

Year 2007
VCE
Mathematical Methods
and
Mathematical Methods
(CAS)
Solutions
Trial Examination 1



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Question 1

a. $f : R \setminus \{3\} \rightarrow R, f(x) = \frac{2}{x-3} + 4$

$$f \quad y = \frac{2}{x-3} + 4 \quad \text{interchanging } y \text{ and } x$$

$$f^{-1} \quad x = \frac{2}{y-3} + 4 \quad \text{transposing to make } y \text{ the subject}$$

$$f^{-1} \quad x - 4 = \frac{2}{y-3}$$

$$f^{-1} \quad y - 3 = \frac{2}{x-4}$$

$$f^{-1}(x) = \frac{2}{x-4} + 3 \quad \text{A1}$$

b. $\text{dom } f^{-1} = R \setminus \{4\}$ A1

Question 2

$$y = \frac{\tan(2x)}{2x} \quad \text{differentiating using the quotient rule}$$

let $u = \tan(2x) \quad v = 2x$

$$\frac{du}{dx} = \frac{2}{\cos^2(2x)} \quad \frac{dv}{dx} = 2 \quad \text{M1}$$

$$\frac{dy}{dx} = \frac{\frac{4x}{\cos^2(2x)} - 2 \tan(2x)}{4x^2}$$

$$\text{when } x = \frac{\pi}{8} \quad \left. \frac{dy}{dx} \right|_{x=\frac{\pi}{8}} = \frac{\frac{\frac{\pi}{2}}{\cos^2\left(\frac{\pi}{4}\right)} - 2 \tan\left(\frac{\pi}{4}\right)}{\frac{\pi^2}{16}} \quad \text{M1}$$

$$\left. \frac{dy}{dx} \right|_{x=\frac{\pi}{8}} = \frac{16(\pi-2)}{\pi^2} \quad \text{A1}$$

Question 3

a. $y = \cos(x)$ into $y = 4\cos\left(2\left(x - \frac{\pi}{3}\right)\right)$

- dilation by a factor of 4 parallel to the y-axis (or away from the x-axis)
- dilation by a factor of $\frac{1}{2}$ parallel to the x-axis (or away from the y-axis)
- translation by $\frac{\pi}{3}$ to the right parallel to the x-axis (or away from the y-axis)

for correct transformations A2

b. $f : [-\pi, \pi] \rightarrow \mathbb{R}, f(x) = 4\cos\left(2\left(x - \frac{\pi}{3}\right)\right)$ the amplitude is 4 and

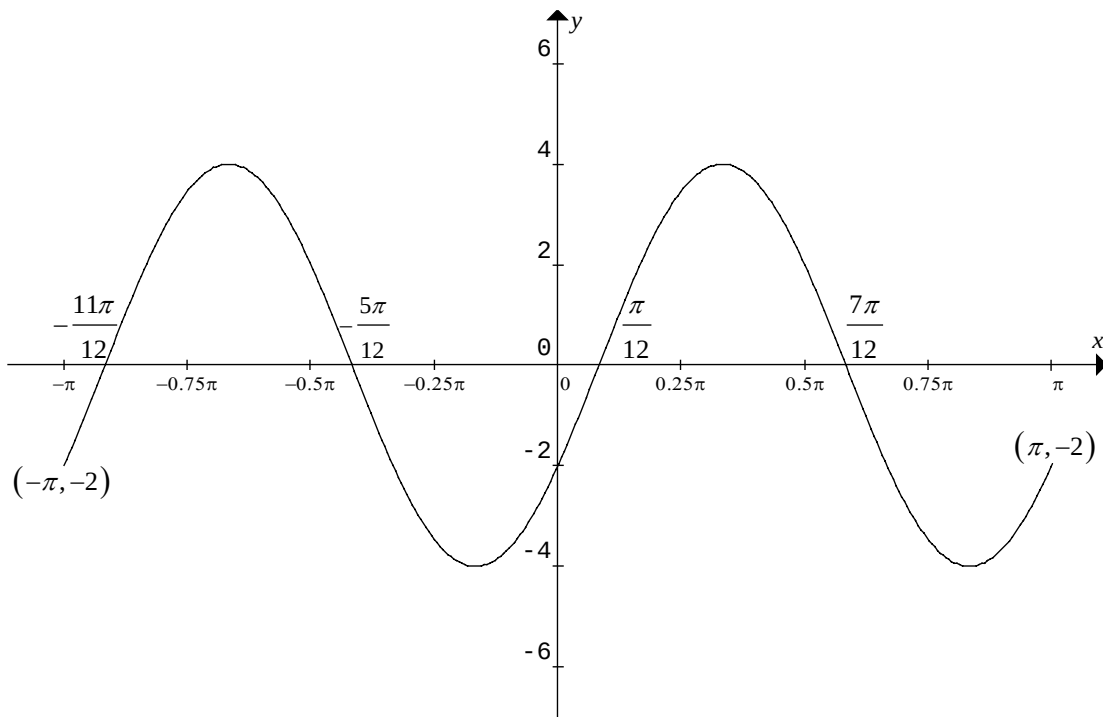
the period is π , so that $f(-\pi) = f(0) = f(\pi) = 4\cos\left(-\frac{2\pi}{3}\right) = -2$ A1

crosses the x-axis when $y = 0$ $\cos\left(2\left(x - \frac{\pi}{3}\right)\right) = 0$

$2\left(x - \frac{\pi}{3}\right) = \frac{\pi}{2}$ at $x - \frac{\pi}{3} = \frac{\pi}{4}$ so $x = \frac{7\pi}{12}$ A1

the other x-intercepts are $\frac{\pi}{2}$ apart, $\left(-\frac{11\pi}{12}, 0\right), \left(-\frac{5\pi}{12}, 0\right), \left(\frac{\pi}{12}, 0\right), \left(\frac{7\pi}{12}, 0\right)$

correct graph A1



Question 4

$$f(x) = \begin{cases} k(x-4)^2 & 1 \leq x \leq 3 \\ 0 & \text{otherwise} \end{cases}$$

a. $\int_1^3 k(x-4)^2 dx = 1$

$$k \left[\frac{(x-4)^3}{3} \right]_1^3 = 1$$

A1

$$k \left[\frac{(-1)^3}{3} - \frac{(-3)^3}{3} \right] = 1$$

$$k \left[-\frac{1}{3} + 9 \right] = 1$$

$$\frac{26k}{3} = 1$$

$$k = \frac{3}{26}$$

b. $\Pr(X < 2) = \int_1^2 \frac{3}{26}(x-4)^2 dx$

$$\Pr(X < 2) = \frac{3}{26} \left[\frac{(x-4)^3}{3} \right]_1^2$$

M1

$$\Pr(X < 2) = \frac{3}{26} \left[\frac{(-2)^3}{3} - \frac{(-3)^3}{3} \right]$$

$$\Pr(X < 2) = \frac{3}{26} \left[-\frac{8}{3} + 9 \right]$$

$$\Pr(X < 2) = \frac{3}{26} \left(\frac{-8+27}{3} \right)$$

$$\Pr(X < 2) = \frac{19}{26}$$

A1

Question 5

X is the weights of chocolate bars

$$X \stackrel{d}{=} N(\mu = 51, \sigma^2 = 4^2)$$

a. $\Pr(X < 50)$

$$= \Pr\left(Z < \frac{50-51}{4}\right)$$

$$= \Pr(Z < -0.25) = \Pr(Z > 0.25)$$

$$= 1 - \Pr(Z < 0.25)$$

$$= 1 - 0.6$$

$$= 0.4$$

A1

b. $\Pr(51 < X < 52)$

$$= \Pr\left(\frac{51-51}{4} < Z < \frac{52-51}{4}\right)$$

$$= \Pr(0 < Z < 0.25) = \Pr(Z < 0.25) - \Pr(Z < 0)$$

$$= 0.6 - 0.5$$

$$= 0.1$$

A1

c. $Y \stackrel{d}{=} Bi(n = 3, p = 0.6)$

$$\Pr(Y \geq 1) = 1 - [\Pr(Y = 0)]$$

$$\Pr(Y \geq 1) = 1 - (0.4)^4$$

M1

$$\Pr(Y \geq 1) = 1 - 0.064$$

$$\Pr(Y \geq 1) = 0.936$$

A1

Question 6

a. $f(g(x)) = \log_e(\cos(2x))$

$$g(x) = \cos(2x) \text{ then } f(x) = \log_e(x)$$

A1

b. $\frac{d}{dx}[f(g(x))] = \frac{d}{dx}[\log_e(\cos(2x))]$

$$= \frac{-2\sin(2x)}{\cos(2x)}$$

$$= -2\tan(2x)$$

A1

c. hence $\int \tan(2x) dx = -\frac{1}{2} \log_e(\cos(2x)) + C$

A1

Question 7

The line $3y + x + k = 0$ $3y = -x - k$ $y = -\frac{x}{3} - \frac{k}{3}$

has a gradient of $-\frac{1}{3}$ so $m_N = -\frac{1}{3}$

so the tangent has a gradient of $m_T = 3$

A1

$$y = x^5 + bx$$

$$\frac{dy}{dx} = 5x^4 + b = 3 \quad \text{at } x = -1$$

M1

$$5(-1)^4 + b = 5 + b = 3$$

$$b = -2$$

So the curve is $y = x^5 - 2x$ at the point $x = -1$

$$y(-1) = (-1)^5 + 2 = -1 + 2 = 1$$

M1

the point $P(-1, 1)$ is also on the line

$$3y + x + k = 0 \quad 3 - 1 + k = 0$$

so $k = -2$

A1

Question 8

a. $f : R \setminus \left\{ \frac{3}{2} \right\} \rightarrow R, f(x) = \frac{-6}{|2x-3|}$

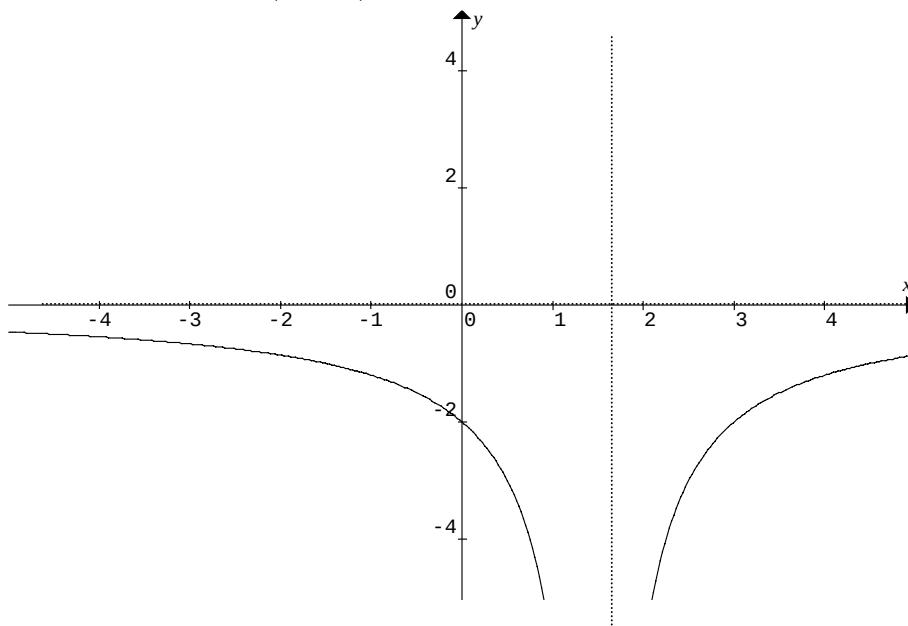
$x = \frac{3}{2}$ is a vertical asymptote and $y = 0$ (the x-axis) is a horizontal asymptote

crosses the y-axis at $x = 0$ $f(0) = \frac{-6}{|-3|} = -2$ at $(0, -2)$

A1

correct graph range $(-\infty, 0)$

A1



b. the area $A = \int_0^1 \frac{-6}{|2x-3|} dx = \int_0^1 \frac{6}{3-2x} dx$ since $|2x-3| = 3-2x$ for $0 < x < 1$

$$A = [-3 \log_e (3-2x)]_0^1 \quad \text{and } A > 0 \quad \text{M1}$$

$$A = [-3 \log_e (3) + 3 \log_e (3)]$$

$$A = 3 \log_e (3) = \log_e (27) \quad \text{A1}$$

Question 9

a. $s = \sqrt{a^2 + b^2}$ but the point $P(a, b)$ lies on the line $y + 2x - 5 = 0$
so $b + 2a - 5 = 0$ $b = 5 - 2a$

$$s = \sqrt{a^2 + (5-2a)^2} = \sqrt{a^2 + 25 - 20a + 4a^2} \quad \text{M1}$$

$$s = \sqrt{5a^2 - 20a + 25} = (5a^2 - 20a + 25)^{\frac{1}{2}}$$

b. for the minimum value of s $\frac{ds}{da} = 0$

differentiating using the chain rule M1

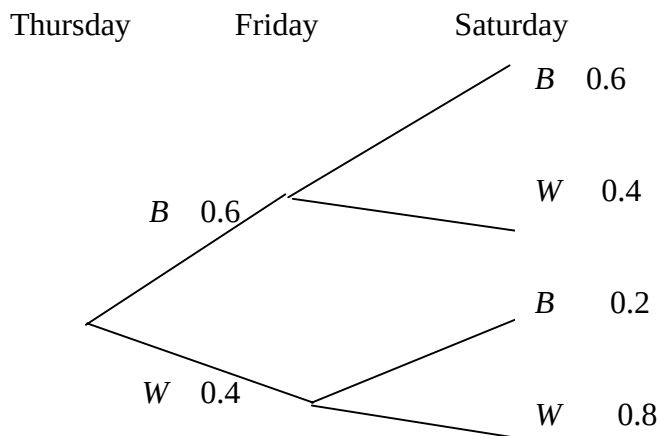
$$\frac{ds}{da} = \frac{\frac{1}{2}(10a - 20)}{\sqrt{5a^2 - 20a + 25}} = 0$$

$$10a = 20 \quad \text{A1}$$

$$a = 2 \quad b = 1$$

$$s = \sqrt{5} \quad \text{A1}$$

Question 10



M1

$$\Pr(\text{beer on Saturday}) = 0.6 \times 0.6 + 0.4 \times 0.2 = 0.36 + 0.08 \quad \text{A1}$$

$$\Pr(\text{beer on Saturday}) = 0.44 \quad \text{A1}$$

Question 11

a. $f : (0, \infty) \rightarrow \mathbb{R}, f(x) = 2x + \frac{8}{x^2} = 2x + 8x^{-2}$

for stationary points $f'(x) = 2 - 16x^{-3} = 2 - \frac{16}{x^3} = 0$ M1

$$2 = \frac{16}{x^3} \quad x^3 = 8$$

$$x = 2 \quad \text{and} \quad f(2) = 4 + \frac{8}{4} = 6$$

the point is $(2, 6)$ A1

b. the points $(a, 10)$ and $(b, 8.5)$ both lie on the line $x + 2y = 21$

$$a + 20 = 21 \quad \text{so that} \quad a = 1$$
 A1

$$b + 17 = 21 \quad \text{so that} \quad b = 4$$

$$\text{let } y_1 = \frac{21-x}{2} \quad \text{and} \quad y_2 = 2x + \frac{8}{x^2}$$

the area between the line and the curve is

$$\int_a^b (y_1 - y_2) dx$$

$$= \int_1^4 \left(\left(\frac{21-x}{2} \right) - \left(2x + \frac{8}{x^2} \right) \right) dx$$
 M1

$$= \int_1^4 \left(\frac{21}{2} - \frac{5x}{2} - \frac{8}{x^2} \right) dx = \int_a^b \left(p + qx + \frac{r}{x^2} \right) dx$$

so that $p = \frac{21}{2} = 10.5$ $q = -\frac{5}{2} = -2.5$ and $r = -8$ A1

END OF SUGGESTED SOLUTIONS