

Mathematical Methods

Written examination 2



2006 Trial Examination

SOLUTIONS

SECTION 1: Multiple-choice questions (1 mark each)

1. D	12. B
2. B	13. C
3. E	14. C
4. E	15. E
5. C	16. D
6. A	17. D
7. A	18. B
8. E	19. A
9. E	20. A
10. D	21. E
11. D	22. B

SECTION 2

Question 1

a.

$$\begin{aligned}f'(x) &= 2(1+x)(3-x) - 1(1+x)^2 \\&= 2(1+x)(3-x) - (1+x)^2 \\&= (1+x)[2(3-x) - (1+x)] \\&= (1+x)(6-2x-1-x) \\&= (1+x)(-3x+5) \\f'(x) &= -(1+x)(3x-5) \\a &= 3, \quad b = -5\end{aligned}$$

3 marks

b.

$$\begin{aligned}f'(x) &= -(1+x)(3x-5) \\x &= -1, \quad x = \frac{5}{3} \\x &= -1; \quad f(x) = (1-1)^2(4) - 4 = -4 \\x &= \frac{5}{3}; \quad f\left(\frac{5}{3}\right) = \left(1 + \frac{5}{3}\right)^2 \left(3 - \frac{5}{3}\right) - 4 \\&= \frac{64}{9} \left(\frac{4}{3}\right) - 4 \\&= \frac{148}{27}\end{aligned}$$

$$\text{Min}(-1, -4), \text{Max}\left(\frac{5}{3}, \frac{148}{27}\right)$$

2 marks

c.

i.

$$f'(-2) = -(1-2)(-6-5) = -11, f(-2) = 1$$

$$g(x) = -x^2 + bx + c$$

$$g'(x) = -2x + b$$

Let ,

$$g'(-2) = -11$$

$$-11 = -2 \times -2 + b$$

$$-15 = b$$

$$g(-2) = 1$$

$$1 = -4 - 2b + c$$

$$5 = 2 \times 15 + c$$

$$c = -25$$

$$\therefore g(x) = -x^2 - 15x - 25$$

4 marks

ii. $g'(x) = -2x - 15 = 0$

$$2x = -15$$

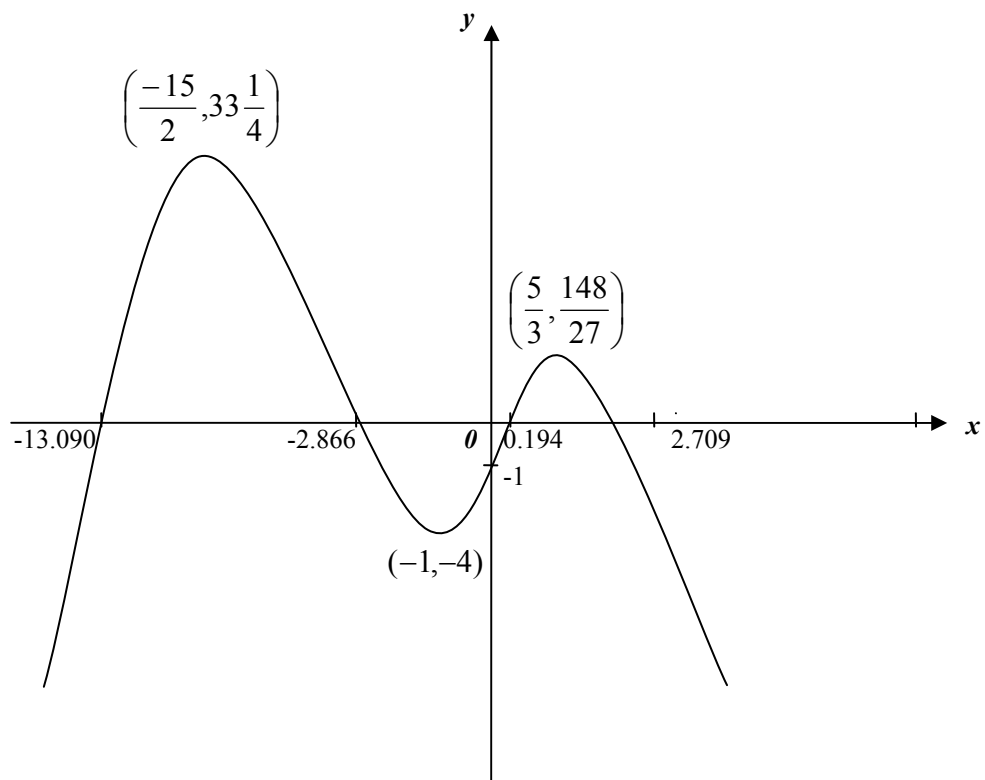
$$x = \frac{-15}{2}$$

$$\begin{aligned} g\left(\frac{-15}{2}\right) &= -\left(\frac{-15}{2}\right)^2 - 15\left(\frac{-15}{2}\right) - 25 \\ &= 31\frac{1}{4} \end{aligned}$$

$$\left(\frac{-15}{2}, 31\frac{1}{4}\right)$$

2 marks

d.



3 marks
Total 14 marks

Question 2

a. $(4,3); 3 = e^{4b}(4^2 - 12)$

$$3 = e^{4b}(4)$$

$$3 = 4e^{4b}$$

$$\frac{3}{4} = e^{4b}$$

$$\log_e \left(\frac{3}{4} \right)^{\frac{1}{4}} = b$$

$$b = \frac{1}{4} \log_e \left(\frac{3}{4} \right)$$

3 marks

b. $y = 0$

$$0 = e^x(x^2 - 3x)$$

$$e^x = 0, x^2 - 3x = 0$$

$$x(x - 3) = 0$$

$$x = 0, x = 3$$

2 marks

c.

i.

$$\frac{dy}{dx} = [e^x(x^2 - 3x) + (2x - 3)e^x]$$

$$= e^x(x^2 - x - 3)$$

4 marks

ii.

Using graphics calculator,

B is at $(-1.30, 1.52)$ and C is at $(2.30, -16.06)$

2 marks

d.

$$\left[e^x(x^2 + px + q) + e^x(2x + p) \right] = e^x(x^2 - 3x)$$

$$e^x[x^2 + px + q + 2x + p] = e^x(x^2 - 3x)$$

$$x^2 + px + q + 2x + p = x^2 - 3x$$

$$x(p + 2) + q + p = -3x$$

$$p + 2 = -3, \quad q + p = 0$$

$$p = -5, \quad q = 5$$

$$\therefore \text{Area} = -\int_0^3 e^x(x^2 - 3x)dx = -\left[e^x(x^2 - 5x + 5) \right]_0^3$$

$$= -e^3(9 - 15 + 5) - -[e^0(5)]$$

$$= e^3 + 5 \text{ units}^2$$

7 marks

Total 18 marks

Question 3

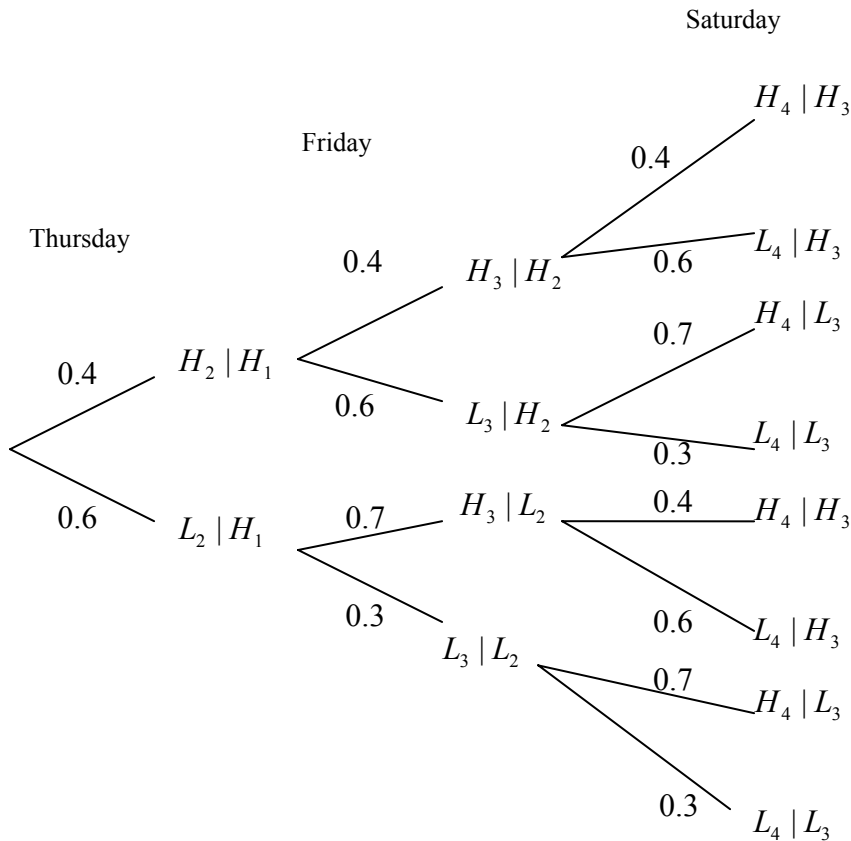
a. Let H_i be that John studies at home one night. Let L_i be that John studies at home one night.

i. $\Pr(H_{i+1} | H_i) = 0.4$

$\Pr(L_{i+1} | L_i) = 0.3$

$\Pr(H_{i+1} | L_i) = 0.7$

$\Pr(L_{i+1} | H_i) = 0.6$



Therefore

$$\Pr(\text{Home on two of the next three nights}) = 0.4 \times 0.4 \times 0.6 + 0.4 \times 0.6 \times 0.7 + 0.6 \times 0.7 \times 0.4 = 0.432$$

3 marks

ii. $\Pr(\text{Home on Friday}) = 0.4^2 + (0.6 \times 0.7) = 0.58$

1 mark

b.

$$\begin{aligned}\int_3^5 \frac{6}{125} t(5-t) dt &= \frac{6}{125} \left[\frac{5}{2} t^2 - \frac{t^3}{3} \right]_3^5 \\ &= \frac{6}{125} \left(\frac{125}{2} - \frac{125}{3} \right) - \frac{6}{125} \left(\frac{45}{2} - 9 \right) = \frac{44}{125}\end{aligned}$$

2 marks

c. $\Pr(X \geq 2) = {}^3C_2 \left(\frac{44}{125} \right)^2 \left(\frac{81}{125} \right)^1 + {}^3C_3 \left(\frac{44}{125} \right)^3 = 0.284$
or use $1 - \text{bincdf}\left(3, \frac{44}{125}, 1\right) = 0.284$

2 marks

d. $0.30 = \frac{6}{125} \int_0^n t(5-t) dt = \frac{6}{125} \left[\frac{5}{2} t^2 - \frac{t^3}{3} \right]_0^n$
 $125 \times 0.3 = 15n^2 - 2n^3 .$

Solve this cubic using CALC INTERSECT on the domain $0 < n < 5$ to get $n = 1.816$ (hours). The required time is 109 minutes.

4 marks
Total 12 marks

Question 4

a. 21m

1 mark

b. 9m

1 mark

c. Solve $17 = 6 \sin \frac{1}{2} \pi (x - 2) + 15$ to obtain $x = 6.216$ using
CALC INTERSECT

2 marks

d.

i. $f'(x) = 3ax^2 + 2bx + c$

$$f'(6) = 108a + 12b + c$$

2 marks

ii.

$$h(x) = 6 \cos \frac{\pi}{2} (x - 2) + 15$$

$$h'(x) = \frac{6}{2} \pi \cos \frac{\pi}{2} (x - 2)$$

$$h'(6) = 3\pi \cos \frac{\pi}{2} (4)$$

$$= 3\pi \cos 2\pi = \frac{3}{2} \pi$$

$$f'(6) = h'(6)$$

$$108a + 12b + c = 3\pi$$

2 marks

iii.

$$f'(x) = 3ax^2 + 2bx + c$$

$$\text{But } f'(0) = 0, \text{ so } c = 0 \quad (1)$$

$$\text{Also } f(0) = 0, \text{ so } d = 0 \quad (2)$$

$$h(6) = 3 \sin \frac{\pi}{2}(4) + 15 = 15$$

$$\therefore f(6) = 15$$

$$15 = 216a + 36b + 6c + d \quad 15 = 216a + 36b \quad (3)$$

$$c = d = 0 \quad 3\pi = 108a + 12b \quad (4)$$

$$(3) \div 3 \quad 5 = 72a + 12b \quad (3a)$$

$$(4) - (3a) \text{ gives } 3\pi - 5 = 36a$$

$$a = \frac{3\pi - 5}{36}$$

Substitute for 72a in (3a)

$$5 = 2(3\pi - 10) + 12b$$

$$\text{so } b = \frac{25 - 6\pi}{12}$$

$$f(x) = \left(\frac{3\pi - 5}{36} \right) x^3 + \left(\frac{25 - 6\pi}{12} \right) x^2$$

4 marks

e.

$$\text{Area} = \int_6^{16} \left(6 \sin \frac{\pi}{2}(x-2) + 15 \right) dx + \int_0^6 \left(\left(\frac{3\pi - 5}{36} \right) x^3 + \left(\frac{25 - 6\pi}{12} \right) x^2 \right) dx$$

$$= \left[-\frac{12}{\pi} \cos \left[\frac{\pi}{2}(x-2) \right] + 15x \right]_6^{16} + \left[\left(\frac{3\pi - 5}{144} \right) x^4 + \left(\frac{25 - 6\pi}{36} \right) x^3 \right]_0^6$$

$$= \left[\left(\frac{-12}{\pi} \times -1 \right) + 240 \right] - \left[\frac{-12}{\pi} + 90 \right] + [9(3\pi - 5) + 150 - 36\pi] - 0$$

$$= \frac{12}{\pi} + 150 + \frac{12}{\pi} + 27\pi - 45 + 150 + 36\pi$$

$$= \left(\frac{24}{\pi} - 9\pi + 300 \right) \text{square units}$$

$$= 279.365 \text{ square units}$$

4 marks

Total 16 marks