

Year 2007

VCE

**Mathematical Methods
and
Mathematical Methods
(CAS)**

Trial Examination 1



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**Victorian Certificate of Education
2007**

STUDENT NUMBER

Figures
Words

Letter

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**MATHEMATICAL METHODS
AND
MATHEMATICAL METHOD (CAS)**

Trial Written Examination 1

Reading time: 15 minutes

Total writing time: 1 hour

QUESTION AND ANSWER BOOK

Structure of book

<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
11	11	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers.
- Students are NOT permitted to bring into the examination room: notes of any kind, blank sheets of paper, white out liquid/tape or a calculator of any type.

Materials supplied

- Question and answer book of 12 pages with a detachable sheet of miscellaneous formulas at the end of this booklet.
- Working space is provided throughout the booklet.

Instructions

- Detach the formula sheet from the end of this book during reading time.
- Write your **student number** in the space provided above this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Instructions

Answer **all** questions in the spaces provided.
 A decimal approximation will not be accepted if an **exact** answer is required to a question.
 In questions where more than one mark is available, appropriate working must be shown.
 Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1

For the function $f : R \setminus \{3\} \rightarrow R$, $f(x) = \frac{2}{x-3} + 4$

- a. Find the rule for the inverse function f^{-1} .

1 mark

- b. Find the domain of the inverse function f^{-1} .

1 mark

Question 2

Let $y = \frac{\tan(2x)}{2x}$. Evaluate $\frac{dy}{dx}$ when $x = \frac{\pi}{8}$.

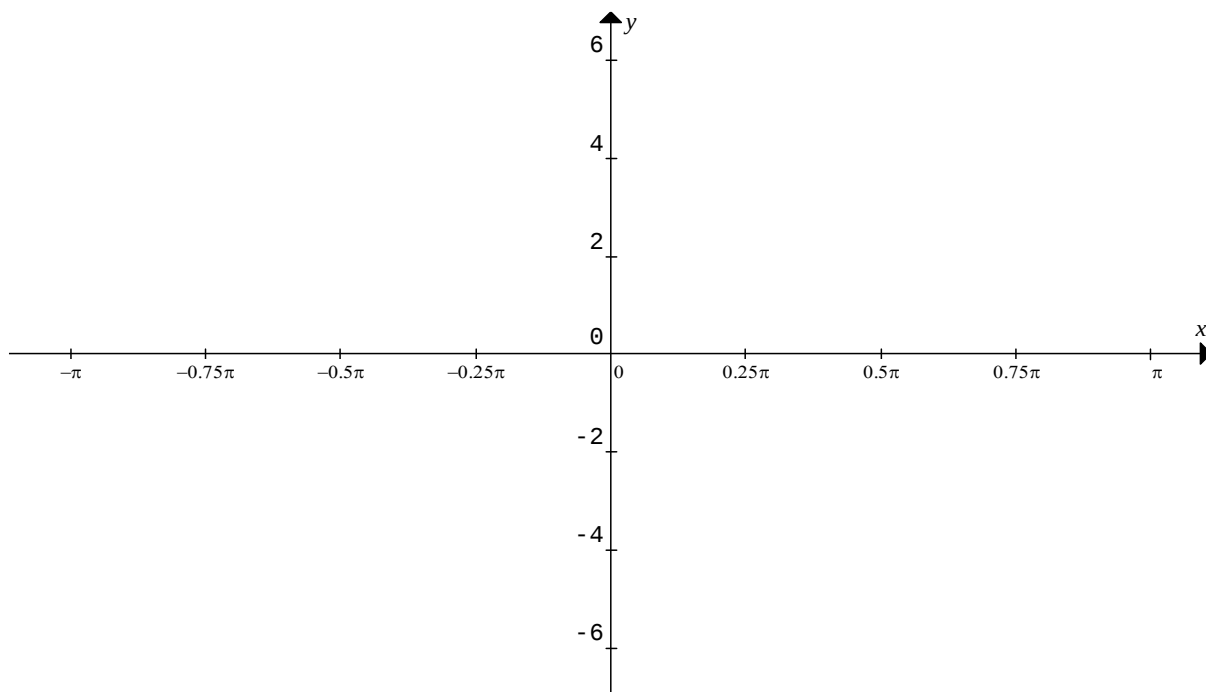
3 marks

Question 3

- a.** State in words, giving scale factors, the transformations required to sketch the graph of $y = 4 \cos\left(2\left(x - \frac{\pi}{3}\right)\right)$ from the graph of $y = \cos(x)$.

2 marks

- b.** Sketch the graph of the function $f : [-\pi, \pi] \rightarrow \mathbb{R}$, $f(x) = 4 \cos\left(2\left(x - \frac{\pi}{3}\right)\right)$ on the set of axes below. Label axes intercepts with their coordinates. Label endpoints with their coordinates.



3 marks

Question 4

The probability density function of a continuous random variable X is given by

$$f(x) = \begin{cases} k(x-4)^2 & 1 \leq x \leq 3 \\ 0 & \text{otherwise} \end{cases}$$

- a.** Show that $k = \frac{3}{26}$.

1 mark

- b.** Find $\Pr(X < 2)$.

2 marks

Question 5

The weights of chocolate bars are normally distributed with a mean of 51 grams and a standard deviation of 4 grams. Let Z be the standard normal random variable.

Given that $\Pr(Z < 0.25) = 0.6$,

- a. Find the probability that a randomly selected chocolate bar has a weight of less than 50 grams. Give your answer correct to one decimal place.

1 mark

- b. Find the probability that a randomly selected chocolate bar has a weight of between 51 and 52 grams. Give your answer correct to one decimal place.

1 mark

- c. One night Lilly eats three such chocolate bars. Find the probability that at least one of the chocolate bars has a weight of more than 50 grams. Give your answer correct to three decimal places.

2 marks

Question 6

Let $g(x) = \cos(2x)$ and $f(g(x)) = \log_e(\cos(2x))$

- a. Write down the rule for the function $f(x)$.

1 mark

- b. Find the derivative of $f(g(x))$.

1 mark

- c. **Hence**, find an anti-derivative of $\tan(2x)$.

1 mark

Question 7

The line $3y + x + k = 0$ is a normal to the curve $y = x^5 + bx$ at the point $x = -1$.

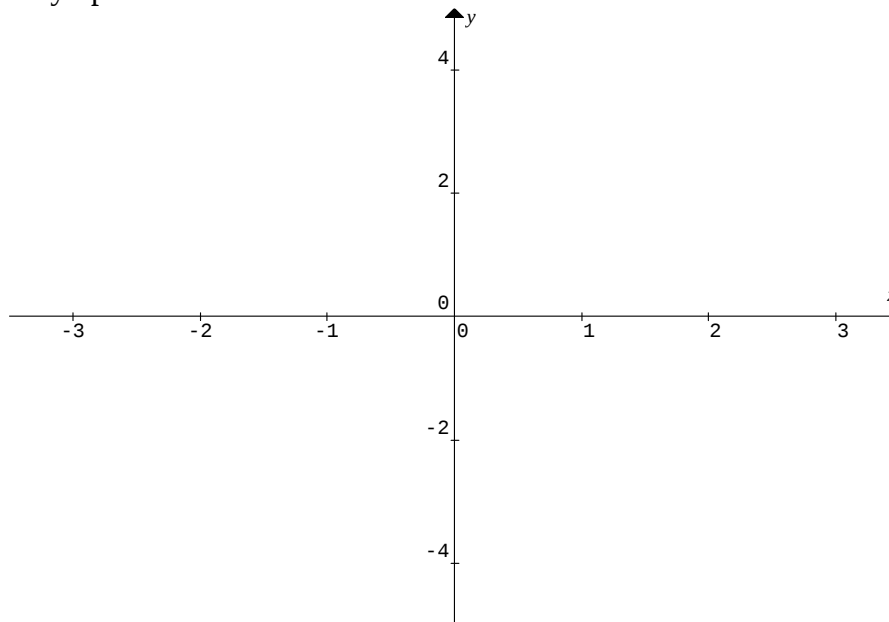
Find the values of b and k .

4 marks

Question 8

- a. Sketch the graph of the function $f : \mathbb{R} \setminus \left\{ \frac{3}{2} \right\} \rightarrow \mathbb{R}$, $f(x) = \frac{-6}{|2x-3|}$

on the axes below, stating the coordinates of any axial intercepts and the equations of all asymptotes.



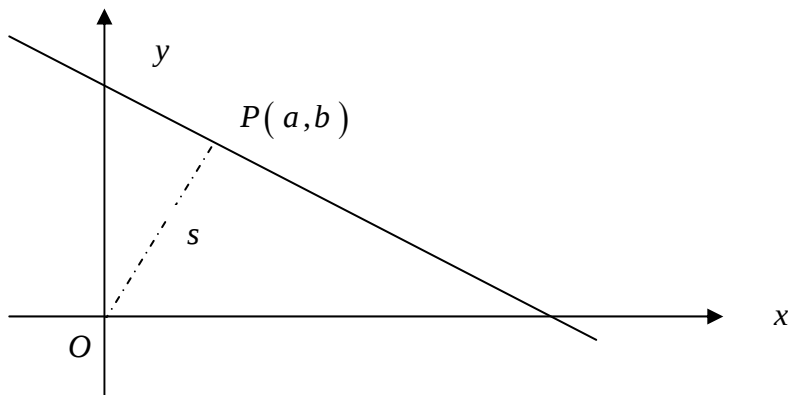
2 marks

- b. Find the area bounded by the graph of $y = \frac{-6}{|2x-3|}$, the coordinates axes and the line $x=1$.

2 marks

Question 9

The point $P(a, b)$ where a and b are positive real numbers lies on the line $y + 2x - 5 = 0$.



- a.** Let s be the distance from the origin O to the point P .
Find the distance s in terms of a .

1 mark

- b.** Find, using **calculus**, the minimum value of s and the value of a for which this occurs.

3 marks

Question 10

Each night at dinner Bill has either a glass of wine or a glass of beer. If he has beer one night, the probability he has a beer the next night is 0.6. If he has wine one night, the probability he has a beer the next night is 0.2. Suppose he has a beer on Thursday night. What is the probability that he has a beer on the following Saturday night?

3 marks

Question 11

- a. Show that the graph of the function $f : (0, \infty) \rightarrow \mathbb{R}$, $f(x) = 2x + \frac{8}{x^2}$ has a single stationary point and find its coordinates.

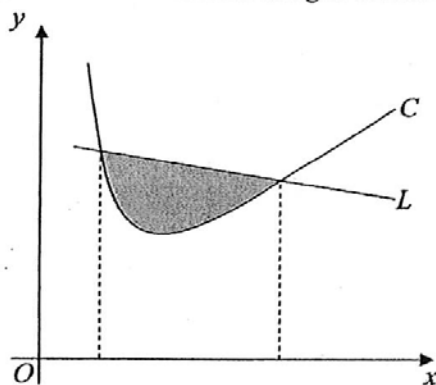
2 marks

- b. The curve C with the equation $y = 2x + \frac{8}{x^2}$ for $x > 0$ is sketched below.

The line L with the equation $x + 2y = 21$ intersects the curve C at the points $(a, 10)$ and $(b, 8.5)$. If the area of the shaded region bounded by the curve C and the line L can be expressed as the definite integral

$$\int_a^b \left(p + qx + \frac{r}{x^2} \right) dx, \text{ find the values of } a, b, p, q \text{ and } r.$$

(There is no need to evaluate the definite integral or find the area.)



3 marks

END OF QUESTION AND ANSWER BOOKLET

EXTRA WORKING SPACE

This image shows a blank sheet of white paper with horizontal ruling lines. The lines are evenly spaced and extend across the width of the page. There are no margins, text, or other markings on the paper.

END OF EXAMINATION

MATHEMATICAL METHODS

Written examination 1

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time.

This formula sheet is provided for your reference.

Mathematical Methods and CAS Formulas

Mensuration

area of a trapezium:	$\frac{1}{2}(a+b)h$	volume of a pyramid:	$\frac{1}{3}Ah$
curved surface area of a cylinder:	$2\pi rh$	volume of a sphere:	$\frac{4}{3}\pi r^3$
volume of a cylinder:	$\pi r^2 h$	area of triangle:	$\frac{1}{2}bc \sin(A)$
volume of a cone:	$\frac{1}{3}\pi r^2 h$		

Calculus

$\frac{d}{dx}(x^n) = nx^{n-1}$	$\int x^n dx = \frac{1}{n+1}x^{n+1} + c, n \neq -1$
$\frac{d}{dx}(e^{ax}) = ae^{ax}$	$\int e^{ax} dx = \frac{1}{a}e^{ax} + c$
$\frac{d}{dx}(\log_e(x)) = \frac{1}{x}$	$\int \frac{1}{x} dx = \log_e x + c$
$\frac{d}{dx}(\sin(ax)) = a \cos(ax)$	$\int \sin(ax) dx = -\frac{1}{a}\cos(ax) + c$
$\frac{d}{dx}(\cos(ax)) = -a \sin(ax)$	$\int \cos(ax) dx = \frac{1}{a}\sin(ax) + c$
$\frac{d}{dx}(\tan(ax)) = \frac{a}{\cos^2(ax)} = a \sec^2(ax)$	

product rule: $\frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}$

quotient rule: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$

Chain rule: $\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}$

approximation: $f(x+h) \approx f(x) + h f'(x)$

Probability

$\Pr(A) = 1 - \Pr(A')$

$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$

$\Pr(A/B) = \frac{\Pr(A \cap B)}{\Pr(B)}$

Mean: $\mu = E(X)$

variance: $\text{var}(X) = \sigma^2 = E((X - \mu)^2) = E(X^2) - \mu^2$

probability distribution		mean	variance
discrete	$\Pr(X = x) = p(x)$	$\mu = E(X)$	$\sigma^2 = \sum (x - \mu)^2 p(x)$
continuous	$\Pr(a < X < b) = \int_a^b f(x) dx$	$\mu = \int_{-\infty}^{\infty} x f(x) dx$	$\sigma^2 = \int_{-\infty}^{\infty} (x - \mu)^2 f(x) dx$