

Trial Examination 2006

VCE Mathematical Methods Units 3 & 4

Written Examination 1

Question and Answer Booklet

Reading time: 15 minutes

Writing time: 1 hour

Student's Name: _____

Teacher's Name: _____

Structure of Booklet

Number of questions	Number of questions to be answered	Number of marks
10	10	40

Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers.

Students are NOT permitted to bring into the examination room: notes of any kind, blank sheets of paper and/or white out liquid/tape, a calculator.

Materials supplied

Question and answer booklet of 9 pages, with a detachable sheet of miscellaneous formulas in the centrefold.

Working space is provided throughout the booklet.

Instructions

Write **your name** and your **teacher's name** in the space provided above on this page.

All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other electronic communication devices into the examination room.

Students are advised that this is a trial examination only and cannot in any way guarantee the content or the format of the 2006 VCE Mathematical Methods Units 3 & 4 Written Examination 1.

Instructions

Answer **all** questions in the spaces provided.

A decimal approximation will not be accepted if an **exact** answer is required to a question.

In questions where more than one mark is available, appropriate working must be shown.

Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.

Question 1

A random variable X has the following probability distribution.

x	0	1	2	3
$\Pr(X = x)$	$3a$	$4a$	$2a$	a

- a. Find the value of a .

- b. Find $\Pr(X \leq 2)$.

- c. Evaluate $E(2X + 1)$.

1 + 1 + 2 = 4 marks

Question 2

- a. If $f(x) = e^{x+1} - 2$ for $x \in R$, find the rule for the inverse function, f^{-1} .

- b. Write down the domain of f^{-1} .

2 + 1 = 3 marks

Question 3

The graph of $f(x) = x^2$ is transformed to the graph of $g(x) = 2x^2 + 4x - 7$.

- a. Write $g(x)$ in the form $2(x + a)^2 + b$, where a and b are integers.

- b. Describe the transformations which map $f(x)$ to $g(x)$.

2 + 2 = 4 marks

Question 4

Find the exact solutions of $\cos(3\pi x) = -\sin(3\pi x)$ for $0 \leq x \leq 1$.

3 marks

Question 5

A curve has equation $y = x^2 e^{-3x}$.

- a. Find $\frac{dy}{dx}$.

- b. Write down the **exact** coordinates of all stationary points.

2 + 2 = 4 marks

Question 7

The function f is defined as

$$f(x) = \begin{cases} k(x+1), & 0 \leq x \leq 2 \\ 0, & x < 0 \text{ or } x > 2 \end{cases}$$

- a.** Find k so that $f(x)$ could be used as a probability density function of a continuous random variable X .

- b.** Find the median of X , giving your answer in exact form.

2 + 3 = 5 marks

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Formula Sheet

Directions to students

Detach this formula sheet during reading time.
This formula sheet is provided for your reference.

MATHEMATICAL METHODS FORMULAS**Mensuration**

area of a trapezium: $\frac{1}{2}(a+b)h$

volume of a pyramid: $\frac{1}{3}Ah$

curved surface area of a cylinder: $2\pi rh$

volume of a sphere: $\frac{4}{3}\pi r^3$

volume of a cylinder: $\pi r^2 h$

area of a triangle: $\frac{1}{2}bc \sin(A)$

volume of a cone: $\frac{1}{3}\pi r^2 h$

Calculus

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$$\int x^n dx = \frac{1}{n+1}x^{n+1} + c, n \neq -1$$

$$\frac{d}{dx}(e^{ax}) = ae^{ax}$$

$$\int e^{ax} dx = \frac{1}{a}e^{ax} + c$$

$$\frac{d}{dx}(\log_e(x)) = \frac{1}{x}$$

$$\int \frac{1}{x} dx = \log_e|x| + c$$

$$\frac{d}{dx}(\sin(ax)) = a \cos(ax)$$

$$\int \sin(ax) dx = -\frac{1}{a} \cos(ax) + c$$

$$\frac{d}{dx}(\cos(ax)) = -a \sin(ax)$$

$$\int \cos(ax) dx = \frac{1}{a} \sin(ax) + c$$

$$\frac{d}{dx}(\tan(ax)) = \frac{a}{\cos^2(ax)} = a \sec^2(ax)$$

product rule: $\frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}$

quotient rule: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$

chain rule: $\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}$

approximation: $f(x+h) \approx f(x) + hf'(x)$

Probability

$$\Pr(A) = 1 - \Pr(A')$$

$$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$$

$$\Pr(A|B) = \frac{\Pr(A \cap B)}{\Pr(B)}$$

mean: $\mu = E(X)$

variance: $\text{Var}(X) = \sigma^2 = E((X - \mu)^2) = E(X^2) - \mu^2$

probability distribution		mean	variance
discrete	$\Pr(X=x) = p(x)$	$\mu = \sum xp(x)$	$\sigma^2 = \sum (x - \mu)^2 p(x)$
continuous	$\Pr(a < X < b) = \int_a^b f(x) dx$	$\mu = \int_{-\infty}^{\infty} xf(x) dx$	$\sigma^2 = \int_{-\infty}^{\infty} (x - \mu)^2 f(x) dx$

END OF FORMULA SHEET

Question 8

Consider the following functions:

$$f(x) = 3 \sin(2x), \quad 0 \leq x \leq \pi$$

$$g(x) = 1 - x^2, \quad x \in \mathbb{R}$$

- a.** Show that the composite function with rule $g(f(x))$ exists.

- b.** Write down the rule for $g(f(x))$ and state the domain of this composite function.

- c.** What is the range of this composite function?

1 + 2 + 1 = 4 marks

Question 9

If $f(x+h) \approx f(x) + hf'(x)$ and $f(x) = \frac{1}{\sqrt{x}}$, find an approximation to the value of $\frac{1}{\sqrt{3.9}}$.
Give your answer in the form $\frac{a}{b}$, where a and b are integers.

This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

4 marks

