

Student Name: _____

Mathematical Methods

Written examination 1



2006 Trial Examination

Reading Time: 15 minutes

Writing Time: 1 Hour

QUESTION AND ANSWER BOOK

Structure of Book

<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
11	11	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers.
- Students are NOT permitted to bring into the examination room: notes of any kind, blank sheets of paper and/or white out liquid/tape, a calculator.

Materials supplied

- Question and answer book of 10 pages.
- Working space is provided throughout the book.

Instructions

- Print your name in the space provided on the top of this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other electronic communication devices into the examination room.

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Instructions

Answer **all** questions in the spaces provided.

A decimal approximation will not be accepted if an **exact** answer is required to a question.

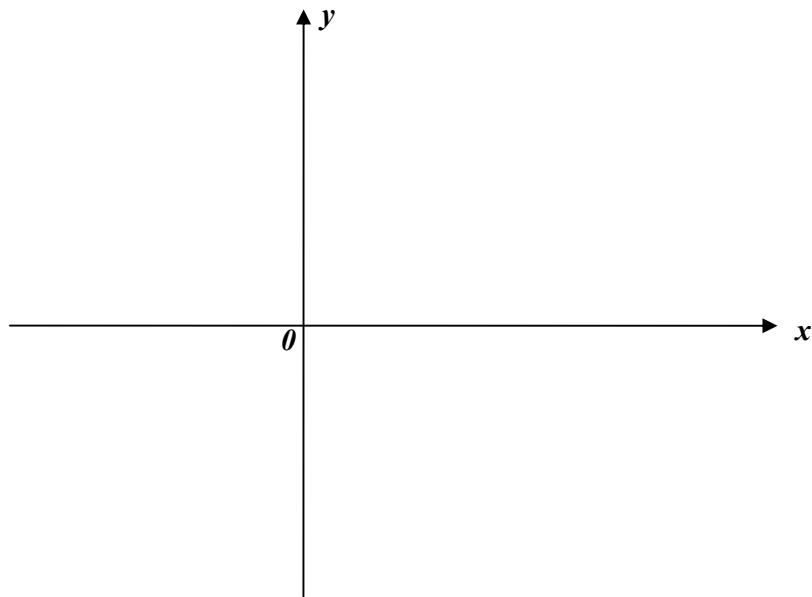
In questions where more than one mark is available, appropriate working must be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1

Sketch the graph of the function $y = 3 - 2|x - 1|$ on the axes below.

Label axes intercepts and the vertex with their coordinates.



3 marks

TURN OVER

Question 2

For the function $f : (-\infty, -2) \rightarrow \mathbb{R}, f(x) = \frac{3}{x+2} - 1$, find:

- a. the rule of the inverse function f^{-1} .

- b. the domain of the inverse function f^{-1} .

2 + 1 = 3 marks

Question 3

- a. Find $\frac{dy}{dx}$ if $y = 4e^{-2x} \cos 5x$.

- b. If $f : [3, \infty) \rightarrow \mathbb{R}$ is such that $f'(x) = \sqrt{x-3}$ and $f(4) = 1$, find the rule of f .

2 + 2 = 4 marks

Question 4

Solve the equation $2 \cos x = -\sqrt{3}$ for $x \in [-\pi, \pi]$, giving exact values in terms of π .

3 marks

Question 5

- a. Obtain (as a single fraction) the rule for the gradient function of $f(x) = \frac{\log_e 3x}{x}$.

- b. Hence determine the exact coordinates of the turning point of this graph.

2 + 2 = 4 marks

TURN OVER

Question 6

Let X be a random variable with a normal distribution with mean 200 and standard deviation 30 and let Z be a random variable with the standard normal distribution.

- a. Calculate the value of X corresponding to the Z value -0.5 .

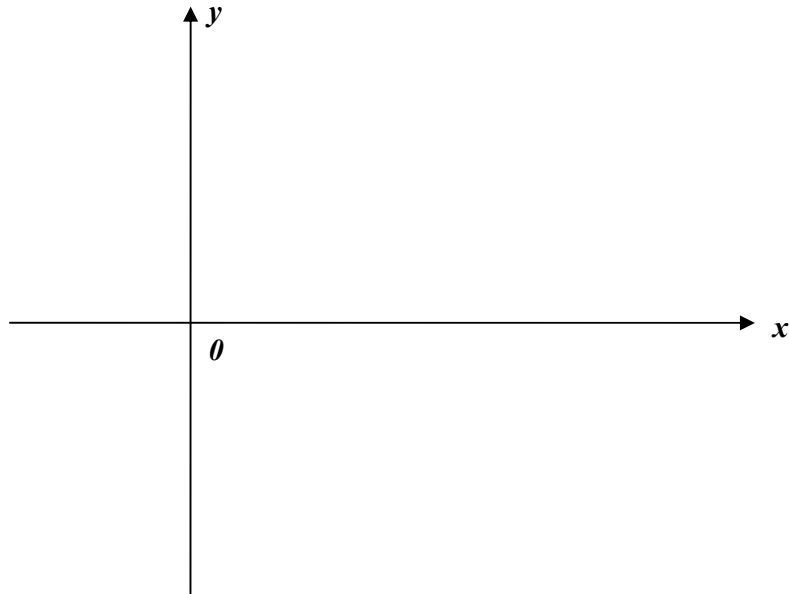
- b. Find the value of k so that $\Pr(160 < X < 240) = 1 - 2 \Pr(Z > k)$.

1 + 1 = 2 marks

Question 7

- a. State the period and amplitude of the function $f : [0, 2] \rightarrow R, f(x) = -4 \sin 3\pi x$.

- b. Sketch the graph of this function on the diagram provided, specifying scales on both axes and giving coordinates of turning points.



- c. Use calculus to determine the area of **one** of the regions lying between the graph and the x-axis.

1 + 2 + 2 = 5 marks

Question 8

A continuous random variable, X has a probability density function:

$$f(x) = \begin{cases} kx(4-x) & , \text{ if } 0 \leq x \leq 4 \\ 0 & , \text{ if } x < 0 \text{ or } x > 4 \end{cases}$$

- a.** Calculate the value of the positive constant k .

- b.** Use calculus to calculate the mean of this probability distribution.

3 + 2 = 5 marks

Question 9

In a best of three game Table Tennis match between Fred and Samantha, each player has an equal probability of winning the first game. However, if Fred wins a game, Samantha has only a 0.4 chance of winning the next game, while if Samantha wins a game, Fred has only a 0.3 chance of winning the next game.

- a. Draw up a tree diagram to illustrate the (four) possible situations: Fred wins in two games, Samantha wins in two games, Fred wins in three games and Samantha wins in three games. Label the branches of the tree with the relevant probabilities.

- b. Calculate the probability that Samantha wins the match.

- c. Given that Samantha wins the match, calculate the probability that the match took three games.

1 + 2 + 1 = 4 marks

Question 10

Let $f(x) = 5x^3$ and $g(x) = \sin x$.

- a. Write down the rule for $f(g(x))$.

- b. Find the derivative of $f(g(x))$.

1 + 1 = 2 marks

Question 11

Determine the values of c which will make the line $y = -2x + c$ tangent to the curve $y = -x^3 + 4x$.

5 marks

END OF QUESTION AND ANSWER BOOK