



# **THE SCHOOL FOR EXCELLENCE (TSFX)**

## **UNIT 4 MATHEMATICAL METHODS 2006**

### **WRITTEN EXAMINATION 1**

Reading Time: 15 minutes

Writing time: 1 hour

### **QUESTION AND ANSWER BOOKLET**

Structure of Booklet

| Number of questions | Number of questions to be answered | Number of marks |
|---------------------|------------------------------------|-----------------|
| 11                  | 11                                 | 40              |

Students are permitted to bring into the examination rooms: pens, pencils, highlighters, erasers, sharpeners, rulers.

Students are **NOT** permitted to bring into the examination room: notes of any kind, a calculator, blank sheets of paper and/or white out liquid/tape.

Students are **NOT** permitted to bring mobile phones and/or any electronic communication devices into the examination room.

All written responses must be in English.

## **COMPLIMENTS OF THE SCHOOL FOR EXCELLENCE**

**Voted Number One For Excellence and Quality in VCE Programs and Tutorials.**

## THE SCHOOL FOR EXCELLENCE (TSFX)

The School For Excellence (TSFX) is a private educational institution that provides educational services to Year 11 and 12 students out of school hours. Our services include intense revision courses before examinations, intense weekly tuition classes, study skills lectures, as well as specialised courses that prepare students in advance of each school term.

The educational programs conducted by TSFX are widely recognised for providing the highest quality programs in Victoria today. Our programs are the result of 15 years of collaborative effort and expertise from dozens of teachers and schools across the state, ensuring the highest possible quality resources and teaching styles for VCE students.

For additional information, join our complimentary mailing list at [www.tsfx.com.au](http://www.tsfx.com.au). Alternatively, please call TSFX on (03) 9663 3311 during business hours.

### FREE VCE RESOURCES AT VCEDGE ONLINE

**VCEdge Online** is a new educational resource designed to provide students the best opportunities to optimise their Year 11 or 12 scores. **VCEdge Online** members receive over \$300 worth of resources at no charge, including:

- Subject notes and course summaries.
- Sample A+ SACS and SATS.
- Trial examinations with worked solutions.
- Weekly study tips and exam advice (in the weeks leading up to the examinations)
- Two **FREE** tickets into an intense examination strategy lecture (valued at \$200!!!).
- Cheat sheets and formula cards.
- Critical VCE updates.
- Free VCE newsletters.
- Information on upcoming educational events.
- And much, much more!!!

**JOIN FREE OF CHARGE AT [WWW.TSFX.COM.AU](http://WWW.TSFX.COM.AU)**

### Instructions

- Answer **all** questions in the spaces provided.
- A decimal approximation will not be accepted if an **exact** answer is required to a question.
- In questions where more than 1 mark is available, appropriate working must be shown.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

### QUESTION 1

Let  $p(x) = 3x^4 + 4x^3 - 14x^2 - 20x - 5$ .

- a. (i) Show that  $3x + 1$  is a linear factor of  $p(x)$ .

---

---

---

---

---

---

---

---

- (ii) **Hence** express  $p(x)$  in the form  $(3x + 1)q(x)$  where  $q(x)$  is a polynomial.

---

---

---

---

2 + 1 = 3 marks

- 
- 
- 
- 
- 
- 

- [illegible]

**Total 6 marks**

## QUESTION 2

A continuous random variable  $X$  has a probability distribution function given by

$$f(x) = \begin{cases} ax & \text{if } 0 < x < b \\ 0 & \text{otherwise} \end{cases}$$

- a. Show that  $ab^2 = 2$ .

---

---

---

---

- b. Find the mean value of  $X$  in terms of  $a$  and  $b$ .

---

---

---

---

- c. Find the values of  $a$  and  $b$  if the mean value of  $X$  is equal to 1.

---

---

---

---

---

---

---

---

1 + 1 + 2 marks

**Total 4 marks**

**QUESTION 3**

Find in exact form all real solutions to the equation  $2^{3x} + 2^{2x} - 2^{-x} = 1$ .

---

---

---

---

---

---

---

---

3 marks

**QUESTION 4**

For the function  $f : (-\infty, 0] \rightarrow \mathbb{R}$  with rule  $f(t) = \frac{3t}{t^2 + 1}$ , find the rule for the inverse function.

---

---

---

---

---

---

---

---

---

---

---

---

3 marks

a. Show that the derivative of  $\frac{\cos^2 x}{x}$  is equal to  $\frac{-2x \sin x \cos x - \cos^2 x}{x^2}$ .

This image shows a blank sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

$$a = \frac{(2n+1)\pi}{2} \text{ and } n \in J.$$

---

---

---

---

---

---

- c. Show that the curve  $y = \frac{\cos^2 x}{x}$  also has stationary points with  $x$ -coordinates given by the solutions to the equation  $2x \tan(x) = -1$ .

---

---

---

---

---

---

---

---

3 + 1 + 1 marks

**Total 5 marks**

**QUESTION 6**

The line  $y = \frac{x}{2} - 1$  is normal to the curve  $y = bx^4 + 1$ . Find the exact value of  $b$ .

---

---

---

---

---

---

---

---

---

---

---

---

---

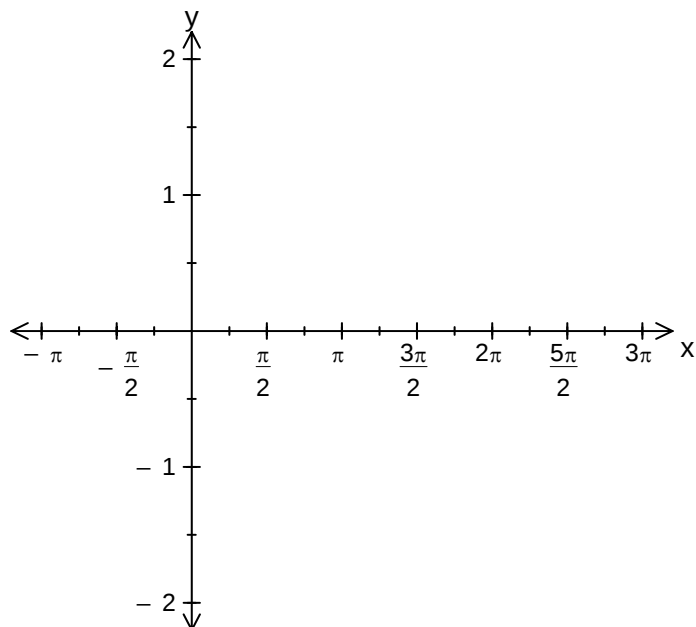
---

---

4 marks

### QUESTION 7

- a. Sketch the graph of the function  $f : [-\pi, 3\pi] \rightarrow \mathbb{R}$  with rule  $f(x) = \cos\left(\frac{x}{2}\right)$  on the set of axes below.



- b. On the same set of axes, sketch the graph of  $y = |2f(x)|$ .
- c. Hence find the exact area of the region bounded by the two curves.

---

---

---

---

---

---

---

---

1 + 1 + 2 marks

**Total 4 marks**

### QUESTION 8

Let  $f(x) = x^4 - x$  and  $g(x) = 16x^4 + 2x + 1$ .

State in correct order the transformations necessary to obtain the graph of  $g(x)$  from the graph of  $f(x)$ .

---

---

---

---

---

---

---

---

---

---

---

---

---

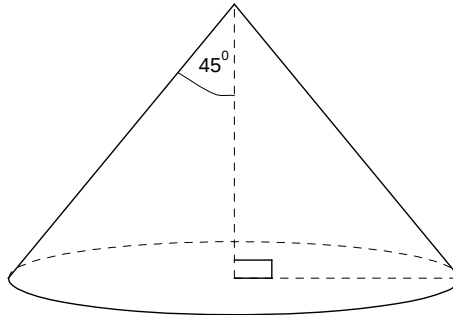
---

---

2 marks

### QUESTION 9

Grain is falling from the chute of a large storage container at a rate of  $0.3 \text{ m}^3 / \text{min}$ . The grain forms a heap on a flat horizontal floor in the shape of a right circular cone of semi-vertical angle  $45^\circ$ .



- a. Show that the exact height of the cone 30 minutes after the chute is first opened is  $\frac{3}{\pi^{1/3}}$ .

---

---

---

---

---

---

---

---

2 marks

- b. Find in the form  $\frac{1}{a\pi^b}$  m/min, where  $a$  and  $b$  are constants, the rate at which the height of the cone is increasing 30 minutes after the shute is first opened.

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

2 + 3 marks

**Total 5 marks**

### QUESTION 10

The population drift from the country to the city has been of concern in recent years. If a person is living in the country one year the probability of that person living in the city the following year is  $\frac{1}{7}$ . If a person is living in the city one year the probability of that person

living in the country the following year is  $\frac{1}{10}$ .

Find the exact probability that a person who is currently living in the country will be living in the city in two years time.

---

---

---

---

---

---

---

---

---

---

---

---

2 marks

**QUESTION 11**

The probability of obtaining a particular number of dots (the score),  $X$ , on the uppermost face of a biased die is shown in the following table.

|              |     |               |      |      |      |     |
|--------------|-----|---------------|------|------|------|-----|
| $x$          | 1   | 2             | 3    | 4    | 5    | 6   |
| $\Pr(X = x)$ | $p$ | $\frac{1}{7}$ | $3p$ | $3p$ | $4p$ | $p$ |

- a. Find the exact value of  $p$ .

---

---

---

---

---

- b. If a score greater than 2 was rolled, find the exact probability that the score was less than 5.

---

---

---

---

---

---

1 + 1 marks

**Total 2 marks**

**END OF QUESTION AND ANSWER BOOKLET**

---

## BONUS QUESTION

### QUESTION 12

- a. Find the derivative of  $xe^{-x}$ .

---

---

---

---

---

---

---

1 mark

- b. Hence find an anti-derivative of  $xe^{-x}$ .

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

2 marks

- c.** A continuous random variable  $X$  has a probability distribution function given by

$$f(x) = \begin{cases} \frac{3}{2}e^{-x} & \text{if } 0 < x < \log_e 3 \\ 0 & \text{otherwise} \end{cases}$$

- (i) Find in the form  $a - \log_e b$ , where  $a$  and  $b$  are constants, the mean value of  $X$ .

[illegible]

3 marks

- (ii) Find in the form  $\log_e c$ , where  $c$  is a constant, the median value of  $X$ .

---

---

---

---

---

---

2 marks

**Total 8 marks**