

Mathematical Methods

Written examination 1



2006 Trial Examination

SOLUTIONS

Question 1

Vertex $(1, 3)$ and correct shape for graph

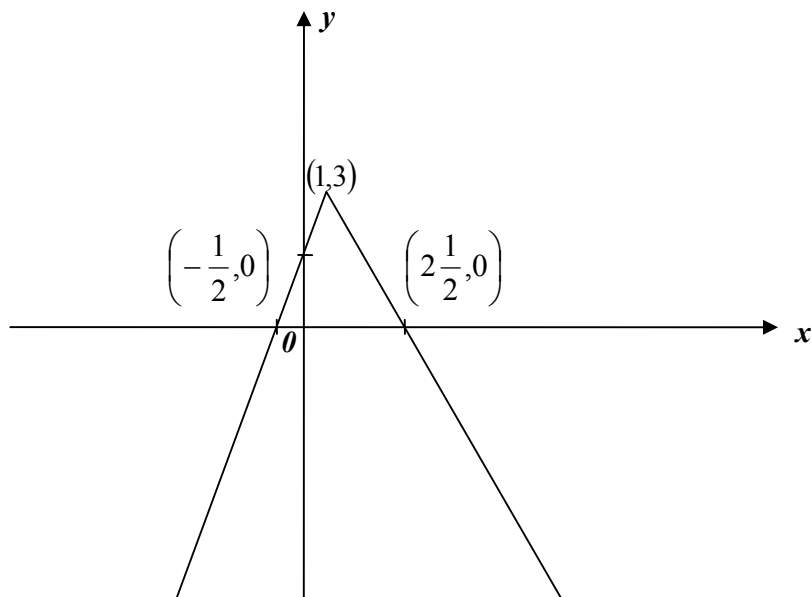
A1

$(0, 1)$ y -intercept marked in or stated

A1

x -intercepts $\left(-\frac{1}{2}, 0\right)$ and $\left(2\frac{1}{2}, 0\right)$ (both marked in or stated)

A1



Question 2

a. Inverse has equation $x + 1 = \frac{3}{y + 2}$ M1

So $y = \frac{3}{x + 1} - 2$ A1

b. Domain of this is $x < -1$. A1

Question 3

a. $f'(x) = -8e^{-2x} \cos 5x - 20e^{-2x} \sin 5x$ M1 product rule and some correct derivatives, A1

b. $f(x) = \frac{2}{3}(x - 3)^{\frac{3}{2}} + c$ A1

$f(4) = 1 \Rightarrow 1 = \frac{2}{3} + c \Rightarrow c = \frac{1}{3}$ A1

Question 4

$\cos x = -\frac{1}{2}\sqrt{3}$. Second or third quadrant, related angle $\frac{\pi}{6}$. M1

$x = \pm \frac{5\pi}{6}$. A1 + A1

Question 5

a. $f'(x) = \frac{x \times \frac{3}{3x} - \ln(3x)}{x^2} = \frac{1 - \ln(3x)}{x^2}$ M1 (quotient rule) + A1

b. Turning point at $\ln(3x) = 1$ so $3x = e, x = \frac{1}{3}e$ M1 (must involve a log equation)

$f\left(\frac{1}{3}e\right) = \frac{3}{e}$, so co ordinates are $\left(\frac{1}{3}e, \frac{3}{e}\right)$. A1

Question 6

a. $x = 200 - \frac{1}{2} \text{ of } 30 = 185$. A1

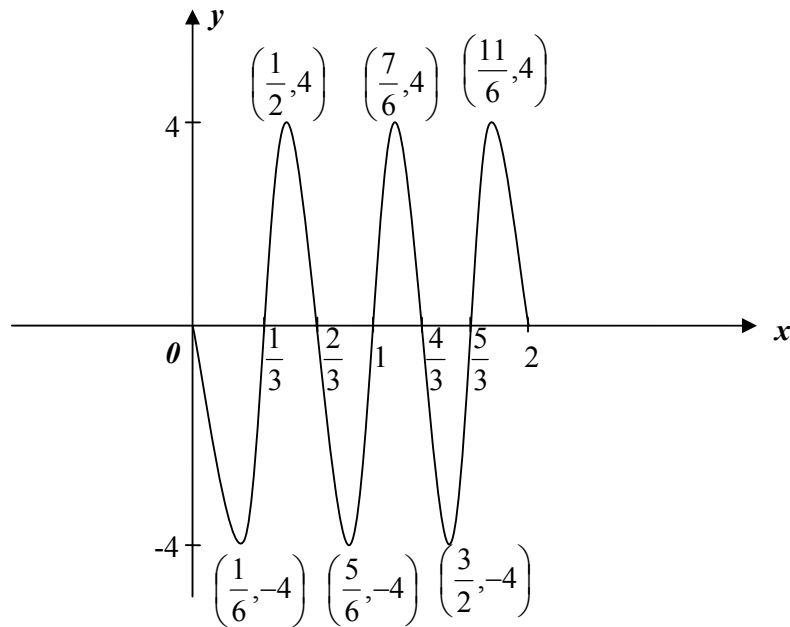
b. By symmetry, k is the Z value corresponding to the X value $240 = 200 + 40$, which is $1\frac{1}{3}$ standard deviation's above the mean, so Z is $1\frac{1}{3}$. A1

Question 7

a. Period $\frac{2}{3}$, amplitude 4. A1

b. Correct axial scales and placement of graph on their scale A1

Correct turning point coordinates and shape of graph A1



c. Area = $-\int_0^{\frac{1}{3}} 4 \sin 3\pi x \, dx$ (or equivalent) A1

$$= \frac{4}{3\pi} [\cos 3\pi x]_0^{\frac{1}{3}}$$

The indefinite integral must be shown!

$$= \frac{4}{3\pi}$$
A1

Question 8

a. $\int_{-\infty}^{\infty} f(x) \, dx = 1 = \int_0^4 k(4x - x^2) \, dx$ M1

$$= k \left[2x^2 - \frac{x^3}{3} \right]_0^4$$
H1

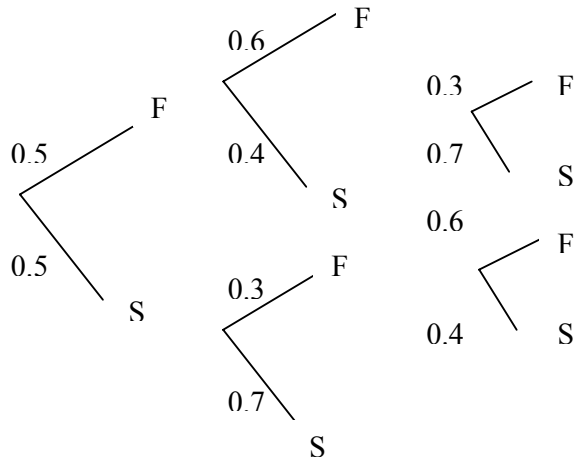
$$= \frac{32k}{3} \quad \text{thus } k = \frac{3}{32}$$
A1

b. $\mu = \int_{-\infty}^{\infty} x f(x) \, dx = \frac{3}{32} \int_0^4 (4x^2 - x^3) \, dx$ H1

$$= \frac{3}{32} \left[\frac{4x^3}{3} - \frac{x^4}{4} \right]_0^4 = 2$$
A1

Question 9

a.



M1 correct format of tree and attempt at probabilities

b. $\Pr(\text{S wins}) = 0.5 \times 0.7 + 0.5 \times 0.4 \times 0.7 + 0.5 \times 0.3 \times 0.4$ H1

$= 0.55$ A1

c. $\Pr(3 \text{ games} \mid \text{S wins}) = \frac{(0.5 \times 0.4 \times 0.7 + 0.5 \times 0.3 \times 0.4)}{0.55} = \frac{4}{11}$ A1

Question 10

a. $f(g(x)) = 5 \sin^3 x$ A1

b. $\frac{d}{dx}[f(g(x))] = 15 \sin^2 x \cos x$ A1

Question 11

Gradients of both curves must be equal. This is so if $-3x^2 + 4 = -2$ M1

Thus $x^2 = 2, x = \pm\sqrt{2}$ A1

If $x = \pm\sqrt{2}$ in the formula $y = -x^3 + 4x, y = \pm 2\sqrt{2}$ A1

Tangent $y = -2x + c$ goes through $(\sqrt{2}, 2\sqrt{2})$ if $c = 4\sqrt{2}$ M1

Other value of c (by symmetry or other) is $c = -4\sqrt{2}$ A1