

1a. $Z = \frac{45-56}{5} = -2.2$

1bi.

$$\Pr(56 < X < 61) = \Pr(0 < Z < 1) = \frac{1}{2} \Pr(-1 < Z < 1) = \frac{1}{2} \times 0.68 = 0.34$$

1bii.

$$\Pr(X < 51 \cup X > 61) \\ = \Pr(Z < -1 \cup Z > 1) = 1 - \Pr(-1 < Z < 1) = 1 - 0.68 = 0.32$$

2a. $\int_0^3 kx dx = 1, \left[\frac{kx^2}{2} \right]_0^3 = 1, \therefore \frac{9k}{2} = 1, k = \frac{2}{9}$

2b. $\Pr(X > 2) = \int_2^3 x dx = \left[\frac{x^2}{2} \right]_2^3 = 1 - \frac{4}{9} = \frac{5}{9}$

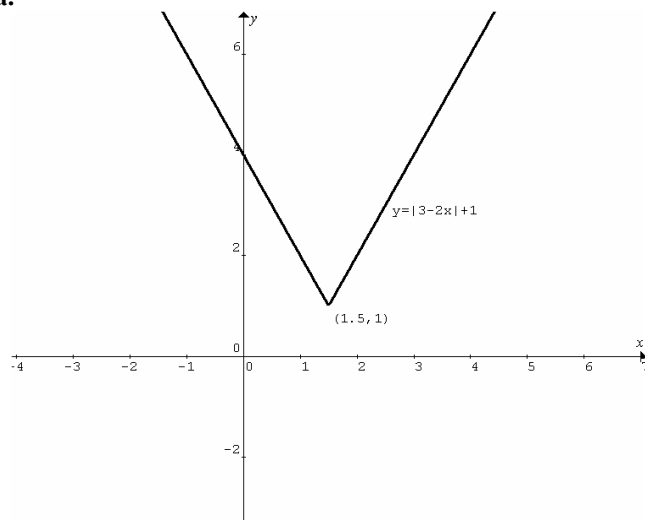
3. Solve $x^3 - 3x^2 - 8x + 24 = 0, x \geq 0$.

$$(x^3 - 3x^2) - (8x - 24) = 0, x^2(x-3) - 8(x-3) = 0,$$

$$(x^2 - 8)(x-3) = 0, (x-2\sqrt{2})(x+2\sqrt{2})(x-3) = 0.$$

Since $x \geq 0, \therefore x = 2\sqrt{2}$ or 3

4a.



4b.

Translate 1 unit downwards.

Translate 1.5 units to the left.

Dilate horizontally by a factor of 2 (or dilate vertically by a factor of $\frac{1}{2}$).

5a. $f(x) = 10^{2\log_{10}(x) - \log_{10}(x+1) - 1} = 10^{\log_{10}(x^2) - \log_{10}(x+1) - \log_{10} 10}$
 $= 10^{\log_{10} \frac{x^2}{10(x+1)}} = \frac{x^2}{10(x+1)} = \frac{x^2}{10x+10}$ where $x > 0$

5b. Equation of the original function: $y = \frac{x^2}{10x+10}$ where $x > 0$

Equation of the inverse: $x = \frac{y^2}{10y+10}$ where $y > 0$

Transpose to make y the subject of the equation:

$$y^2 - 10xy - 10x = 0,$$

$$y = \frac{10x + \sqrt{100x^2 + 40x}}{2} \quad (\text{quadratic formula: } a = 1, b = -10x, c = -10x),$$

$$\therefore y = 5x + \sqrt{25x^2 + 10x} \quad (\text{Note: Only '+' results in } y > 0)$$

$$\therefore f^{-1}(x) = 5x + \sqrt{25x^2 + 10x}.$$

6. $\sin(3\theta) + \sqrt{3} \cos(3\theta) = 0, -\pi \leq \theta \leq \pi$

$$\frac{\sin(3\theta) + \sqrt{3} \cos(3\theta)}{\cos(3\theta)} = 0, -3\pi \leq 3\theta \leq 3\pi$$

$$\frac{\sin(3\theta)}{\cos(3\theta)} + \sqrt{3} = 0, \tan(3\theta) = -\sqrt{3},$$

$$\therefore 3\theta = -\frac{7\pi}{3}, -\frac{4\pi}{3}, -\frac{\pi}{3}, \frac{2\pi}{3}, \frac{5\pi}{3}, \frac{8\pi}{3}$$

$$\therefore \theta = -\frac{7\pi}{9}, -\frac{4\pi}{9}, -\frac{\pi}{9}, \frac{2\pi}{9}, \frac{5\pi}{9}, \frac{8\pi}{9}.$$

7a. $e^{2x} - e^x - 2 = 0, (e^x - 2)(e^x + 1) = 0.$

Since $e^x + 1 > 0, \therefore e^x - 2 = 0, e^x = 2, x = \log_e 2.$

7b. When $e^x = 2, x = \log_e 2$ and $y = 2^2 - 2 - 2 = 0, \therefore (\log_e 2, 0).$

Gradient function = $\frac{dy}{dx} = 2e^{2x} - e^x.$

When $e^x = 2, \text{ gradient of tangent} = m_T = 2(2^2) - 2 = 6.$

$$\therefore \text{gradient of normal} = m_N = -\frac{1}{m_T} = -\frac{1}{6}.$$

Equation of normal at $(\log_e 2, 0):$

$$y - y_1 = m_N(x - x_1),$$

$$y = -\frac{1}{6}(x - \log_e 2),$$

$$x + 6y = \log_e 2.$$

8. Area = $-\int_p^{-1} \frac{2}{x} dx$, $-\int_p^{-1} \frac{2}{x} dx = 4$, where $p < 0$.

$$- [2 \log_e |x|]_p^{-1} = 4, \quad -2 \log_e |-1| + 2 \log_e |p| = 4,$$

$$-2 \log_e 1 + 2 \log_e (-p) = 4,$$

$$2 \log_e (-p) = 4, \quad \log_e (-p) = 2,$$

$$-p = e^2, \quad \therefore p = -e^2.$$

9a. $y = f(x) = -3x^4 - 4x^3 + 6x^2 + 12x - 3$,

$$f'(x) = -12x^3 - 12x^2 + 12x + 12.$$

At stationary points, $f'(x) = 0$, $-12x^3 - 12x^2 + 12x + 12 = 0$,

$$-12(x^3 + x^2 - x - 1) = 0, \quad -12((x^3 + x^2) - (x + 1)) = 0,$$

$$-12(x^2(x + 1) - 1(x + 1)) = 0,$$

$$-12(x^2 - 1)(x + 1) = 0,$$

$$-12(x - 1)(x + 1)^2 = 0,$$

$$\therefore x = 1 \text{ and } y = 8,$$

$$\text{or } x = -1 \text{ and } y = -8.$$

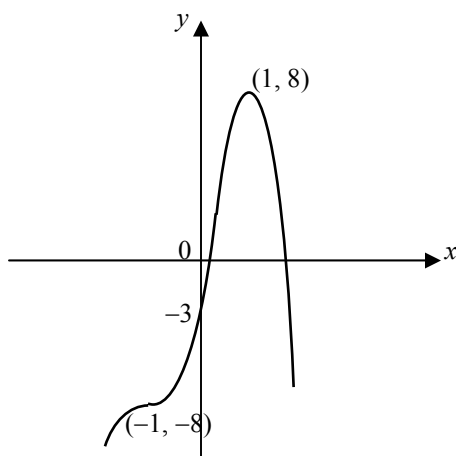
Stationary points are $(-1, -8)$ and $(1, 8)$.

9b. Check the gradients on both sides of stationary points:

x	-2	-1	0	1	2
$f'(x)$	+	0	+	0	-

$\therefore (-1, -8)$ is an inflection point and $(1, 8)$ is a local maximum point.

9c.



10a. $f(x) = \log_2 x = \frac{\log_e x}{\log_e 2}$, $f'(x) = \frac{1}{\log_e 2} \times \frac{1}{x} = \frac{1}{x \log_e 2}$.

10b. $f(a + h) \approx f(a) + h \times f'(a)$, let $a = 18$ and $h = 1$.

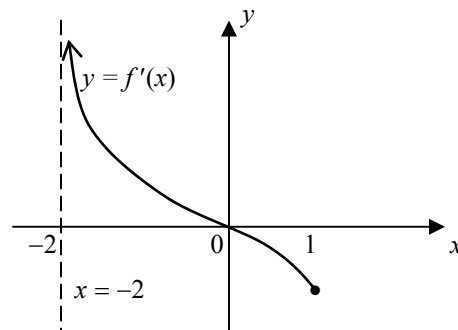
$$\therefore f(19) \approx f(18) + 1 \times f'(18), \text{ i.e. } \log_2 19 \approx \log_2 18 + \frac{1}{18 \log_e 2}.$$

$$\therefore \log_2 19 \approx 4.17 + \frac{1}{18 \times 0.69} = 4.17 + 0.08 = 4.25$$

11a. Equation of circle: $x^2 + y^2 = 2^2$, $\therefore y = \sqrt{4 - x^2}$.

$$f: [-2, 1] \rightarrow R, f(x) = \sqrt{4 - x^2}.$$

11b.



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