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Mathematical Methods

2007

Trial Examination 1

Instructions

Answer **all** questions. Do **not** use calculators.

A decimal approximation will not be accepted if an **exact** answer is required to a question.

In questions where more than one mark is available, appropriate working must be shown.

Unless otherwise indicated, the diagrams in this exam are **not** drawn to scale.

Question 1 X is a normally distributed random variable with a mean of 56 and a standard deviation of 5. Z is the standard normal random variable.

a. Find the value of Z that corresponds to $X = 45$. 1 mark

b. Given that $\Pr(-1 < Z < 1) = 0.68$,
i. find $\Pr(56 < X < 61)$, correct to 2 decimal places 1 mark

ii. find $\Pr(X < 51 \cup X > 61)$, correct to 2 decimal places. 1 mark

Question 2 The probability density function of a continuous random variable X is given by

$$p(x) = \begin{cases} kx & 0 \leq x \leq 3 \\ 0 & \text{elsewhere} \end{cases}$$

a. Find the exact value of k . 2 marks

b. Find the exact value of $\Pr(X > 2)$. 1 mark

Question 3 Find the exact x -intercepts of the graph of $f(x) = x^3 - 3x^2 - 8x + 24$ for $x \geq 0$. 2 marks

Question 4 Given $f(x) = |3 - 2x| + 1$,

a. sketch the graph of $y = f(x)$. 2 marks

b. State a sequence of exactly three transformations, which changes $y = f(x)$ to $y = |x|$. 2 marks

Question 5 Given $f(x) = 10^{2\log_{10}(x) - \log_{10}(x+1) - 1}$,

a. show that $y = f(x)$ can be simplified to $y = \frac{x^2}{10x + 10}$. 2 marks

b. Find $f^{-1}(x)$. 2 marks

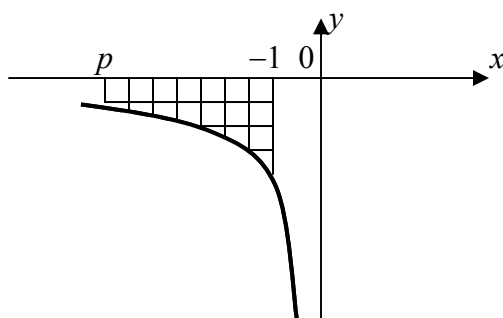
Question 6 Solve $\sin(3\theta) + \sqrt{3}\cos(3\theta) = 0$ for the exact values of θ , where $-\pi \leq \theta \leq \pi$. 3 marks

Question 7 Consider $f(x) = e^{2x} - e^x - 2$.

a. Solve $e^{2x} - e^x - 2 = 0$ for x in exact form. 2 marks

b. Find the equation of the normal to the curve with equation $y = e^{2x} - e^x - 2$ where $e^x = 2$. 3 marks

Question 8 The area of the shaded region is 4 in the following graph of $y = \frac{2}{x}$. Find the exact value of p . 3 marks



Question 9 Consider $f(x) = -3x^4 - 4x^3 + 6x^2 + 12x - 3$.

a. Find the coordinates of the stationary points.

2 marks

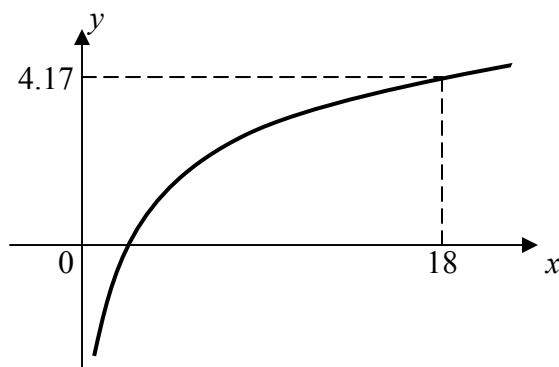
b. Determine the nature of each stationary point found in part a.

2 marks

c. Sketch the graph of $y = f(x)$. (Note: Do not determine the values of the x -intercepts)

2 marks

Question 10 Given $f(x) = \log_2 x$ and the graph of $f(x)$ is shown below.



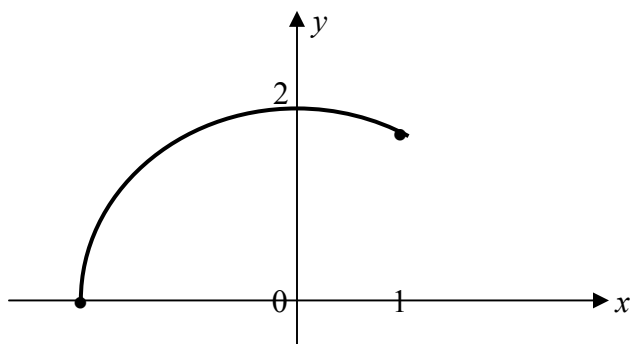
a. Show that $f'(x) = \frac{1}{x \log_e 2}$.

1 mark

b. Use Euler's linear approximation method to estimate $\log_2 19$ to 2 decimal places, given $\log_e 2 = 0.69$.

2 marks

Question 11 The following graph shows a portion of a circle with radius of 2 units and centred at the origin.



a. Use function notation to specify the portion of the circle shown above. 2 marks

b. Sketch the graph of the gradient function from the graph shown above. Pay particular attention to the end points.

2 marks

End of exam 1