

INSIGHT

Trial Exam Paper

2007

MATHEMATICAL METHODS

Written examination 1

STUDENT NAME:

QUESTION AND ANSWER BOOK

Reading time: 15 minutes

Writing time: 1 hour

Structure of book

<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
11	11	40

- Students are permitted to bring the following items into the examination: pens, pencils, highlighters, erasers, sharpeners and rulers.
- Students are NOT permitted to bring notes of any kind, sheets of paper, white out liquid/tape or a calculator into the examination.

Materials provided

- The question and answer book of 12 pages, with a separate sheet of miscellaneous formulas.
- Working space is provided throughout the question book.

Instructions

- Write your **name** in the box provided.
- Remove the formula sheet during reading time.
- You must answer the questions in English.

Students are NOT permitted to bring mobile phones or any other electronic devices into the examination.

This trial examination produced by Insight Publications is NOT an official VCAA paper for the 2007 Mathematical Methods written examination 1.

This examination paper is licensed to be printed, photocopied or placed on the school intranet and used only within the confines of the purchasing school for examining their students. No trial examination or part thereof may be issued or passed on to any other party including other schools, practising or non-practising teachers, tutors, parents, websites or publishing agencies without the written consent of Insight Publications.

Copyright © Insight Publications 2007

This page is blank.

Instructions

Answer **all** questions in the spaces provided.
 A decimal approximation will not be accepted if an **exact** answer is required to a question.
 In questions where more than one mark is available, appropriate working must be shown.
 Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1

Let $f(x) = 2x - 5$ and $g(x) = \cos x$. Write down the rule of $f(g(x))$.

1 mark

Question 2

For the function $f : (2, \infty) \rightarrow R$, $f(x) = 2 \log_e(x - 1)$,

a. find the rule for the inverse function f^{-1} .

2 marks

b. find the domain of the inverse function f^{-1} .

1 mark

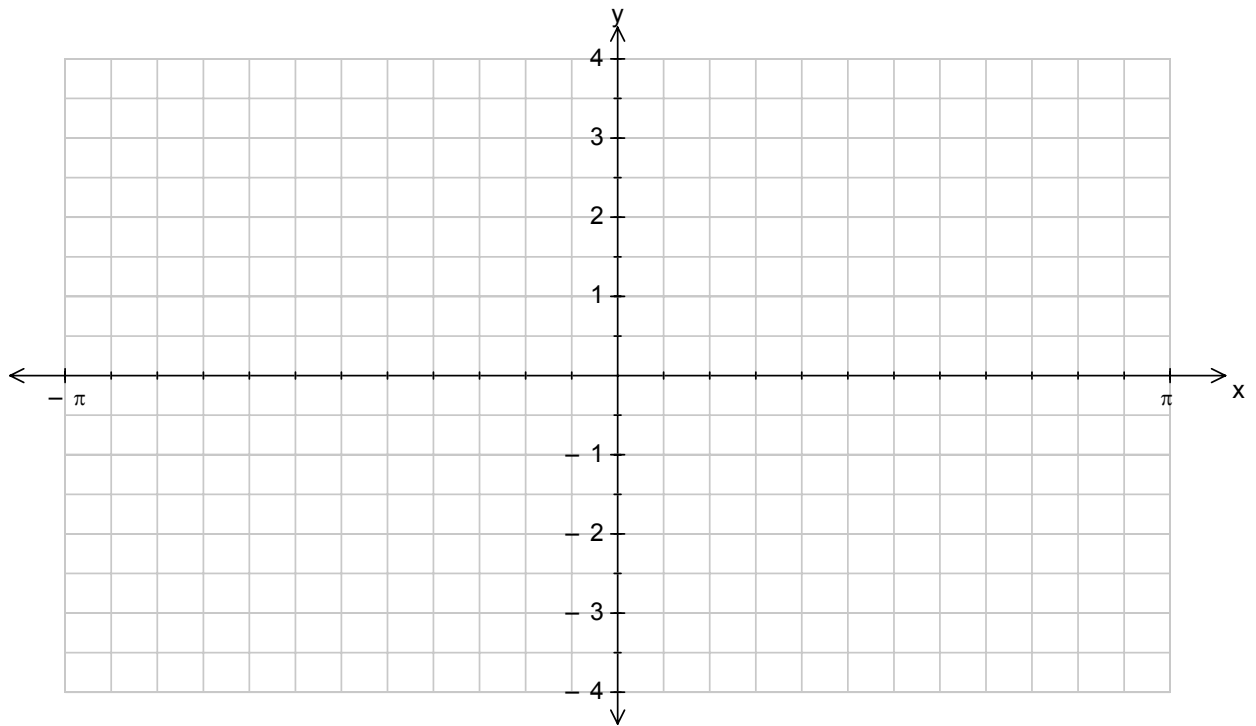
Question 3

For the function $f : [-\pi, \pi] \rightarrow \mathbb{R}$, $f(x) = -2\sin(3(x - \frac{\pi}{4}))$

- a.** write down the amplitude and period of the function.

2 marks

- b.** on the set of axes below, sketch the graph of the function f . Label the axis intercepts with their coordinates. Label the end-points of the graph with their coordinates.



3 marks

Question 4

a. Let $f(x) = \log_e(\sin(x))$. Find $f'(x)$.

1 mark

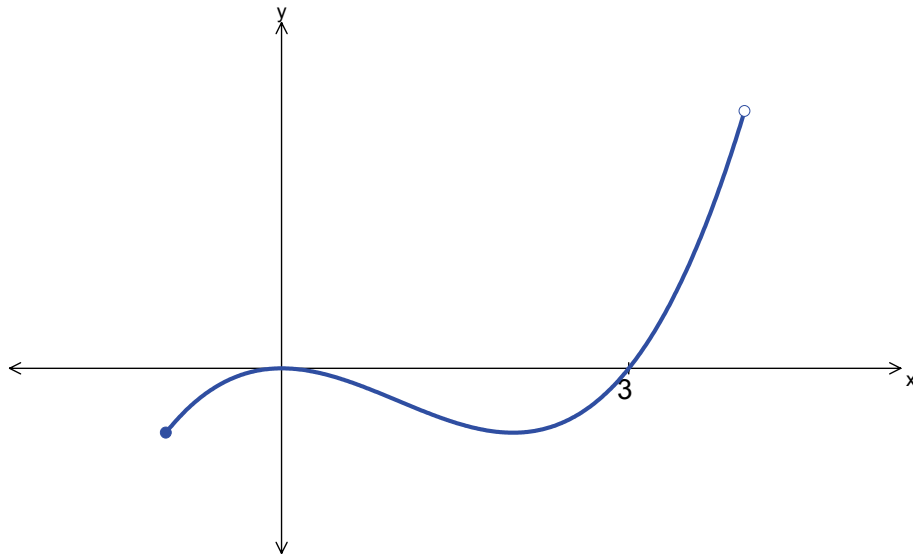
b. Let $y = x^2 \cos(x)$. Evaluate $\frac{dy}{dx}$ when $x = \frac{\pi}{3}$.

2 marks

CONTINUED PLEASE TURN OVER

Question 5

The graph of $f : [-1, 4] \rightarrow \mathbb{R}$ where $f(x) = x^3 - 3x^2$ is shown below.



- a.** Let $g(x) = |f(x)|$. On the same set of axes, sketch the graph of g .

2 marks

- b.** State the domain of the derivative function g' .

1 mark

Question 6

Solve the equation $\sin(2x) - \sqrt{3} \cos(2x) = 0$ for $x \in [0, 2\pi]$, giving exact values in terms of π .

3 marks

CONTINUED PLEASE TURN OVER

Question 7

The probability density function of a continuous random variable X is given by

$$f(x) = \begin{cases} \frac{x}{k} & 2 \leq x \leq 6 \\ 0 & \text{otherwise} \end{cases}$$

- a.** Show that $k = 16$.

2 marks

- b.** Find $\Pr(X > 4)$

2 marks

- c.** Find the median of X

2 marks

Question 8

The random variable X has the following probability distribution:

x	-1	0	1	2
$Pr(X = x)$	$a + b$	$2a - b$	$3a$	0.4

- a.** Find the value of a .

1 mark

- b.** If $E(X) = 0.95$, find the value of b .

2 marks

Question 9

The random variable X is normally distributed with mean 50 and standard deviation 5. The random variable Z is normally distributed with mean 0 and standard deviation 1.

- a.** If $\Pr(X < 56) = \Pr(Z < a)$, find the value of a .

2 marks

- b.** If $\Pr(50 < X < b) = 0.5 - \Pr(Z > 2)$, find the value of b .

2 marks

Question 10

A hemispherical bowl is being filled with water at a constant rate of $150\pi \text{ cm}^3/\text{min}$. When the depth of the water in the bowl is h cm, the volume, $V \text{ cm}^3$, of the water is given by

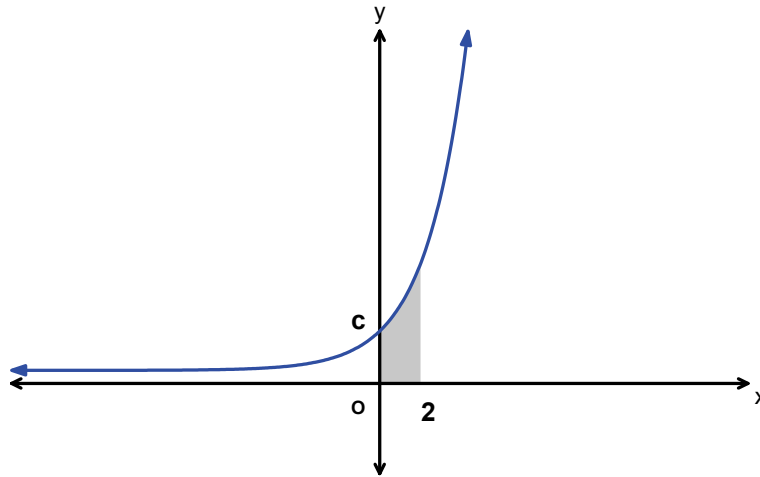
$V = \pi h^2 (30 - \frac{2}{3}h)$. Find the rate at which the depth of the water is increasing when the depth is 5 cm.

4 marks

CONTINUED PLEASE TURN OVER

Question 11

Part of the graph of the function $f : \mathbb{R} \rightarrow \mathbb{R}$, $f(x) = ae^{2x} + b$ is shown below. If the shaded area is $3e^4 + 1$ square units, find one set of possible values for a , b and c , where c is the y -intercept of the graph $y = f(x)$.



5 marks

END OF QUESTION AND ANSWER BOOK