Insight Publications

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Trial Exam Paper

2007

MATHEMATICAL METHODS

Written examination 2

STUDENT NAME:

QUESTION AND ANSWER BOOK

Reading time: 15 minutes
Writing time: 2 hours

Structure of book

Section	*Number of questions*	*Number of questions to be answered*	*Number of marks*
1	22	22	22
2	4	4	58
		Total	80

- Students are permitted to bring the following items into the examination: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, one approved **graphics** calculator (memory DOES NOT need to be cleared) and, if desired, one scientific calculator, one bound reference.
- Students are NOT permitted to bring the following items into the examination: blank sheets of paper and/or white out liquid/tape.

Materials provided
- The question and answer book of 20 pages, with a separate sheet of miscellaneous formulas.
- An answer sheet for multiple-choice questions.

Instructions
- Write your **name** in the box provided and on the answer sheet for multiple-choice questions.
- Remove the formula sheet during reading time.
- You must answer the questions in English.

At the end of the exam
- Place the answer sheet for multiple-choice question inside the front cover of this question book.

Students are NOT permitted to bring mobile phones or any other electronic devices into the examination.

SECTION 1

Instructions for Section 1

Answer **all** questions on the multiple-choice answer sheet provided.

Choose the **correct** response – a correct answer scores 1, an incorrect answer scores zero.

Marks will **not** be deducted for an incorrect answer and no marks will be given if more than one response is recorded for any one question.

Question 1

The graph with the equation $y = x^3$ is translated 3 units down and then reflected in the x-axis. The resulting graph has the equation

A. $y = x^3 + 3$

B. $y = -x^3 - 3$

C. $y = -x^3 + 3$

D. $y = x^3 - 3$

E. $y = -(x-3)^3$

Question 2

The range of the function $f : [-1, 4) \to R,\ f(x) = x^2 - 2$ is

A. $[-1,\ 14)$

B. $(-1, 14]$

C. $[-2, 14)$

D. $(-2, 14)$

E. R

Question 3

The range of the function $f : R \to R,\ f(x) = -2\cos(x - \pi) + 3$ is

A. $[-2, 3]$

B. $[1, 5]$

C. $[-5, -1]$

D. $[0, 3]$

E. $[-\pi,\ 3]$

Question 4

The largest set of real values of t for which $|t-2| > 4t$ is

A. $t > \frac{2}{5}$

B. $t > -\frac{2}{3}$

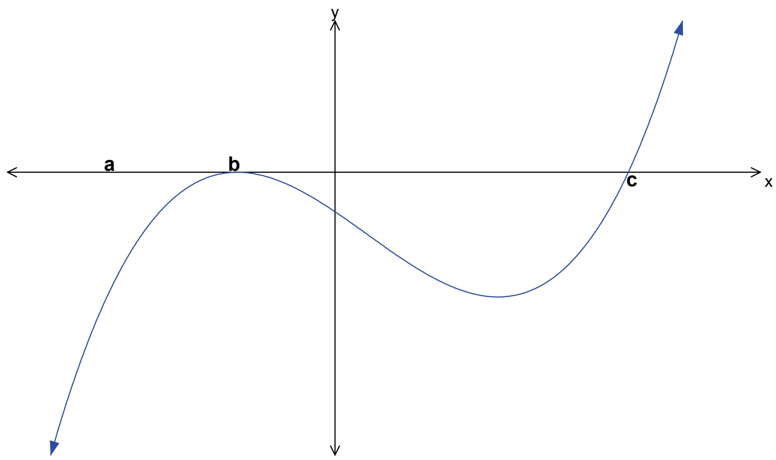
C. $t < \frac{-2}{3}$ or $t > \frac{2}{5}$

D. $t < \frac{2}{5}$

E. $\frac{2}{5} < t < 2$

Question 5

Part of the graph of the function f is shown below.



The total area bounded by the curve of $y = f(x)$ and the x-axis over the interval (a,c) is given by

A. $\int_a^c f(x)dx$

B. $\int_c^a f(x)dx$

C. $-\int_c^a f(x)dx$

D. $\int_a^b f(x)dx - \int_b^c f(x)dx$

E. $\int_a^b f(x)dx + \int_b^c f(x)dx$

Question 6

The function f has the rule $f(x) = \log_e|2x-1|$. The maximal domain of f is

A. $(-1,\infty)$

B. $(\frac{1}{2},\infty)$

C. $(1,\infty)$

D. $R\setminus\left\{\frac{1}{2}\right\}$

E. R^+

Question 7

A bag contains three red and six blue balls. Three balls are drawn one at a time from the bag without replacement. The probability that at least one is red is

A. $\frac{15}{28}$

B. $\frac{5}{21}$

C. $\frac{16}{21}$

D. $\frac{5}{28}$

E. $\frac{83}{84}$

Question 8

The function $f:[a,\infty)\to R$, with the rule $y = x^{\frac{2}{3}}$, will have an inverse function if

A. $a<1$

B. $a>-1$

C. $a>0$

D. $a<-1$

E. $a<0$

Question 9

The graph of the function $y = -e^{(x+2)} - 5$ is obtained from the graph of $y = e^x$ by

A. a reflection in the x-axis, a translation by 2 units up and a translation 5 units down.
B. a reflection in the x-axis, a translation by 5 units right and a translation 2 units left.
C. a reflection in the y-axis, a translation by 5 units right and a translation 2 units down.
D. a reflection in the x-axis and a dilation by a factor of 5 from the x-axis.
E. a reflection in the x-axis, a translation by 2 units left and a translation 5 units down.

Question 10

If $y = 2a^{\frac{x}{3}} - b$ then x is equal to

A. $3\log_a(\frac{y+b}{2})$

B. $3\log_a(\frac{y-b}{2})$

C. $3\log_a \frac{y}{2} + b$

D. $\frac{1}{3}\log_a(\frac{y+b}{2})$

E. $\frac{1}{3}\log_a(\frac{2y+b}{2})$

Question 11

A fair coin is tossed 20 times. The probability, correct to four decimal places, of getting fewer than 12 heads is

A. 0.1201
B. 0.8684
C. 0.7483
D. 0.1316
E. 0.2517

Question 12

The function $f:(5,\infty)\to R$ has the rule $f(x)=\dfrac{2x+1}{x-5}$. The rule for the inverse function is

A. $f^{-1}(x)=\dfrac{11}{x-2}+5$

B. $f^{-1}(x)=\dfrac{11}{x+2}-5$

C. $f^{-1}(x)=\dfrac{11}{x+5}-2$

D. $f^{-1}(x)=\dfrac{1}{x+5}-2$

E. $f^{-1}(x)=5-\dfrac{11}{x+2}$

Question 13

If $4^x=e^{kx}$ for all $x\in R$, then k equals

A. 4

B. $\log_e 4$

C. $\log_e 4x$

D. $\dfrac{4}{e}$

E. $4x$

Question 14

A function $f:R\to R$ is such that:

$f'(x)=0$ at $x=-2$ and $x=3$
$f'(x)>0$ for $x<-2$ and $-2<x<3$
$f'(x)<0$ for $x>3$

Which one of the following is true?

A. The graph has a local maximum turning point at $x=-2$.
B. The graph has a stationary point of inflection at $x=-2$.
C. The graph has a local minimum at $x=-2$.
D. The graph has a local minimum at $x=3$.
E. The graph has a stationary point of inflection at $x=3$.

Question 15

If $f'(x) = 4x - g'(x)$, $f(0) = 3$ and $g(0) = -2$, then $f(x)$ is given by

A. $f(x) = 4x - g(x) + 1$

B. $f(x) = 4x - g(x) - 5$

C. $f(x) = 2x^2 - g(x) + 1$

D. $f(x) = 2x^2 - g(x) - 5$

E. $f(x) = 4x - g'(x)$

Question 16

The derivative of $\dfrac{\cos(3x)}{2x - e^{4x}}$ with respect to x is

A. $\dfrac{-3\sin(3x)}{2 - 4e^{4x}}$

B. $\dfrac{\cos(3x)(2 - 4e^{4x}) - 3(2x - e^{4x})\sin(3x)}{4x^2 - 4xe^{4x} + e^{8x}}$

C. $\dfrac{-3(2x - e^{4x})\sin(3x) - \cos(3x)(2 - 4e^{4x})}{4x^2 + e^{8x}}$

D. $\dfrac{-3(2x - e^{4x})\sin(3x) - \cos(3x)(2 - 4e^{4x})}{4x^2 - 4xe^{4x} + e^{8x}}$

E. $\dfrac{(2x - e^{4x})\cos(3x) - \sin(3x)(2 - 4e^{4x})}{4x^2 - 4xe^{4x} + e^{8x}}$

Question 17

If $y = |\log_e(x)|$, then the rate of change of y with respect to x at $x = k$, $0 < k < 1$, is

A. $\dfrac{1}{k}$

B. $-\dfrac{1}{k}$

C. $-\log_e k$

D. $\log_e k$

E. e^k

Question 18

If $f : R \to R$ be a differentiable function, then for all $x \in R$, the derivative of $f(e^{4x})$ with respect to x is equal to

A. $4e^{4x} f'(x)$

B. $e^{4x} f'(x)$

C. $f'(e^{4x})$

D. $4e^{4x} f'(e^{4x})$

E. $e^{4x} f'(e^{4x})$

Question 19

An antiderivative of $e^{3x} + 3x + 3$ is

A. $9e^{3x} + 6x + 3$

B. $\frac{1}{3}e^{3x} + 3x^2 + 3x$

C. $3e^{3x} + 3x^2 + 3$

D. $\frac{1}{3}e^{3x} + \frac{3}{2}x^2 + 3x$

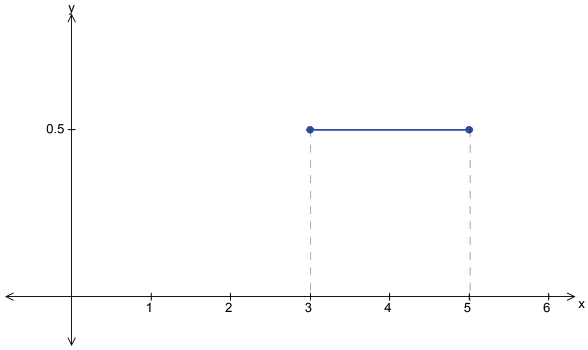
E. $3e^{3x} + 3$

Question 20

A continuous random variable X has a probability density function given by

$$f(x) = \frac{1}{2} \text{ for } 3 < x < 5$$

The graph of f is shown below.



The mean of X is

A. 0.25

B. 0.5

C. 3

D. 4

E. 5

Question 21

A probability density function is given by

$$f(x) = \frac{1}{8}(2 + 2x) \qquad \text{for } 0 < x < k$$

The value of k is

A. 1

B. 2

C. 4

D. 8

E. 12

Question 22

The masses of '55 gram' chocolate bars are normally distributed with a mean of 55.1 g and a standard deviation of 0.15 g. It would be expected that about two-thirds of the chocolate bars produced would be between

A. 54.65 g and 55.55 g

B. 55.10 g and 55.40 g

C. 54.95 g and 55.25 g

D. 54.80 g and 55.40 g

E. 51.10 g and 55.55 g

END OF SECTION 1
TURN OVER

SECTION 2

Instructions for Section 2

Answer **all** questions in the spaces provided.

A decimal answer will not be accepted if an **exact** answer is required to a question.

In questions where more than one mark is available, appropriate working must be shown.

Where an instruction to **use calculus** is stated for a question, you must show an appropriate derivative or antiderivative.

Unless otherwise stated diagrams are not drawn to scale.

Question 1

Speak-Easy is a mobile phone service provider, with more than two million customers, and has two mobile phone plans, the Quickchat Plan and the Gasbagger Plan.

70% of customers have the Quickchat plan and 30% of customers have the Gasbagger Plan.

a. If 12 Speak-Easy customers are selected at random

i. what is the probability, correct to four decimal places, that exactly nine customers are on the Quickchat plan?

ii. how many customers would be expected to be on the Quickchat plan?

2 + 1 = 3 marks

b. The sales manager for Speak-Easy has calculated that the monthly time in hours used by Quickchat customers is a random variable with the probability density function given by

$$f(t) = \begin{cases} 0.04e^{-0.04t} & \text{for } t > 0 \\ 0 & \text{for } t \le 0 \end{cases}$$

i. What percentage, correct to the nearest per cent, of Quickchat customers use more than 20 hours in a month?

ii. What is the probability, correct to three decimal places, that a randomly chosen Quickchat customer uses more than 30 hours per month, given that the customer uses more than 20 hours per month?

iii. Find the mean number of hours used in a month by a randomly selected Quickchat customer.

iv. Find the median number of hours, correct to 2 decimal places, used in a month by a randomly selected Quickchat customer.

2 + 2 + 2 + 2 = 8 marks

c. To try to increase profits, Speak-easy is conducting a marketing campaign to persuade customers to switch to the more profitable Gasbagger mobile phone plan. The marketing manager estimates that each month 30% of customers on the Quickchat plan will switch to the Gasbagger plan, and 5% of the customers on the Gasbagger plan will switch to the Quickchat plan. No existing customers of Speak-Easy are estimated to stop using Speak-Easy.

According to the estimates, after two months, what proportion of the present customers will be on the Quickchat plan (correct to two decimal places)?

3 marks

Total 14 marks

Question 2

Consider the function $f: R \rightarrow R, f(x) = x(x-2)^3 + 2$.

a. If $f'(x) = (ax+b)(x-2)^2$, where a and b are constants, use calculus to find the values of a and b.

2 marks

b. The coordinates of the stationary points of the graph of $y = f(x)$ are $(u,2)$ and $(v,\frac{5}{16})$.

Find the values of u and v.

2 marks

c. Find the values of x for which $f'(x) \leq 0$.

2 marks

SECTION 2 – Question 2 – continued
TURN OVER

d. Find the real values of k for which both solutions to the equation $f(x+k)=2$ are positive.

1 mark

The function *f(x)* can also be written as $f(x)=x^4-6x^3+12x^2-8x+2$.

e. Use calculus to find the area bounded by the graph of $y=f(x)-2$ and the x-axis.

3 marks

f. **i.** Describe a sequence of transformations which maps the graph of $y=f(x)$ on to the graph of $y=f(2x)-2$.

ii. Find the x-axis intercepts of the graph of $y=f(2x)-2$.

iii. Use the answers to 2e., 2fi. and 2fii. above to write down the area of the region bounded by the graph of $y = f(2x) - 2$ and the x-axis, correct to two decimal places.

2 + 1 + 1 = 4 marks

g. Find the real values of p for which the equation $\left|f(2x) - 2\right| = p$ has exactly four solutions.

2 marks

Total 16 marks

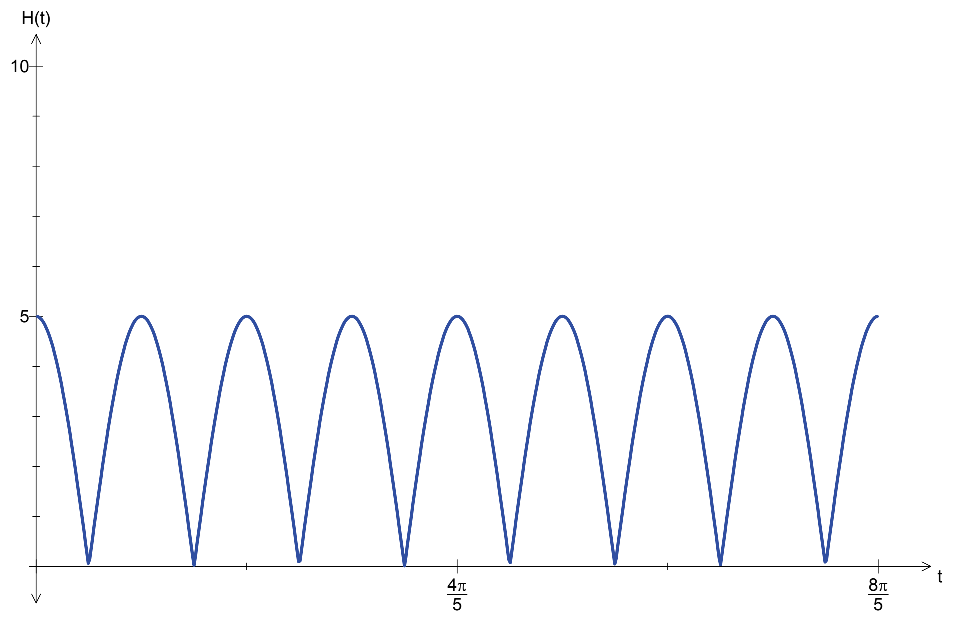
Question 3

A ball is dropped from a height of 5 metres and bounces continuously in a regular motion for the next $\frac{8\pi}{5}$ seconds. The height of the ball above the ground is given by

$$H(t) = |5\cos(5t)|$$

where H is the height in metres and t is the time in seconds after the ball is dropped.

The graph of the motion as modelled by the equation $H(t) = |5\cos(5t)|$ for $t \in [0, \frac{8\pi}{5}]$ is shown below.



a. Find when the ball first hits the ground.

1 mark

b. Find when the ball first rebounds to maximum height.

1 mark

Realistically, the ball cannot rebound to its previous height. Instead, the rebound height on each bounce will diminish with each bounce. It is found that the height of a ball released from a height of 5 metres can be more accurately modelled by the equation

$$H(t) = \left|5e^{-0.8t}\cos(5t)\right|$$

c. Find the first time, correct to 2 decimal places, when the ball is exactly 3 metres above the ground.

1 mark

d. How many times during the motion as described by the model is the ball exactly at a height of 2 metres above the ground?

1 mark

e. **i.** Find an expression for $\frac{dH}{dt}$ for values of t such that $\frac{\pi}{10} < t < \frac{3\pi}{10}$.

ii. Find $\frac{dH}{dt}$ for $t = \frac{\pi}{5}$. Give your answer correct to two decimal places.

iii. Using your answer to **ei**., write down an equation the solution of which gives the value of t when the ball has rebounded fully from each bounce. Find this value of t for the first rebound, correct to two decimal places. Also find, according to the model, the height of the first full rebound, correct to two decimal places.

3 + 1 + 3 = 7 marks

SECTION 2 – Question 3 – continued
TURN OVER

f. The company that manufactures the balls can modify the elastic component of the balls and produce balls that bounce back to different rebound heights according to the model

$$H(t) = \left|5e^{-at}\cos(5t)\right| \quad \text{for } a > 0$$

Find the exact value of t that gives the time when a ball rebounds fully after the first bounce.

3 marks

g. A ball is considered flat if the first full rebound height is less than 2 metres. Find, correct to 3 decimal places, the smallest value of a for a ball to be considered flat.

3 marks

Total 17 marks

Question 4

Two street lights A and B, of power 40 units and 320 units respectively, are placed 20 metres apart. The intensity of light illumination, I units, along the straight road between the two lights is given by

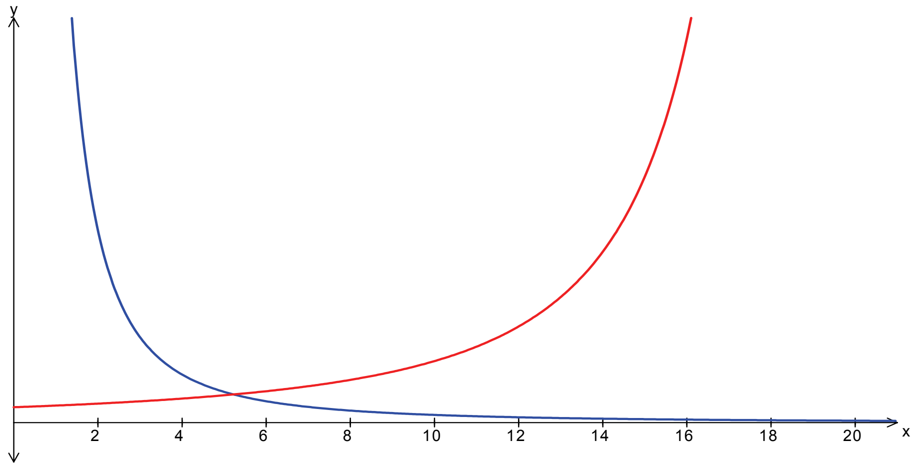
$$I(x) = \frac{40}{x^2} + \frac{320}{(20-x)^2},$$

where x is the distance in metres from light A.

a. What is the intensity of light illumination at a point 5 metres from light B? Express the answer correct to 2 decimal places.

1 mark

b. The graphs of $y_1 = \dfrac{40}{x^2}$ and $y_2 = \dfrac{320}{(20-x)^2}$ are shown below. On the same set of axes, sketch the graph of I for $0 < x < 20$.



3 marks

c. Diana stands on the straight road between lights A and B. Where should she stand to ensure that the light illumination intensity is more than 18 units? Express your answer correct to two decimal places.

2 marks

d. Use calculus to find the exact point on the straight road between the two lights that has the least light illumination intensity.

5 marks

Total 11 marks

END OF QUESTION AND ANSWER BOOK