)}$$

$$= \frac{-1}{x(x-1)}$$

M1

$$= \frac{1}{x(1-x)}$$

$$f'\left(-\frac{1}{2}\right) = \frac{1}{-\frac{1}{2}\left(1+\frac{1}{2}\right)}$$

M1

$$= -\frac{4}{3}$$

A1

Thus the gradient of the tangent is $-\frac{4}{3}$, so the gradient of the normal is $\frac{3}{4}$.

A1

Question 5

$$\begin{aligned}
 \int_5^1 2(h(x) - 1)dx &= \int_5^1 (2h(x) - 2)dx \\
 &= -2 \int_1^5 h(x)dx - \int_5^1 2dx \\
 &= -2[4] - [2x]_5^1 \\
 &= -8 - (2 - 10) \\
 &= 0
 \end{aligned}$$

M1

A1

Question 6

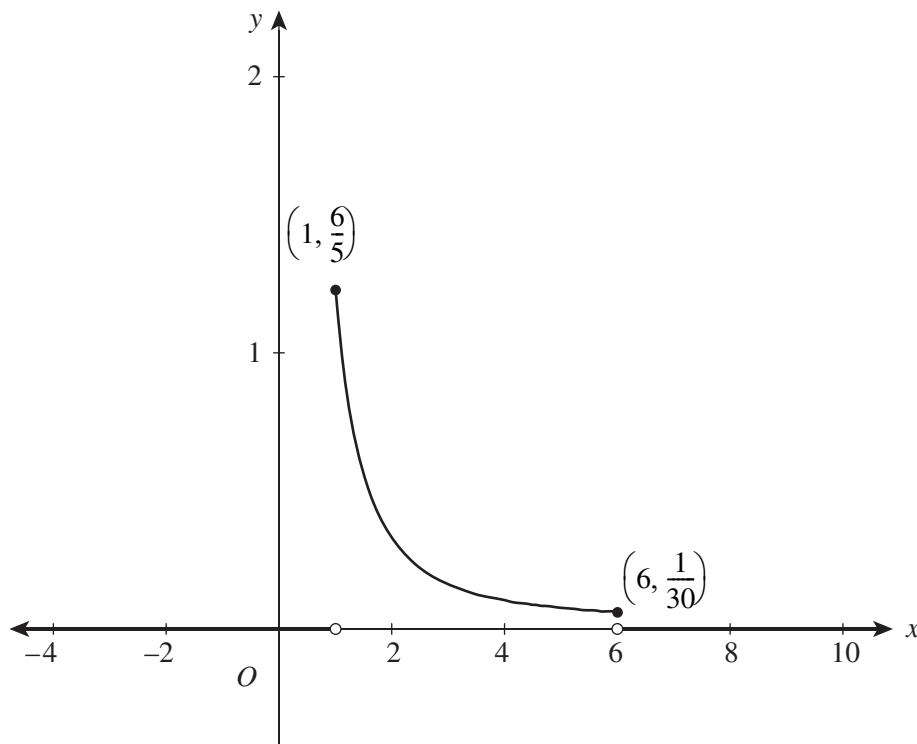
a. As $f(x)$ is a probability density function, $\int_1^6 \frac{k}{x^2} dx = 1$.

M1

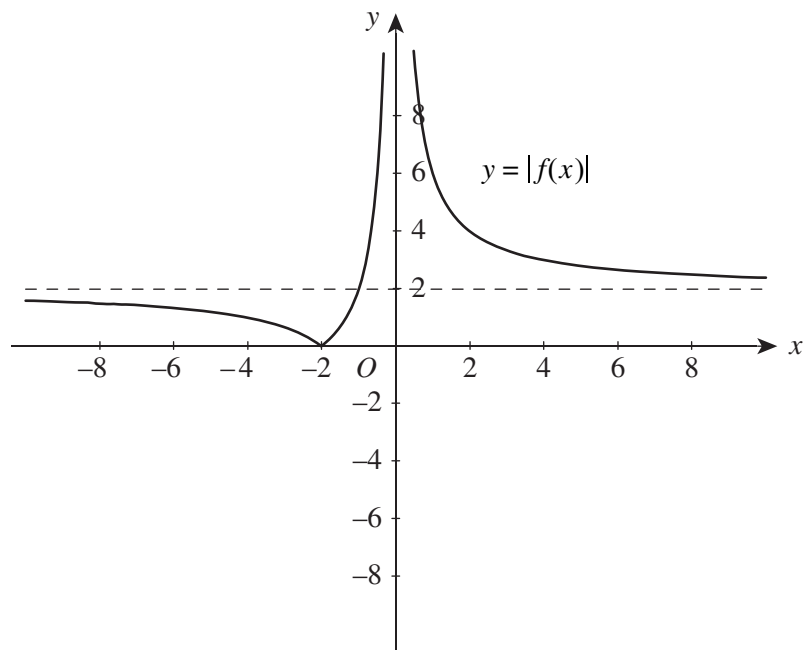
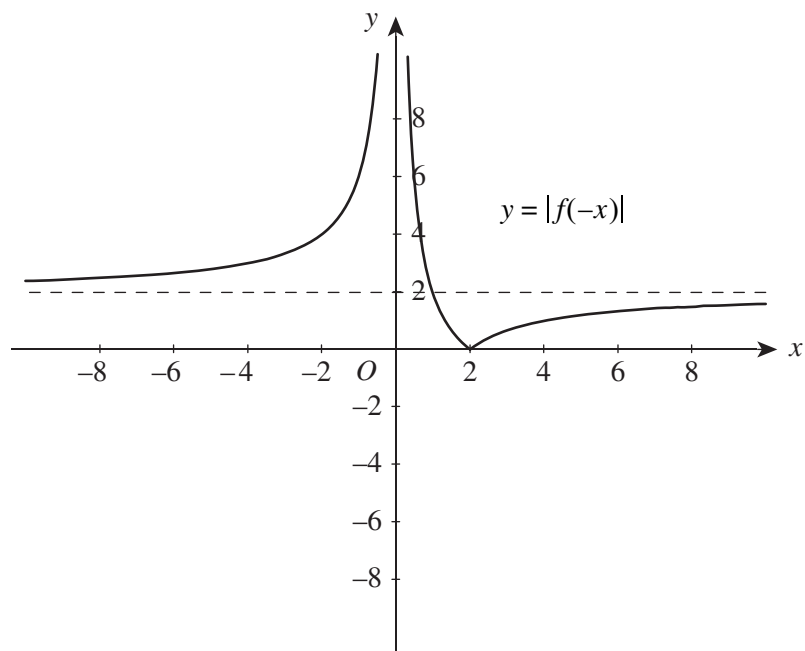
$$\begin{aligned}
 \left[-\frac{k}{x} \right]_1^6 &= 1 \\
 -\frac{k}{6} + \frac{k}{1} &= 1 \\
 k &= \frac{6}{5}
 \end{aligned}$$

A1

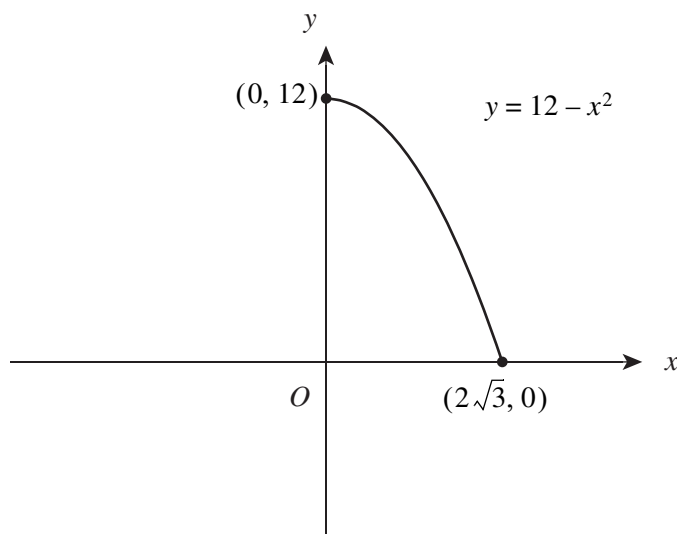
b.



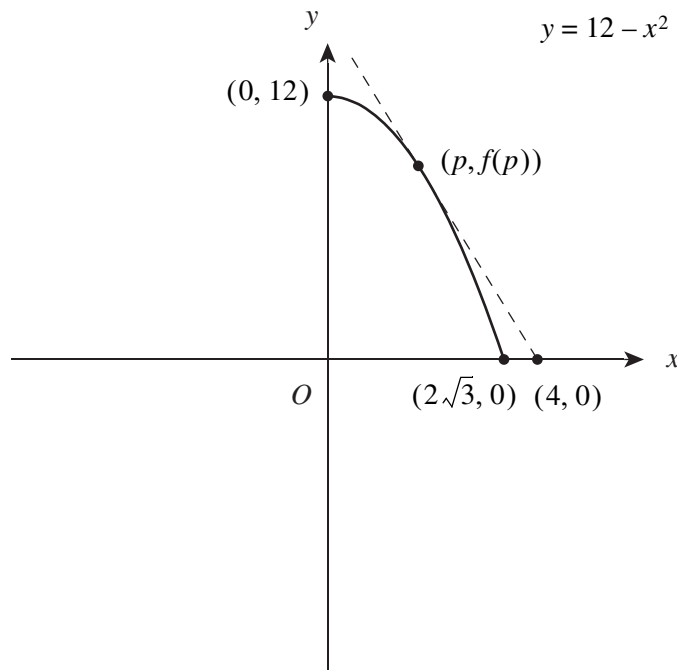
Correct endpoints A1
Correct graph on $(-\infty, \infty)$ A1

Question 7**a.**Correct graph of $|f(x)|$ A1**b.**Correct graph of $|f(-x)|$ A1**c.** range : $\mathbb{R}^+ \cup \{0\}$

A1

Question 8**a.**

Correct graph A1

b.

The gradient of the tangent at $x = p$ is given by $f'(p)$.

$$f'(x) = -2x \text{ so } f'(p) = -2p$$

$$\text{Equation of the tangent: } y - y_1 = m(x - x_1)$$

$$\therefore y - (12 - p^2) = -2p(x - p)$$

$$y = -2px + p^2 + 12$$

M1 A1

As the tangent passes through the point $(4, 0)$

$$0 = -8p + p^2 + 12$$

$$0 = (p - 6)(p - 2)$$

M1

$$p = 6 \text{ or } p = 2$$

$$\text{Clearly } 0 < p < 2\sqrt{3} \text{ so } p = 2$$

A1

Question 9

a. $\frac{dr}{dt} = \frac{1}{25}, V = \frac{4}{3}\pi r^3, \frac{dV}{dr} = 4\pi r^2$

Using the chain rule,

$$\frac{dV}{dt} = \frac{dV}{dr} \times \frac{dr}{dt}$$

$$\frac{dV}{dt} = 4\pi r^2 \times \frac{1}{25} = \frac{4\pi r^2}{25}$$

M1

At $r = 10$,

$$\frac{dV}{dt} = \frac{4\pi \times 100}{25} = 16\pi \text{ cm}^3/\text{s}$$

A1

b. The area of a cross-section through the centre of the sphere is $A = \pi r^2$

$$\frac{dA}{dr} = 2\pi r$$

$$\frac{dA}{dt} = \frac{6\pi}{25}$$

$$\frac{dA}{dt} = \frac{dA}{dr} \times \frac{dr}{dt}$$

M1

$$\frac{6\pi}{25} = 2\pi r \times \frac{1}{25}$$

$$r = 3$$

At $r = 3$,

$$V = \frac{4}{3}\pi \times 3^3 = 36\pi \text{ cm}^3$$

A1

Question 10

Point of intersection:

$$\sin(x) = \cos(x)$$

$$\tan(x) = 1$$

$$x = \frac{\pi}{4}$$

A1

$$\text{area} = \int_0^{\frac{\pi}{4}} (\cos(x) - \sin(x)) dx$$

$$= [\sin(x) + \cos(x)]_0^{\frac{\pi}{4}}$$

M1

$$= \sin\left(\frac{\pi}{4}\right) + \cos\left(\frac{\pi}{4}\right) - (0 + 1)$$

$$= \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} - 1$$

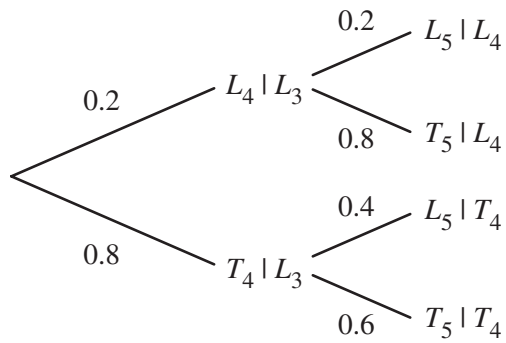
$$= \frac{2 - \sqrt{2}}{\sqrt{2}}$$

$$= \sqrt{2} - 1$$

A1

Question 11

Let T_i be the event ‘on time for school on the i th day of the week’, and let L_i be the event ‘late for school on the i th day of the week’.



Tree diagram or table M1
Correct probabilities A1

$$\begin{aligned}\Pr(L_5) &= 0.2 \times 0.2 + 0.8 \times 0.4 \\ &= 0.36\end{aligned}$$

A1