

SECTION 1

1	2	3	4	5	6	7	8	9	10	11
D	C	D	E	A	B	B	B	D	E	E

12	13	14	15	16	17	18	19	20	21	22
B	C	A	D	D	E	A	D	C	C	C

Q1 $(2,0)$, $0 = \sqrt{2a+b}$, $\therefore 2a+b=0$ (1)

$(4,4)$, $4 = \sqrt{4a+b}$, $\therefore 4a+b=16$ (2)

Solve (1) and (2) simultaneously, $a=8$ and $b=-16$.

Q2 $\log_5 10 = \frac{\log_{10} 10}{\log_{10} 5} = \frac{1}{\log_{10} 5} = 1.431$.

Q3 Use graphics calc to sketch $y = |\cos(3x)|$ and $y = 0.15$. The number of intersections in $-\pi \leq x \leq \pi$ is 12.

Q4 $g(x)$ is the result of $f(x)$ undergoing reflection in the y -axis, horizontal dilation by a factor of $\frac{1}{2}$ and downward translation.

Q5 Domain of $f(x)$ is $(-1, \infty)$. For $f[g(x)]$ to be defined, $g(x) \in (-1, \infty)$ and hence $x \in (1, \infty)$.

Q6 Use graphics calc to sketch $y = \frac{x^2 e^x}{(2\pi x)}$ and $y = 1$. The x -coordinate of the intersection is 1.46

Q7 The repeated factors $(2x+a)^3$ and $(x-2b)^2$ indicate that $f(x)$ has a stationary inflection point on the x -axis at $x = -\frac{a}{2}$ and a turning point on the x -axis at $x = 2b$. $\therefore f'(x) = 0$ at $x = -\frac{a}{2}$ and $x = 2b$.

Q8 The range of f is $(-\infty, 1] \cup (2, \infty)$. This becomes the domain of f^{-1} .

Equation of f is $y = \frac{1}{x+1} + 2$, $\therefore$ equation of f^{-1} is

$$x = \frac{1}{y+1} + 2. \text{ Express } y \text{ as the subject, } y = \frac{1}{x-2} - 1,$$

$$\therefore f^{-1}(x) = \frac{1}{x-2} - 1.$$

Q9 Remainder theorem:

$$R = P\left(-\frac{1}{2}\right) = 32\left(-\frac{1}{2}\right)^5 + 8\left(-\frac{1}{2}\right)^3 + 2\left(-\frac{1}{2}\right) + 1 = -2$$

Q10 $y = \sin\left(\frac{\pi x}{2}\right) - \frac{\pi x}{2}$, $-2\pi \leq x \leq 2\pi$. The tangent is parallel to the x -axis when $\frac{dy}{dx} = \frac{\pi}{2} \cos\left(\frac{\pi x}{2}\right) - \frac{\pi}{2} = 0$, i.e. $\cos\left(\frac{\pi x}{2}\right) = 1$.

Hence $\frac{\pi x}{2} = -2\pi, 0, 2\pi$, $\therefore x = -4, 0, 4$.

Q11 Let $y = e^{\sqrt{1+x^2}}$ and $u = \sqrt{1+x^2}$.

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = e^{\sqrt{1+x^2}} \times \frac{x}{\sqrt{1+x^2}} = \frac{xe^{\sqrt{1+x^2}}}{\sqrt{1+x^2}}.$$

Q12 Rate of change = gradient of $y = \frac{\log_e |x^3 + 3|}{x^3 + 3}$.

Use graphics calc to find $\frac{dy}{dx}$ at $x = -2$.

$$\frac{dy}{dx} = -0.29, \therefore \text{rate of decrease} = 0.29.$$

Q13 Use rectangles (left or right) to estimate.

$$\begin{aligned} \text{Q14 } \int_0^2 \frac{1}{2x-5} dx &= \left[\frac{\log_e |2x-5|}{2} \right]_0^2 \\ &= \frac{\log_e |-1|}{2} - \frac{\log_e |-5|}{2} = -\frac{1}{2} \log_e 5 = -\log_e \sqrt{5}. \end{aligned}$$

$$\begin{aligned} \text{Q15 } \int_1^2 [2f(x) - 3] dx &= 2 \int_1^2 f(x) dx - \int_1^2 3 dx \\ &= 2[F(x)]_1^2 - [3x]_1^2 = 2(F(2) - F(1)) - (6 - 3) = 2(4) - 3 = 5. \end{aligned}$$

Q16

$$\int_0^{\frac{\sqrt{\pi}}{2}} x \sin(2x^2) dx = \left[\frac{1}{2} \sin^2(x^2) \right]_0^{\frac{\sqrt{\pi}}{2}} = \frac{1}{2} \sin^2\left(\frac{\pi}{4}\right) = \frac{1}{2} \left(\frac{1}{\sqrt{2}}\right)^2 = \frac{1}{4}.$$

$$\text{Q17 } f'(x) = -\frac{x-p}{2\sqrt{2-(x-p)^2}}, \quad f'(p+1) = -\frac{1}{2}.$$

Q18 The difference of the results can be 0, 1, 2, 3, 4 or 5. It is a random variable.

Q19 Mode = 2,

$$\begin{aligned} \text{mean} &= 1 \times 0.1 + 2 \times 0.3 + 3 \times 0.2 + 4 \times 0.1 + 5 \times 0.2 + 6 \times 0.1 = 3.3, \\ \text{median} &= 3. \end{aligned}$$

Q20 $X < 2 \Rightarrow X = 0, 1$, i.e. one success or none.

$\therefore p = 0.1$ and $n = 1 + 5 = 6$.

Q21 Area under $p(x) = 1$, $\therefore \frac{1}{2}(3 + 7)(b - a) = 1$,

$\therefore b - a = 0.2$.

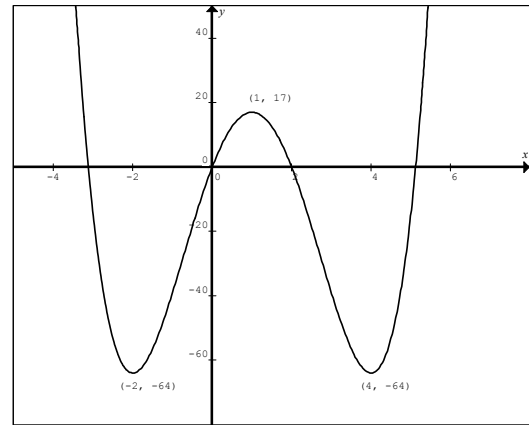
Q22 $\Pr(a \leq X < 2) = 0.8$, $\therefore \Pr(X < 2) - \Pr(X < a) = 0.8$,

$\therefore \Pr(X < a) = \Pr(X < 2) - 0.8 = 0.977 - 0.8 = 0.177$

$\therefore a = -0.93$.

SECTION 2

Q1a Use graphics calc, find x -intercepts at $x = -3.12, 0, 2, 5.12$



Q1b $g(x) = f(x) + p$ has exactly two x -intercepts when $p = 64$ or $p < -17$, i.e. $f(x)$ is translated upwards by 64 units or downwards by more than 17 units.

Q1c $h(x) = \frac{1}{4}[f(x) - x^4 + 4x^3] = \frac{1}{4}(-12x^2 + 32x) = -3x^2 + 8x$
 $k(x) = -h(1-x) + 2 = -[-3(1-x)^2 + 8(1-x)] + 2 = 3x^2 + 2x - 3$.

Q1di Use graphics calc to find the area of the three regions.
 Area = $123.3485 \times 2 + 22.4 = 269.10 \text{ unit}^2$.

Q1dii The regions are dilated vertically by a factor of $\frac{1}{2}$ and horizontally by a factor of 2. The area remains the same, i.e. 269.10 unit^2 . Reflection and horizontal translation do not change the area.

Q2a $(0, 6)$ gives $p + q = 6$.

$(\log_e 25, 1.6)$ gives $pe^{\frac{-\log_e 25}{2}} + q = 1.6$.

Q2b $e^{\frac{-\log_e 25}{2}} = (e^{\log_e 25})^{\frac{-1}{2}} = (25)^{\frac{-1}{2}} = \frac{1}{\sqrt{25}} = 0.2$.

Solve $p + q = 6$ and $0.2p + q = 1.6$ simultaneously to obtain
 $p = 5.5$ and $q = 0.5$

Q2c As $x \rightarrow \infty$, $e^{\frac{-x}{2}} \rightarrow 0$, $\therefore y \rightarrow 0.5$, $\therefore$ asymptote is $y = 0.5$

Q2d When $x = 0$, $y = 6$. When $x = 5$, $y = 5.5e^{-2.5} + 0.5$.

Average gradient = $\frac{5.5e^{-2.5} + 0.5 - 6}{5 - 0} = -1.01$

Q2e $\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$, $\frac{dy}{dt} = -2.75e^{\frac{-x}{2}} \times \frac{dx}{dt}$.

$$\text{Q2fi } \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{-1.1}{0.8} = -1.375$$

$$\text{Q2fii } \frac{dy}{dx} = -2.75e^{\frac{-x}{2}}, -1.375 = -2.75e^{\frac{-x}{2}}, \therefore e^{\frac{-x}{2}} = \frac{1}{2},$$

$$e^{\frac{x}{2}} = 2, \frac{x}{2} = \log_e 2, x = 2 \log_e 2.$$

$$y = 5.5e^{\frac{-x}{2}} + 0.5 = 5.5 \times \frac{1}{2} + 0.5 = 3.25$$

Coordinates $(2 \log_e 2, 3.25)$.

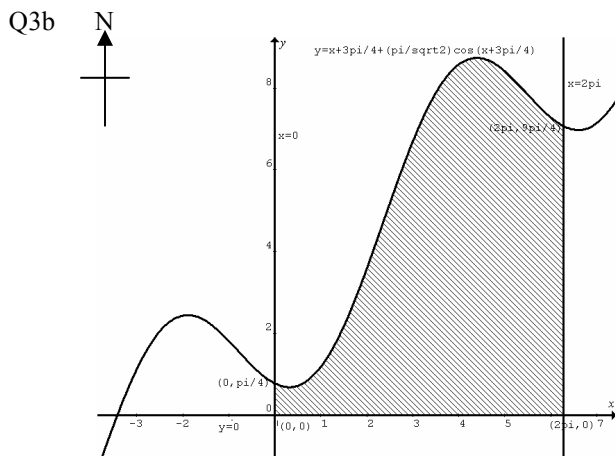
Q2gi Horizontal dilation is required, so change parameter r .

Q2gii Increase the dilation factor r .

$$\text{Q3a } x=0, y = \frac{3\pi}{4} + \frac{\pi}{\sqrt{2}} \cos\left(\frac{3\pi}{4}\right) = \frac{3\pi}{4} - \frac{\pi}{2} = \frac{\pi}{4} \cdot \left(0, \frac{\pi}{4}\right)$$

$$x=2\pi, y = 2\pi + \frac{3\pi}{4} + \frac{\pi}{\sqrt{2}} \cos\left(2\pi + \frac{3\pi}{4}\right) = \frac{9\pi}{4} \cdot \left(2\pi, \frac{9\pi}{4}\right)$$

$(0,0), (2\pi,0)$.



$$\text{Q3ci } \int_0^{2\pi} \left[x + \frac{3\pi}{4} + \frac{\pi}{\sqrt{2}} \cos\left(x + \frac{3\pi}{4}\right) \right] dx$$

$$\text{Q3cii } = \left[\frac{x^2}{2} + \frac{3\pi x}{4} + \frac{\pi}{\sqrt{2}} \sin\left(x + \frac{3\pi}{4}\right) \right]_0^{2\pi}$$

$$= \left[2\pi^2 + \frac{3\pi^2}{2} + \frac{\pi}{2} \right] - \left[\frac{\pi}{2} \right] = \frac{7\pi^2}{2}.$$

$$\text{Land area} = \frac{7\pi^2}{2} \times 100^2 \text{ m}^2 = 35000\pi^2 \text{ m}^2.$$

Q3d Using graphics calc, the local minimum in $[0, 2\pi]$ is $(0.3185, 0.6910)$.

$\therefore$ the shortest distance between the north and the south boundaries is $0.6910 \times 100 = 69.10$ m. Take off 15 m clearance from each boundary. The floor area is $(69.10 - 15 \times 2)^2 = 1529 \text{ m}^2$.

$$\text{Q4a } \Pr(\text{fail}) = \Pr(X \leq 49) = \text{normalcdf}(-E99, 49, 50, 4) = 0.401$$

$$\text{Q4bi } \int_0^a k e^{-kx} dx = \left[-e^{-kx} \right]_0^a = -e^{-ka} + 1.$$

$$\text{As } a \rightarrow \infty, e^{-ka} \rightarrow 0, \therefore \int_0^a k e^{-kx} dx \rightarrow 1.$$

$$\text{Q4bii } \Pr(0 \leq X \leq 50) = \int_0^{50} k e^{-kx} dx = \left[-e^{-kx} \right]_0^{50} = -e^{-k50} + 1 = 0.5,$$

$$e^{-50k} = 0.5, -50k = \log_e 0.5, \therefore k = 0.0139.$$

Q4biii Since $\Pr(0 \leq X \leq 50) = 0.5$, the median of X is 50.

$$\text{Q4c } \Pr(\text{fail}) = \Pr(0 \leq X \leq 49) = \int_0^{49} 0.0139 e^{-0.0139x} dx = 0.494.$$

Q4di Let X be the random variable – number of broken rods repaired with superglue A. $p = \frac{4}{10} = 0.4, \therefore q = 0.6$.

$$\Pr(\text{more with A than with B}) = \Pr(X=3) + \Pr(X=4)$$

$$= {}^5C_3 (0.4)^3 (0.6)^2 + {}^5C_4 (0.4)^4 (0.6)^1 = 0.31.$$

$$\text{Or } = \text{binompdf}(5, 0.4, 3) + \text{binompdf}(5, 0.4, 4) = 0.31.$$

$$\text{Q4dii } \Pr(X=2 | X=0, 1 \text{ or } 2) = \frac{\Pr(X=2 \cap X=0,1,2)}{\Pr(X=0,1,2)}$$

$$= \frac{\Pr(X=2)}{\Pr(X=0,1,2)} = \frac{\text{binompdf}(5, 0.4, 2)}{\text{binomcdf}(5, 0.4, 2)} = 0.51.$$

$$\text{Q4e } \Pr(\text{fail}) = 0.401 \times 0.4 + 0.494 \times 0.6 = 0.46.$$

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