



Trial Examination 2006

VCE Mathematical Methods Units 3 & 4

Written Examination 1

Suggested Solutions

Question 1

a. $3a + 4a + 2a + a = 1$

$$10a = 1$$

$$a = \frac{1}{10}$$

A1

b. $\Pr(X \leq 2) = (3 + 4 + 2) \times \frac{1}{10}$

$$= \frac{9}{10}$$

A1

c. $E(2X + 1) = E(2X) + E(1)$

$$= 2E(X) + 1$$

M1

$$= 2 \times \sum x \Pr(X = x) + 1$$

$$= 2 \left[0 \times \frac{3}{10} + 1 \times \frac{4}{10} + 2 \times \frac{2}{10} + 3 \times \frac{1}{10} \right] + 1$$

$$= 2(0.4 + 0.4 + 0.3) + 1$$

$$= 2 \times 1.1 + 1$$

$$= 3.2$$

A1

Alternatively:

$$E(2X + 1) = \sum (2x + 1) \Pr(X = x)$$

$$= (2 \times 0 + 1) \times 0.3 + (2 \times 1 + 1) \times 0.4 + (2 \times 2 + 1) \times 0.2 + (2 \times 3 + 1) \times 0.1$$

$$= 0.3 + 1.2 + 1.0 + 0.7$$

$$= 3.2$$

Question 2

a. $f: y = e^{x+1} - 2 \quad \text{dom}_f: \mathbb{R}, \text{ran}_f: (-2, \infty)$

$$f^{-1}: x = e^{y+1} - 2$$

M1

$$e^{y+1} = x + 2$$

$$y + 1 = \log_e(x + 2)$$

$$y = \log_e(x + 2) - 1$$

$$f^{-1}(x) = \log_e(x + 2) - 1$$

A1

b. $\text{dom } f^{-1} \Leftrightarrow \text{ran } f = (-2, \infty)$

A1

Question 3

- a. $g(x) = 2x^2 + 4x - 7$
 $= 2\left(x^2 + 2x - \frac{7}{2}\right)$
 $= 2\left[(x^2 + 2x + 1) - 1 - \frac{7}{2}\right]$ M1
 $= 2\left[(x + 1)^2 - \frac{9}{2}\right]$
 $= 2(x + 1)^2 - 9$ A1
- b. Dilation by a factor of two parallel to the y-axis (or from the x-axis). A1
 Translation of one unit in the negative x direction (left)
 and nine units in the negative y direction (down). A1

Question 4

$$\cos(3\pi x) = -\sin(3\pi x) \quad 0 \leq x \leq 1 \Rightarrow 0 \leq 3\pi x \leq 3\pi$$

$$\tan(3\pi x) = -1$$
 M1

The reference angle = $\frac{\pi}{4}$, with solutions in quadrants two and four.

$$3\pi x = \pi - \frac{\pi}{4}, 2\pi - \frac{\pi}{4}, 3\pi - \frac{\pi}{4}$$
 A1
$$= \frac{3\pi}{4}, \frac{7\pi}{4}, \frac{11\pi}{4}$$

$$x = \frac{1}{4}, \frac{7}{12}, \frac{11}{12}$$
 A1

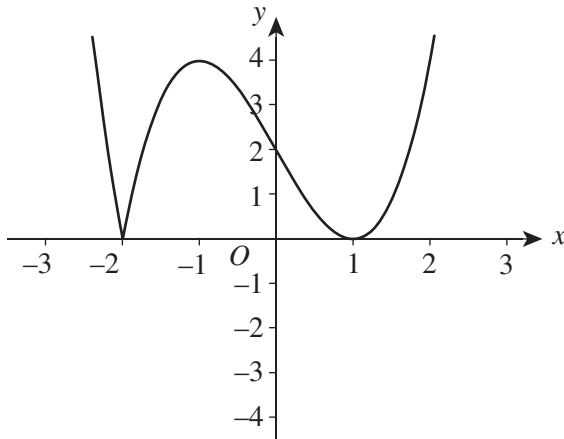
Question 5

- a. $f(x) = x^2 e^{-3x}$
 $f'(x) = 2xe^{-3x} + x^2(-3e^{-3x})$ (apply product rule) M1
 $= xe^{-3x}(2 - 3x)$ A1
- b. $f'(x) = 0$
 $xe^{-3x}(2 - 3x) = 0$
 $x = 0, \frac{2}{3}$
 $f(0) = 0$ and $f\left(\frac{2}{3}\right) = \frac{4}{9}e^{-2} = \frac{4}{9e^2}$
 $\therefore$ stationary points at $(0, 0)$ and $\left(\frac{2}{3}, \frac{4}{9e^2}\right)$

$4 \times A \frac{1}{2}$ for each x and y value
 rounded down to the nearest integer.

Question 6

a. $g(x) = |f(x)|$



A1

b. $R \setminus \{-2\}$

A1

c. $\text{Area} = \int_{-2}^1 g(x) dx \quad \left(\text{or} = \left| \int_{-2}^1 f(x) dx \right| \text{ etc.} \right)$

$$g(x) = |f(x)|$$

$$= -f(x) \text{ when } -2 \leq x \leq 1$$

$$= (x+2)(x-1)^2$$

$$= (x+2)(x^2 - 2x + 1)$$

$$= x^3 - 2x^2 + x + 2x^2 - 4x + 2$$

$$= x^3 - 3x + 2$$

A1

$$\therefore \text{Area} = \int_{-2}^1 (x^3 - 3x + 2) dx$$

$$= \left[\frac{x^4}{4} - \frac{3x^2}{2} + 2x \right]_{-2}^1$$

M1

$$= \left(\frac{1^4}{4} - \frac{3(1)^2}{2} + 2(1) \right) - \left(\frac{(-2)^4}{4} - \frac{3(-2)^2}{2} + 2(-2) \right)$$

$$= \frac{1}{4} - \frac{3}{2} + 2 - 4 + 6 + 4$$

$$= \frac{27}{4} \text{ square units}$$

A1

Question 7

a. Require $\int_0^2 k(x+1)dx = 1$ M1

$$k \int_0^2 (x+1)dx = 1$$

$$k \left[\frac{x^2}{2} + x \right]_0^2 = 1$$

$$k \left[\left(\frac{4}{2} + 2 \right) - (0 + 0) \right] = 1$$

$$4k = 1$$

$$k = \frac{1}{4} \quad \text{A1}$$

b. Require m such that $\int_0^m \frac{1}{4}(x+1)dx = \frac{1}{2}$ M1

$$\int_0^m (x+1)dx = 2$$

$$\left[\frac{x^2}{2} + x \right]_0^m = 2$$

$$\left(\frac{m^2}{2} + m \right) - (0 + 0) = 2$$

$$\frac{m^2}{2} + m = 2$$

$$m^2 + 2m = 4$$

$$m^2 + 2m - 4 = 0 \quad \text{A1}$$

$$m = \frac{-2 \pm \sqrt{2^2 - 4(1)(-4)}}{2(1)}$$

$$= \frac{-2 \pm \sqrt{20}}{2}$$

$$= \frac{-2 \pm 2\sqrt{5}}{2}$$

$$= -1 \pm \sqrt{5}$$

but $0 < m < 2 \therefore m = -1 + \sqrt{5}$ A1

Question 8

- a. For $g(f(x))$ to exist, $\text{ran}_f \subseteq \text{dom}_g$.
 dom_g is $\mathbb{R}$.
 $\therefore \text{ran}_f \subseteq \text{dom}_g$ always $\therefore g(f(x))$ exists.

A1

- b. $g(f(x)) = 1 - (3 \sin(2x))^2$
 $= 1 - 9 \sin^2(2x)$

A1

Domain = dom_f (in this case): $[0, \pi]$

A1

- c. $[-8, 1]$

A1

Question 9

$$f(x) = x^{-\frac{1}{2}}$$

$$f'(x) = -\frac{1}{2}x^{-\frac{3}{2}}$$

$$= -\frac{1}{2x\sqrt{x}}$$

M1

$$x = 4, h = -\frac{1}{10}$$

A1

$$f(3.9) = f\left(4 - \frac{1}{10}\right)$$

$$\approx f(4) - \frac{1}{10}f'(4)$$

$$= \frac{1}{\sqrt{4}} - \frac{1}{10}\left(-\frac{1}{2(4)\sqrt{4}}\right)$$

M1

$$= \frac{1}{2} + \frac{1}{(10)(8)(2)}$$

$$= \frac{1}{2} + \frac{1}{160}$$

$$= \frac{80}{160} + \frac{1}{160}$$

$$= \frac{81}{160}$$

A1

Question 10

$$y = 4x^3 + 1$$

$$\frac{dy}{dx} = 12x^2$$

At $x = a$, the gradient of the tangent, $m_T = 12a^2$ M1

$\therefore$ the tangent has an equation of the form $y = 12a^2x + c$

The tangent passes through the origin $\therefore c = 0$

$$\therefore y = 12a^2x$$
 A1

$$\text{At } x = a, \quad y = 4a^3 + 1$$

$$\therefore 4a^3 + 1 = 12a^3$$
 M1

$$8a^3 = 1$$

$$a^3 = \frac{1}{8}$$

$$a = \frac{1}{2}$$

$$\therefore \text{the tangent has the equation } y = 12\left(\frac{1}{2}\right)^2 x$$

$$y = 3x$$
 A1