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**MATHS METHODS 3 & 4
TRIAL EXAMINATION 2
SOLUTIONS
2007**

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- | | | |
|------|-------|-------|
| 1. D | 9. D | 17. B |
| 2. B | 10. C | 18. C |
| 3. C | 11. E | 19. C |
| 4. C | 12. B | 20. E |
| 5. D | 13. D | 21. D |
| 6. B | 14. A | 22. E |
| 7. E | 15. A | |
| 8. A | 16. E | |
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Section 1 – Multiple-choice solutions

Question 1

For $f(x) = \frac{2}{\sqrt{4x-3}}$, the maximal domain is given by

$$4x - 3 > 0$$

$$4x > 3$$

$$x > \frac{3}{4}$$

The answer is D.

Question 2

$$\begin{aligned} f(x) &= 3 \cos\left(2x + \frac{\pi}{2}\right) + 1 \\ &= 3 \cos\left(2\left(x + \frac{\pi}{4}\right)\right) + 1 \end{aligned}$$

The period is $\frac{2\pi}{2} = \pi$ and the amplitude is 3.

The answer is B.

Question 3

$$4 \sin(2x) - 2\sqrt{3} = 0 \quad 0 \leq x \leq 2\pi$$

$$4 \sin(2x) = 2\sqrt{3} \quad 0 \leq 2x \leq 4\pi$$

$$\sin(2x) = \frac{\sqrt{3}}{2}$$

$$2x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{7\pi}{3}, \frac{8\pi}{3}$$

$$x = \frac{\pi}{6}, \frac{2\pi}{6}, \frac{7\pi}{6}, \frac{8\pi}{6}$$

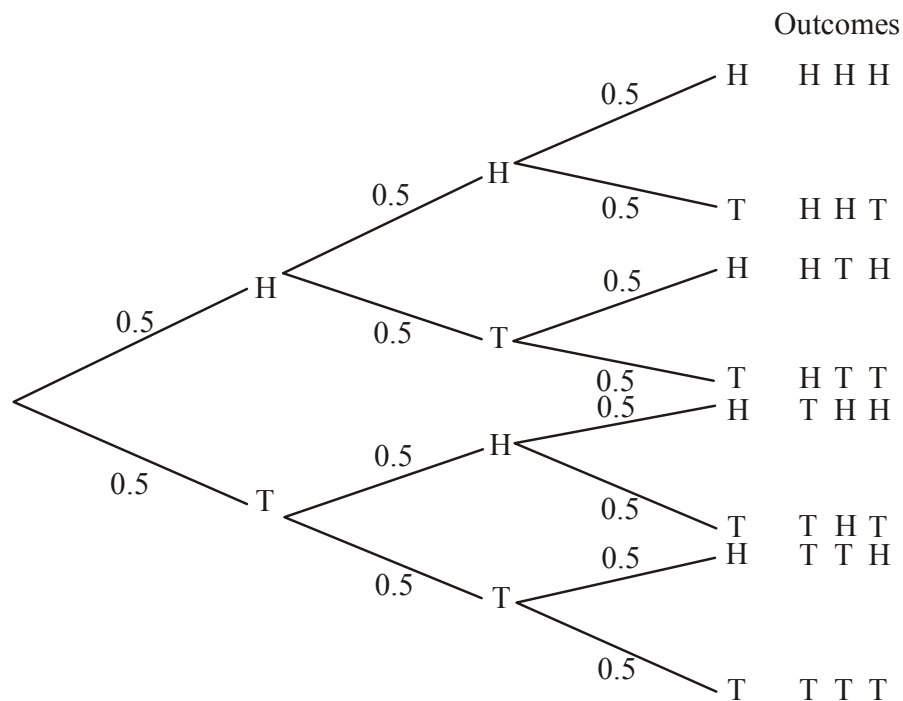
S	A
T	C

The sum is $\frac{18\pi}{6}$ or 3π .

The answer is C.

Question 4

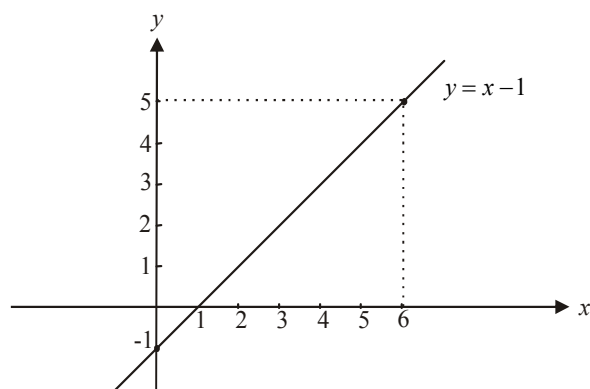
Draw a quick tree diagram.



There are 8 possible outcomes,
 1 of those is to get 3 heads,
 3 of those are to get 2 heads,
 3 of those are to get 1 head,
 and 1 of those is to get 0 heads.
 The answer is C.

Question 5

Sketch the graph of $y = x - 1$.



For the function f , $r_f = [-1, 5)$.

Looking at the graph when $y = -1$, $x = 0$. (Check $y = x - 1$, $-1 = 0 - 1$).

Again looking at the graph when $y = 5$, $x = 6$. (Check $y = x - 1$, $5 = 6 - 1$).

So the domain of $f = [0, 6)$

The answer is D.

Question 6

A translation of 2 units to the left changes the rule $y = 1 - e^x$ to become $y = 1 - e^{(x+2)}$.

A reflection in the y -axis changes the rule to $y = 1 - e^{(-x+2)}$

The answer is B.

Question 7

$g(f(x))$ exists if $r_f \subseteq d_g$.

Now $d_g = [0, \infty)$.

So we require that $r_f \subseteq [0, \infty)$.

For option **A**. $r_f = \mathbb{R}$.

For option **B**. $r_f = \mathbb{R}$.

For option **C**. $r_f = \mathbb{R}$.

For option **D**. $r_f = [-1, \infty)$.

For option **E**. $r_f = [0, \infty)$.

The answer is E.

Question 8

Let $y = 2e^{-x} + 1$

Swap x and y

$$x = 2e^{-y} + 1$$

$$x - 1 = 2e^{-y}$$

$$\frac{x-1}{2} = e^{-y}$$

$$\log_e\left(\frac{x-1}{2}\right) = -y$$

$$y = -\log_e \frac{(x-1)}{2}$$

$$\text{So } f^{-1}(x) = -\log_e \frac{(x-1)}{2}$$

The answer is A.

Question 9Method 1

$$|3p - 2| = p + 1$$

$$3p - 2 = p + 1$$

$$2p = 3$$

$$p = \frac{3}{2}$$

$$\text{if } 3p - 2 > 0$$

$$3p > 2$$

$$p > \frac{2}{3}$$

$$-(3p - 2) = p + 1$$

$$-3p + 2 = p + 1$$

$$-4p = -1$$

$$p = \frac{1}{4}$$

$$\text{if } 3p - 2 < 0$$

$$3p < 2$$

$$p < \frac{2}{3}$$

The answer is D.

Method 2

Sketch $y_1 = \text{abs}(3x - 2)$ and $y_2 = x + 1$.

Points of intersection occur at $x = \frac{1}{4}$ and at $x = \frac{3}{2}$, so, $p = \frac{1}{4}$ or $p = \frac{3}{2}$.

The answer is D.

Question 10

$$y = e^{-x} \sin(3x)$$

$$= -e^{-x} \sin(3x) + e^{-x} \times 3 \cos(3x)$$

$$= e^{-x} (3 \cos(3x) - \sin(3x))$$

The answer is C.

Question 11Method 1

$$\begin{aligned}
 y &= \sqrt{x + 2x^2} \\
 &= (x + 2x^2)^{\frac{1}{2}} \\
 &= u^{\frac{1}{2}} \quad \text{where } u = x + 2x^2 \\
 \frac{dy}{du} &= \frac{1}{2} u^{-\frac{1}{2}} \quad \frac{du}{dx} = 1 + 4x \\
 \frac{dy}{dx} &= \frac{dy}{du} \cdot \frac{du}{dx} \quad (\text{chain rule}) \\
 &= \frac{1}{2} u^{-\frac{1}{2}} (1 + 4x) \\
 &= \frac{1}{2} \cdot \frac{1}{\sqrt{u}} (1 + 4x) \\
 &= \frac{1 + 4x}{2\sqrt{x + 2x^2}}
 \end{aligned}$$

At $x = 1$

$$\frac{dy}{dx} = \frac{5}{2\sqrt{3}}$$

The answer is E.

Method 2Sketch $y = (x + 2x^2)^{\frac{1}{2}}$.Calculate $\frac{dy}{dx}$ when $x = 1$.

$$\begin{aligned}
 \frac{dy}{dx} &= 1.4433757 \\
 &= \frac{5}{2\sqrt{3}}
 \end{aligned}$$

The answer is E.

Question 12Let v equal the volume of the cylinder.

$$v = \pi r^2 h$$

since $r = 5$

$$v = 25\pi h$$

$$\frac{dv}{dh} = 25\pi$$

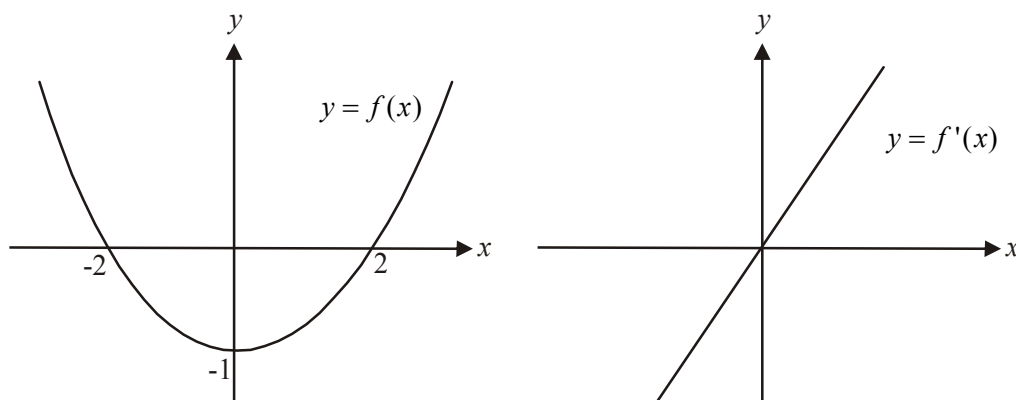
$$\text{Also } \frac{dh}{dt} = \frac{1}{t}$$

We are required to find $\frac{dv}{dt}$ when $t = 10$.

$$\begin{aligned}
 \text{So } \frac{dv}{dt} &= \frac{dv}{dh} \cdot \frac{dh}{dt} \quad (\text{Chain rule}) \\
 &= 25\pi \times \frac{1}{t}
 \end{aligned}$$

$$\text{When } t = 10, \frac{dv}{dt} = 2.5\pi$$

The answer is B.

Question 13

$f(x)$ is positive for $x < -2 \cup x > 2$.

$f'(x)$ is the gradient of $f(x)$ and is positive for $x > 0$.

So $f(x)$ and $f'(x)$ are both positive for $x > 2$.

The answer is D.

Question 14

Let the antiderivative function of f be called g .

So $g'(x) = f(x)$.

For $x \in (-a, a)$, $f(x) > 0$ so the graph of g will have a positive gradient.

Note that for there to be a stationary point (which may be a local min/max or point of inflection) $f(x) = 0$. For $x \in (-a, a)$, $f(x) \neq 0$.

The answer is A.

Question 15

$$\begin{aligned}\int (2x-1)^6 dx &= \frac{1}{2 \times 7} (2x-1)^7 + c \\ &= \frac{1}{14} (2x-1)^7 + c\end{aligned}$$

The answer is A.

Question 16

$$h'(x) = 2x - 3g'(x)$$

$$\int h'(x) dx = \int 2x dx - 3 \int g'(x) dx$$

$$h(x) = x^2 - 3g(x) + c$$

Now $h(2) = 4$ and $g(2) = 3$

So when $x = 2$,

$$4 = 4 - 9 + c$$

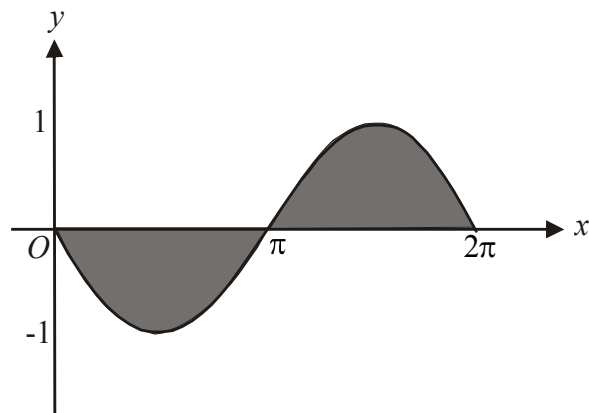
$$c = 9$$

So $h(x) = x^2 - 3g(x) + 9$

The answer is E.

Question 17

Sketch the graph.



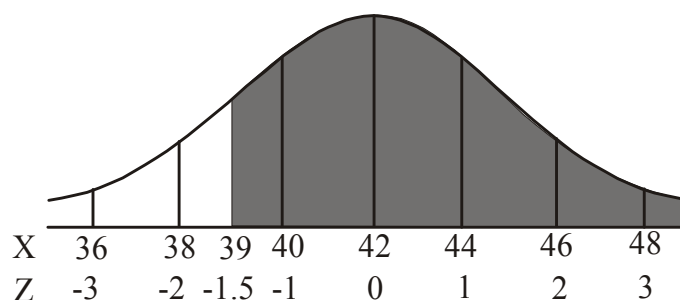
$$\begin{aligned} \text{Area} &= -\int_0^{\pi} \sin(x) dx + \int_{\pi}^{2\pi} \sin(x) dx \\ &= 2 \int_0^{\pi} \sin(x) dx \text{ because of the symmetry of the sine curve} \end{aligned}$$

The answer is B.

Question 18

$$\begin{aligned} &2 \log_2(3) - \log_2(6) \\ &= \log_2(3^2) - \log_2(6) \\ &= \log_2\left(\frac{9}{6}\right) \\ &= \log_2(1.5) \\ &= \frac{\log_{10}(1.5)}{\log_{10}(2)} \quad \text{OR} \quad \frac{\log_e(1.5)}{\log_e(2)} \quad (\text{change of base rule}) \\ &= 0.5850 \text{ (to 4 dec. places)} \quad = 0.5850 \text{ (to 4 dec. places)} \end{aligned}$$

The answer is C.

Question 19

The shaded area is given by $\text{normalcdf}(39, 100, 42, 2) = 0.93319\dots$
 $= 93\%$ to the nearest percent

(Alternatively, $\text{normalcdf}(-1.5, 100, 0, 1) = 0.93319\dots$)

The answer is C.

Question 20

This represents a binomial distribution because there is a fixed probability of an event happening i.e. 0.78 and there is a fixed number of “trials” i.e. 4 and we want a particular number of events to occur i.e. 3 or 4.

Method 1

$$\begin{aligned} & \Pr(3 \text{ goals}) + \Pr(4 \text{ goals}) \\ &= {}^4C_3 (0.78)^3 (0.22)^1 + {}^4C_4 (0.78)^4 (0.22)^0 \\ &= 0.7878 \end{aligned}$$

The answer is E.

Method 2

$$\begin{aligned} & \text{binompdf}(4, 0.78, 3) + \text{binompdf}(4, 0.78, 4) \\ &= 0.7878 \end{aligned}$$

The answer is E.

Question 21

The mode is the value of x for which the maximum value of f occurs.

Now, $y = \frac{3}{4}(1 - (x - 1)^2)$ for $0 \leq x \leq 2$, describes the graph of an inverted parabola with a

maximum turning point at $\left(1, \frac{3}{4}\right)$.

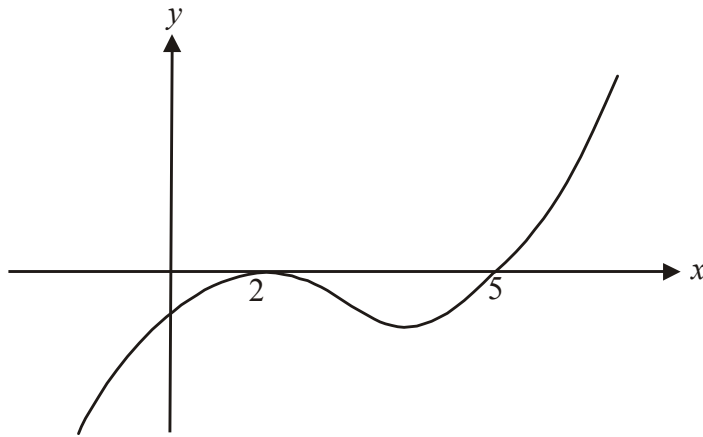
So the maximum value of f is $\frac{3}{4}$ and it occurs at $x = 1$.

The mode (that is the most popular value) is 1.

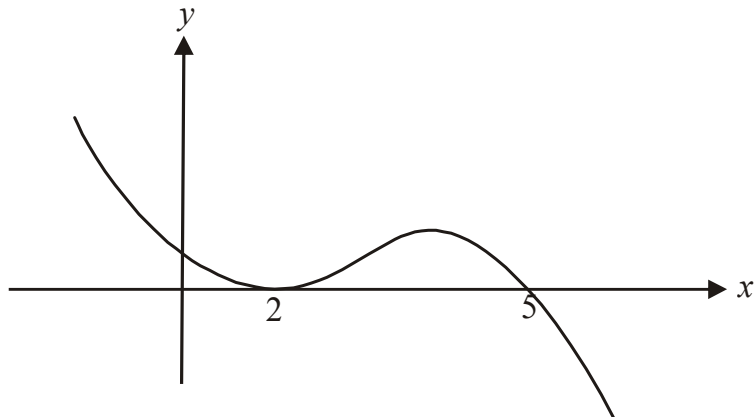
The answer is D.

Question 22

The graph of $y = f(x)$ could be



or



Both of these graphs produce the graph of $y = |f(x)|$.

The equation of the first could be $y = (x - 2)^2(x - 5)$. This is option E.

The equation of the second could be $y = -(x - 2)^2(x - 5)$. This is not offered.

The answer is E.

SECTION 2

Question 1

a. $y = 6x^2 \log_e(2x)$

x -intercept occurs when $y = 0$

$$0 = 6x^2 \log_e(2x)$$

Either $6x^2 = 0$

$x = 0$ but we reject this

since $d_f = (0, \infty)$

or $\log_e(2x) = 0$

$$e^0 = 2x$$

$$2x = 1$$

$$x = \frac{1}{2}$$

The x -intercept occurs at $\left(\frac{1}{2}, 0\right)$ as required

(1 mark)

- b. The function $f(x) = 6x^2 \log_e(2x)$ is only defined for values of x such that $2x > 0$; that is, for $x > 0$ because $\log_e(2x)$ is only defined for these values.

(1 mark)

- c. Use a graphics calculator to locate the turning point. It is the point $(0.3, -0.3)$ where each coordinate is correct to one decimal place.

(1 mark)

- d. From part c. the range is $[-0.2759..., \infty)$ or $[-0.3, \infty)$ correct to one decimal place.

(1 mark)

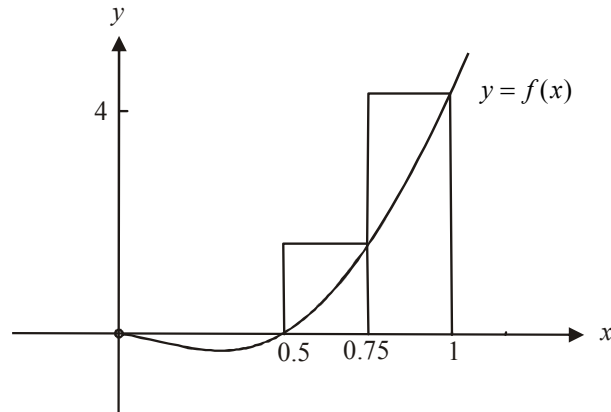
- e. i. The function f does not have an inverse function because it is not a 1:1 function.

(1 mark)

- ii. The function g must be a 1:1 function since g^{-1} exists.
The maximal domain of g is therefore $[0.3032..., \infty)$ where the value of a is the x -coordinate of the turning point of the curve. The answer is $a = 0.3$ where a is correct to one decimal place.

(1 mark)

f. i.



(1 mark)

Approximate area using right rectangles method

$$= 0.25 \times f(0.75) + 0.25 \times f(1)$$

$$= 1.3818\dots$$

$$= 1.38 \text{ square units (correct to 2 decimal places)}$$

(1 mark)

- ii. Since the right rectangles extend above the function f the approximation will be greater than the exact area.

(1 mark)

g. i.

$$y = x^3 \log_e(2x)$$

$$\frac{dy}{dx} = 3x^2 \log_e(2x) + x^3 \times \frac{2}{2x} \quad (\text{product rule})$$

$$= 3x^2 \log_e(2x) + x^2$$

$$\text{So } a = 3 \text{ and } b = 1$$

(1 mark)

(1 mark)

- ii. Hence means use what you have already found.

$$\text{From part i., we found that } \frac{d}{dx}(x^3 \log_e(2x)) = 3x^2 \log_e(2x) + x^2$$

If we antidifferentiate each term on both sides, we obtain

$$x^3 \log_e(2x) + c_1 = \int 3x^2 \log_e(2x) dx + \int x^2 dx \quad \text{where } c_1 \text{ is a constant.}$$

Rearranging, we obtain

$$\int 3x^2 \log_e(2x) dx = x^3 \log_e(2x) + c_1 - \int x^2 dx$$

$$= x^3 \log_e(2x) - \frac{x^3}{3} + c$$

(1 mark)

(Note that c = the sum of c_1 and the constant that arose from $\int x^2 dx$.)

$$\text{So } \int 6x^2 \log_e(2x) dx = 2 \left(x^3 \log_e(2x) - \frac{x^3}{3} + c \right)$$

$$= 2x^3 \log_e(2x) - \frac{2x^3}{3} + 2c$$

Now the exact area

$$= \int_{0.5}^1 6x^2 \log_e(2x) dx \quad (1 \text{ mark})$$

$$= \left[2x^3 \log_e(2x) - \frac{2x^3}{3} \right]_{0.5}^1 \quad (1 \text{ mark})$$

$$= \left\{ \left(2 \log_e(2) - \frac{2}{3} \right) - \left(\frac{1}{4} \log_e(1) - \frac{1}{12} \right) \right\}$$

$$= 2 \log_e(2) - \frac{2}{3} + \frac{1}{12}$$

$$= 2 \log_e(2) - \frac{7}{12} \text{ square units}$$

(1 mark)

(Note – if you have time, check your answer using a calculator:

$$\int_{0.5}^1 6x^2 \log_e(2x) dx = 0.80296...$$

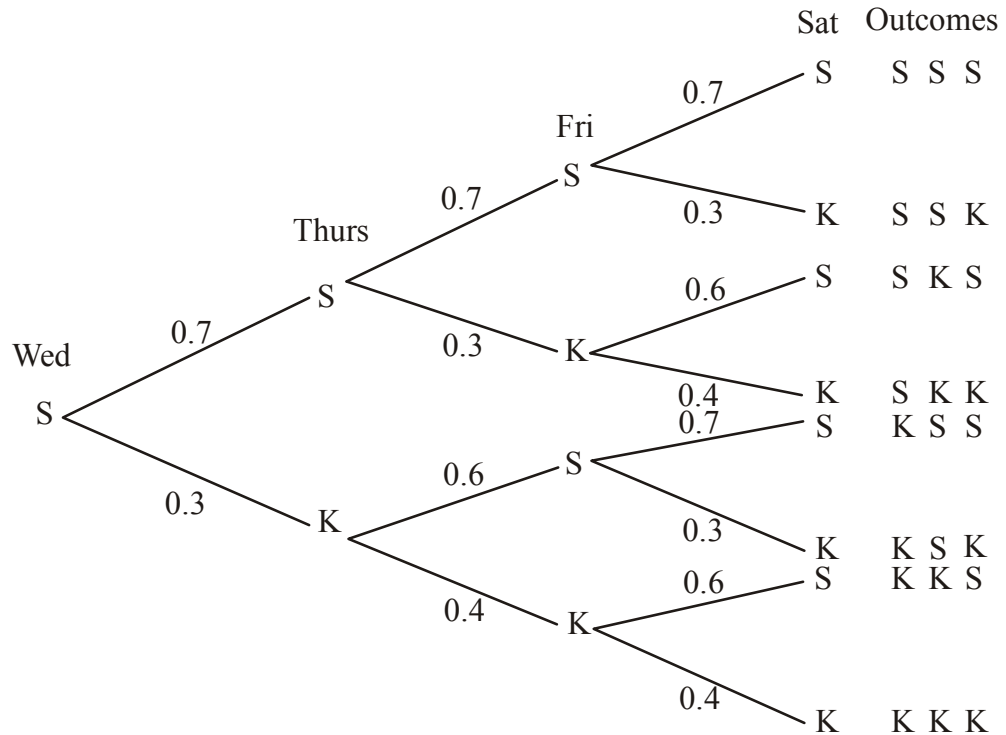
$$\text{and } 2 \log_e(2) - \frac{7}{12} = 0.80296...$$

Total 15 marks

Question 2

- a. i. The probability that Sealord's provides the seafood on Thursday, Friday and Saturday given that it supplied it on Wednesday is $0.7 \times 0.7 \times 0.7 = 0.343$.
(1 mark)

- ii. Draw a tree diagram to see all the possible outcomes.

**(1 mark)**

Looking at the possible outcomes the probability that Sealord's supplies on at least two of the next three nights

$$\begin{aligned}
 &= \Pr(SSS) + \Pr(SSK) + \Pr(SKS) + \Pr(KSS) \\
 &= (0.7 \times 0.7 \times 0.7) + (0.7 \times 0.7 \times 0.3) + (0.7 \times 0.3 \times 0.6) + (0.3 \times 0.6 \times 0.7) \\
 &= 0.343 + 0.147 + 0.126 + 0.126 \\
 &= 0.742
 \end{aligned}$$

(1 mark)

- iii. From the tree diagram, the probability that the first time Kingfisher supplies the restaurant is on Saturday or on Sunday.
 $= \Pr(SSK) + \Pr(SSSK)$ (extend the tree by 1 more branch)
 $= (0.7 \times 0.7 \times 0.3) + (0.7 \times 0.7 \times 0.7 \times 0.3)$
 $= 0.147 + 0.1029$
 $= 0.2499$

(1 mark)

- b. $\Pr(\text{Marco spends more than 5 hours in the kitchen on a particular night})$

$$\begin{aligned}
 &= -\frac{6}{125} \int_5^6 (t-1)(t-6) dt \\
 &= 0.104
 \end{aligned}$$

(1 mark)

- c. This is a binomial distribution with $p = 0.104$ (from part b.), $n = 4$ and $x = 3$.

Method 1

Required probability $= {}^nC_x p^x q^{n-x}$

$$= {}^4C_3 (0.104)^3 (0.896)^1$$

$$= 0.00403\dots$$

$$= 0.004 \text{ (correct to 3 decimal places)}$$

Method 2

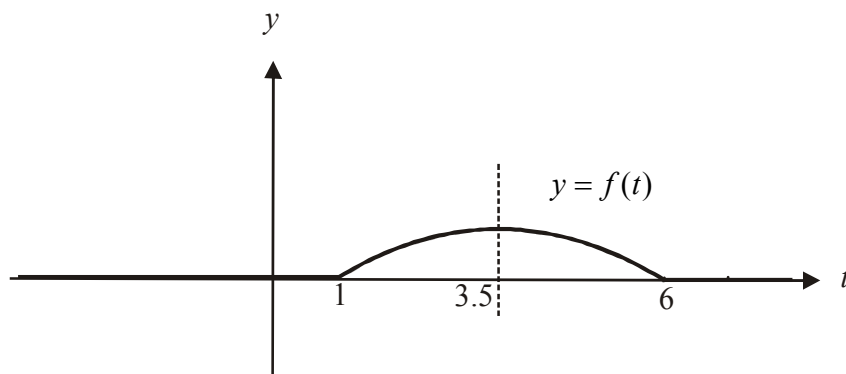
Using a calculator, $\text{binompdf}(4, 0.104, 3) = 0.004$ (correct to 3 decimal places)

(1 mark) for correct answer

(1 mark) for recognition that it was a binomial distribution

- d. Method 1

Sketching the graph.



(1 mark)

We have an inverted parabola with its axis of symmetry located at $t = 3.5$.

So the median time spent in the kitchen on a night by Marco is 3.5 hours since the area under the graph for $t < 3.5$ is equal to the area under the graph for $t > 3.5$.

(1 mark)

Method 2

Let m be the median.

We require that

$$\int_1^m f(t) dt = 0.5$$

$$- \frac{6}{125} \int_1^m (t-1)(t-6) dt = 0.5$$

(1 mark)

Use your calculator and a trial and error method to find the upper limit m .

It is 3.5.

(1 mark)

Method 3

Let m be the median.

We require that

$$\int_1^m f(t) dt = 0.5$$

$$-\frac{6}{125} \int_1^m ((t-1)(t-6)) dt = 0.5 \quad (1 \text{ mark})$$

$$-\frac{6}{125} \int_1^m (t^2 - 7t + 6) dt = 0.5$$

$$-\frac{6}{125} \left[\frac{t^3}{3} - \frac{7t^2}{2} + 6t \right]_1^m = 0.5$$

$$-\frac{6}{125} \left\{ \left(\frac{m^3}{3} - \frac{7m^2}{2} + 6m \right) - \left(\frac{1}{3} - \frac{7}{2} + 6 \right) \right\} = 0.5$$

$$\frac{m^3}{3} - \frac{7m^2}{2} + 6m - 2.8\dot{3} = -10.41\dot{6}$$

$$\frac{m^3}{3} - \frac{7m^2}{2} + 6m + 7.58\dot{3} = 0$$

Solve this by graphing $y = \frac{x^3}{3} - \frac{7x^2}{2} + 6x + 7.58\dot{3}$ and see where it intersected with

$y = 0$, that is, the x - axis, between $x = 1$ and $x = 6$ using a graphics calculator.

It intersects at $x = 3.5$

So $m = 3.5$

(1 mark)

Total 12 marks

Question 3

- a. i. At point A, $x = 0$.

$$f(0) = \frac{1}{56} \{(0-2)^3(0+5) - 72\}$$

$$= -2$$

A is the point $(0, -2)$.

(1 mark)

- ii. At point C, $y = 0$.

Use a graphics calculator to solve $0 = \frac{1}{56} \{(x-2)^3(x+5) - 72\}$.

The answers are $x = -5.1934\dots$ and $x = 4$.

C is the point $(4, 0)$.

(1 mark)

- b. $|f(0)| = |-2|$ from part a.i.
 $= 2$

(1 mark)

- c. Method 1 – expanding then differentiating

$$f(x) = \frac{1}{56} \{(x-2)^3(x+5) - 72\}$$

$$= \frac{1}{56} \{(x^2 - 4x + 4)(x-2)(x+5) - 72\}$$

$$= \frac{1}{56} \{(x^2 - 4x + 4)(x^2 + 3x - 10) - 72\}$$

$$= \frac{1}{56} (x^4 + 3x^3 - 10x^2 - 4x^3 - 12x^2 + 40x + 4x^2 + 12x - 40 - 72)$$

$$= \frac{1}{56} (x^4 - x^3 - 18x^2 + 52x - 112)$$

(1 mark)

$$f'(x) = \frac{1}{56} (4x^3 - 3x^2 - 36x + 52)$$

(1 mark)

Method 2 – using the product rule

$$f(x) = \frac{1}{56} \{(x-2)^3(x+5) - 72\}$$

$$f'(x) = \frac{1}{56} \{(x-2)^3 \times 1 + 3(x-2)^2(x+5)\}$$

$$= \frac{1}{56} \{(x-2)^2((x-2) + 3(x+5))\}$$

$$= \frac{1}{56} \{(x-2)^2(4x+13)\}$$

$$= \frac{1}{56} \{(x^2 - 4x + 4)(4x+13)\}$$

$$= \frac{1}{56} (4x^3 - 3x^2 - 36x + 52)$$

(1 mark)**(1 mark)**

- d. i. At point B , $f'(x) = 0$.

Method 1 – following on from Method 1 in part c.

$$f'(x) = \frac{1}{56}(4x^3 - 3x^2 - 36x + 52) = 0$$

Use a graphics calculator to solve this. $x = 2$ is a solution that corresponds to B 's position.

Method 2 – following on from Method 2 in part c.

$$\frac{1}{56}\{(x-2)^2(4x+13)\} = 0$$

$$x = 2 \text{ or } x = -\frac{13}{4}, \text{ but } x > 0 \text{ so reject this.}$$

Now,

$$\begin{aligned} f(2) &= \frac{1}{56}(0 - 72) \\ &= -\frac{9}{7} \end{aligned}$$

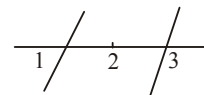
$$B \text{ is the point } \left(2, -\frac{9}{7}\right)$$

(1 mark)

- ii. B is a stationary point because at B , $f'(x) = 0$

$$\begin{aligned} \text{At } x = 1, f'(x) &= \frac{1}{56}(4 - 3 - 36 + 52) \\ &= \frac{17}{56} \\ &> 0 \end{aligned}$$

$$\begin{aligned} \text{At } x = 3, f'(x) &= \frac{1}{56}(108 - 27 - 108 + 52) \\ &= \frac{25}{56} \\ &> 0 \end{aligned}$$



Since the gradient to either side of point B is positive but at point B the gradient is zero, there is a stationary point of inflection at B .

(1 mark)

(1 mark) for first derivative test

- e. Area of garden bed

$$= -\int_0^4 f(x) dx \quad (\text{negative because the area falls below the } x\text{-axis})$$

$$= -\frac{1}{56} \int_0^4 (x^4 - x^3 - 18x^2 + 52x - 112) dx$$

(1 mark) – correct terminals

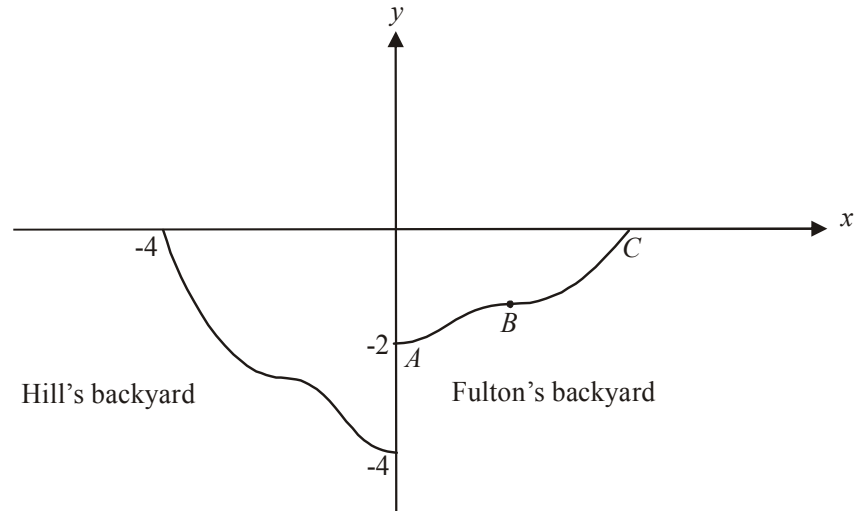
(1 mark) – correct function

- f. i. The Fulton's garden border is described by $y = \frac{1}{56} \{(x-2)^3(x+5) - 72\}$ so the George's garden border is described by $y = \frac{-1}{56} \{(x-2)^3(x+5) - 72\}$.
(1 mark)

- ii. Since C is the point $(4,0)$ (from part a. ii.) $d_g = [0, 4]$.

(1 mark)

- g. i.



(1 mark)

- ii. The rule for the garden border in Fulton's garden is given by $y = \frac{1}{56} \{(x-2)^3(x+5) - 72\}$.

When this function is reflected in the y -axis the rule becomes

$$y = \frac{1}{56} \{(-x-2)^3(5-x) - 72\}$$

When this function is then dilated by a factor of 2 from the x -axis the rule becomes

$$\frac{y}{2} = \frac{1}{56} \{(-x-2)^3(5-x) - 72\}$$

$$\text{So } h(x) = \frac{1}{28} \{(-x-2)^3(5-x) - 72\}$$

(1 mark)

- h. i. The function $y = r(x)$ is the inverse function of the function $y = f(x)$ and vice-versa.

(1 mark)

- ii. The coordinates of the point B' are $\left(-\frac{9}{7}, 2\right)$ because the coordinates of B are $\left(2, -\frac{9}{7}\right)$ from part d.i.

(1 mark)

Total 16 marks

Question 4

- a. Since $d_f = [-1, 1]$ and since the point A is at the right hand endpoint of the graph,

$$x = 1.$$

$$f(1) = \tan(1)$$

$$= 1.56 \text{ (correct to 2 decimal places)}$$

A is the point $(1, 1.56)$

(1 mark)

- b. The function f is not defined for $x = \pm \frac{\pi}{2}$, since an asymptote exists on the graph of

$$y = \tan(x) \text{ at } x = -\frac{\pi}{2} \text{ and } x = \frac{\pi}{2}.$$

Since $\frac{\pi}{2} = 1.57\dots$, this would be within 2km in the directions east and west of the intersection.

(1 mark)

- c. $y = \tan(x)$
 $\frac{dy}{dx} = \sec^2(x)$

At $x = 0$

$$\frac{dy}{dx} = \sec^2(0)$$

$$= \frac{1}{\cos^2(0)}$$

$$= \frac{1}{1^2}$$

$$= 1$$

(1 mark)

So at the point of intersection (i.e. $(0,0)$), the gradient of the function $y = \tan(x)$ is 1 and so the rail line makes an angle of 45° or $\frac{\pi}{4}$ with the road heading north (and of course with the road heading east).

(1 mark)

d. At point B ,

$$\frac{dy}{dx} = \frac{4}{3}$$

Since point B lies on the main rail line, we have

$$y = \tan(x)$$

$$\text{So } \frac{dy}{dx} = \sec^2(x)$$

$$\text{So } \frac{4}{3} = \sec^2(x) \quad (1 \text{ mark})$$

$$\frac{4}{3} = \frac{1}{\cos^2(x)}$$

$$\cos^2(x) = \frac{3}{4}$$

$$\cos(x) = \pm \frac{\sqrt{3}}{2}$$

$$x = \frac{\pi}{6}, \frac{5\pi}{6}, \dots \quad (1 \text{ mark})$$

Since $d_f = [-1, 1]$ and $\frac{5\pi}{6} = 2.6179\dots$ then $x = \frac{5\pi}{6}$ is outside the domain.

The value of x is $\frac{\pi}{6}$. (1 mark)

Since $f(x) = \tan(x)$

$$\begin{aligned} f\left(\frac{\pi}{6}\right) &= \tan\left(\frac{\pi}{6}\right) \\ &= \frac{1}{\sqrt{3}} \end{aligned}$$

So B is the point $\left(\frac{\pi}{6}, \frac{1}{\sqrt{3}}\right)$. (1 mark)

e. The gradient of the shunting line is $\frac{4}{3}$ and it passes through $\left(\frac{\pi}{6}, \frac{1}{\sqrt{3}}\right)$.

The equation of the shunting line is $y - \frac{1}{\sqrt{3}} = \frac{4}{3}\left(x - \frac{\pi}{6}\right)$. (1 mark)

The road running in the east-west direction is represented by the x -axis along which $y = 0$.

$$\text{So } 0 - \frac{1}{\sqrt{3}} = \frac{4}{3}\left(x - \frac{\pi}{6}\right)$$

$$\frac{-3}{\sqrt{3}} = 4x - \frac{2\pi}{3}$$

$$4x = \frac{2\pi}{3} - \sqrt{3}$$

$$\begin{aligned} x &= \frac{\pi}{6} - \frac{\sqrt{3}}{4} \\ &= 0.0905\dots \end{aligned}$$

(1 mark)

Since $x > 0$, the point of intersection occurs on the east side of the dangerous intersection.

(1 mark)

- f. i. The curves with equations $y = \tan(x)$ and $y = a - b\sqrt{2} \cos(x)$ meet smoothly at point C where $x = \frac{\pi}{4}$.

This means firstly that at $x = \frac{\pi}{4}$

$$\tan(x) = a - b\sqrt{2} \cos(x)$$

$$\text{that is, } \tan\left(\frac{\pi}{4}\right) = a - b\sqrt{2} \cos\left(\frac{\pi}{4}\right)$$

$$1 = a - b\sqrt{2} \times \frac{1}{\sqrt{2}}$$

$$1 = a - b \quad - (1)$$

(1 mark)

Secondly, this means that if

$$y = \tan(x)$$

$$\frac{dy}{dx} = \sec^2(x)$$

and if $y = a - b\sqrt{2} \cos(x)$

$$\frac{dy}{dx} = b\sqrt{2} \sin(x)$$

At $x = \frac{\pi}{4}$, the gradients are equal

(1 mark)

$$\text{so } \sec^2(x) = b\sqrt{2} \sin(x)$$

$$\text{that is, } \sec^2\left(\frac{\pi}{4}\right) = b\sqrt{2} \sin\left(\frac{\pi}{4}\right)$$

$$\frac{1}{\cos^2\left(\frac{\pi}{4}\right)} = b\sqrt{2} \times \frac{1}{\sqrt{2}}$$

$$\frac{1}{\left(\frac{1}{\sqrt{2}}\right)^2} = b$$

$$b = 1 \div \frac{1}{2}$$

$$= 2$$

$$\text{From (1) } 1 = a - b$$

$$\text{so } a = 3$$

Have shown.

(1 mark)

- ii. We are looking for the y-intercept of the graph of $y = 3 - 2\sqrt{2} \cos(x)$.

When $x = 0$,

$$y = 3 - 2\sqrt{2} \cos(0)$$

$$= 3 - 2\sqrt{2}$$

$$= 0.17157... \text{ km}$$

$$= 172 \text{ metres (to the nearest metre)}$$

(1 mark)

Total 15 marks