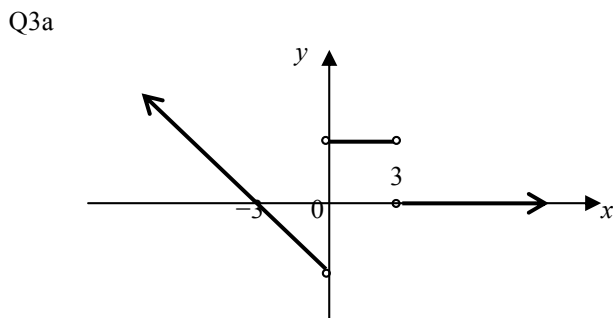


Q1 $f(x) = \frac{x^3}{\sin x}$,
 $f'(x) = \frac{(\sin x)(3x^2) - x^3 \cos x}{\sin^2 x} = \frac{x^2(3 \sin x - x \cos x)}{\sin^2 x}$.

Q2a $\log_e(3x+5) + \log_e 2 = 2$,
 $\log_e 2(3x+5) = 2$, $\log_e(6x+10) = 2$,
 $6x+10 = e^2$, $x = \frac{1}{6}(e^2 - 10)$

Q2b Let $u = \tan x$.
 $g'(x) = \frac{d}{dx} \log_e(\tan x) = \frac{d}{du} \log_e u \times \frac{du}{dx}$
 $= \frac{1}{u} \sec^2 x = \frac{\sec^2 x}{\tan x}$
 $\therefore g'\left(\frac{\pi}{4}\right) = \frac{\sec^2\left(\frac{\pi}{4}\right)}{\tan\left(\frac{\pi}{4}\right)} = \frac{(\sqrt{2})^2}{1} = 2$.



Q3b $R \setminus \{0, 3\}$

Q4 Given $\frac{dV}{dt} = +8 \text{ cm}^3 \text{ s}^{-1}$, $V = 4x^{\frac{3}{2}}$.

$\frac{dV}{dx} = 6x^{\frac{1}{2}} = 12$ when $x = 4$.

$\frac{dV}{dt} = \frac{dV}{dx} \times \frac{dx}{dt}$, $\therefore +8 = 12 \frac{dx}{dt}$, $\frac{dx}{dt} = +\frac{2}{3} \text{ cms}^{-1}$.

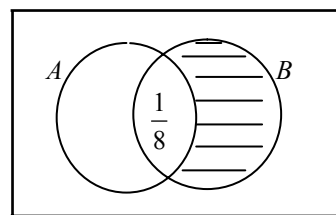
The rate of increase is $\frac{2}{3} \text{ cms}^{-1}$.

Q5 Binomial, $n = 4$, $p = \frac{1}{2}$, $\therefore q = \frac{1}{2}$.

$\Pr(X > 2) = \Pr(X = 3) + \Pr(X = 4)$

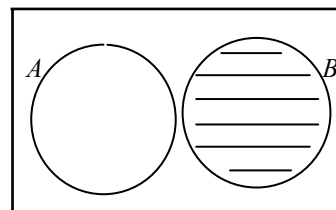
$= {}^4C_3 \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^1 + {}^4C_4 \left(\frac{1}{2}\right)^4 \left(\frac{1}{2}\right)^0 = 4 \times \frac{1}{16} + 1 \times \frac{1}{16} = \frac{5}{16}$.

Q6a



$\Pr(A' \cap B) = \Pr(B) - \Pr(A \cap B) = \frac{1}{3} - \frac{1}{8} = \frac{5}{24}$.

Q6b



$\Pr(A' \cap B) = \Pr(B) - \Pr(A \cap B) = \frac{1}{3} - 0 = \frac{1}{3}$.

Q7 $f'(x) = \cos 3x - 3x \sin 3x$
 $\int f'(x) dx = \int \cos 3x dx - \int 3x \sin 3x dx$
 $f(x) = \frac{1}{3} \sin 3x - 3 \int x \sin 3x dx$
 $x \cos 3x = \frac{1}{3} \sin 3x - 3 \int x \sin 3x dx$
 $3 \int x \sin 3x dx = \frac{1}{3} \sin 3x - x \cos 3x$
 $\int x \sin 3x dx = \frac{1}{9} \sin 3x - \frac{1}{3} x \cos 3x$.

Q8a $\sin \frac{2\pi x}{3} = -\frac{\sqrt{3}}{2}$, $0 \leq x \leq 3$, i.e. $0 \leq \frac{2\pi x}{3} \leq 2\pi$.

$\frac{2\pi x}{3} = \frac{4\pi}{3}$ or $\frac{5\pi}{3}$, $\therefore x = 2$ or $\frac{5}{2}$.

Q8b Since $\sin \frac{2\pi x}{3} = 1$, $0 \leq x \leq 3$, $x = \frac{3}{4}$.

The maximum of $f(x) = \sin \frac{2\pi x}{3}$ is at $x = \frac{3}{4}$.

Let $h(x) = 3f(x) + 2$. The transformations on $f(x)$ do not change the x-coordinate of the maximum point, i.e. $x = \frac{3}{4}$.

$\therefore$ for $g(x) = 3f(x-1) + 2$, the maximum point is translated to the right by 1 unit. Hence $x = \frac{3}{4} + 1 = \frac{7}{4}$.

Q9a $f(x) = e^{\frac{x}{2}} + 1, x \in \mathbb{R}$. When $x = 0$, $y = f(0) = e^0 + 1 = 2$.

$f'(x) = \frac{1}{2}e^{\frac{x}{2}}$. At $x = 0$, $f'(0) = \frac{1}{2}e^0 = \frac{1}{2}$, which is the gradient of the tangent m_T at $(0,2)$.

$\therefore$ the gradient of the normal at $(0,2)$ is $m_N = -\frac{1}{m_T} = -2$.

Equation the normal at $(0,2)$:

$$y - 2 = -2(x - 0), \text{ i.e. } y = -2x + 2.$$

Q9b x -intercept of the normal: $y = 0, x = 1$.

$$\text{Area of the shaded region} = \int_0^1 \left[\left(e^{\frac{x}{2}} + 1 \right) - (-2x + 2) \right] dx$$

$$= \int_0^1 \left(e^{\frac{x}{2}} + 2x - 1 \right) dx = \left[2e^{\frac{x}{2}} + x^2 - x \right]_0^1$$

$$= 2 \left(e^{\frac{1}{2}} - 1 \right) \text{ square units.}$$

$$\text{Q10 } \int_0^9 kx^{\frac{1}{2}} dx = 27, k \int_0^9 x^{\frac{1}{2}} dx = 27, k \left[\frac{2}{3} x^{\frac{3}{2}} \right]_0^9 = 27.$$

$$\therefore 18k = 27, k = \frac{3}{2}.$$

Q11a Let T be 'on time' and F be 'fine'.

Given $\Pr(T | F) = 0.8$, $\Pr(T | F') = 0.6$ and $\Pr(F) = 0.4$.

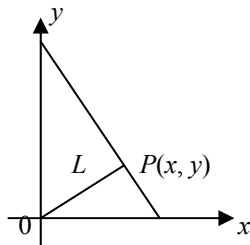
$$\therefore \Pr(F') = 0.6, \Pr(T \cap F) = \Pr(T | F) \Pr(F) = 0.8 \times 0.4 = 0.32$$

$$\text{and } \Pr(T \cap F') = \Pr(T | F') \Pr(F') = 0.6 \times 0.6 = 0.36.$$

$$\therefore \Pr(T) = \Pr(T \cap F) + \Pr(T \cap F') = 0.32 + 0.36 = 0.68$$

$$\text{Q11b } \Pr(F | T) = \frac{\Pr(F \cap T)}{\Pr(T)} = \frac{0.32}{0.68} = \frac{8}{17}.$$

Q12



Let L be the length of OP .

$$\therefore L = \sqrt{x^2 + y^2}$$

Since $y = -2x + 10$,

$$\therefore L = \sqrt{x^2 + 4x^2 - 40x + 100} = \sqrt{5x^2 - 40x + 100}.$$

$$\frac{dL}{dx} = \frac{10x - 40}{2\sqrt{5x^2 - 40x + 100}} = \frac{5x - 20}{\sqrt{5x^2 - 40x + 100}}.$$

$$\text{Let } \frac{dL}{dx} = 0, \therefore 5x - 20 = 0, x = 4 \text{ and } y = 2. \therefore P(4,2).$$

$$L_{\min} = \sqrt{4^2 + 2^2} = 2\sqrt{5} \text{ units.}$$

Instead of using calculus, other methods can be used.

Please inform mathline@itute.com re conceptual, mathematical and/or typing errors