

Year 2006

Mathematical Methods

Examination 2

Suggested Solutions



KILBAHA MULTIMEDIA PUBLISHING
PO BOX 2227
KEW VIC 3101
AUSTRALIA

TEL: (03) 9817 5374
FAX: (03) 9817 4334
chemas@chemas.com
www.chemas.com

© Kilbaha Pty Ltd 2006

ABN 47 065 111 373

IMPORTANT COPYRIGHT NOTICE

- This material is copyright. Subject to statutory exception and to the provisions of the relevant collective licensing agreements, no reproduction of any part may take place without the written permission of Kilbaha Multimedia Publishing.
- The contents of this work are copyrighted. Unauthorised copying of any part of this work is illegal and detrimental to the interests of the author.
- For authorised copying within Australia please check that your institution has a licence from **Copyright Agency Limited**. This permits the copying of small parts of the material, in limited quantities, within the conditions set out in the licence.

Reproduction and communication for educational purposes The Australian Copyright Act 1968 (the Act) allows a maximum of one chapter or 10% of the pages of this work, to be reproduced and/or communicated by any educational institution for its educational purposes provided that educational institution (or the body that administers it) has given a remuneration notice to Copyright Agency Limited (CAL) under the Act.

For details of the CAL licence for educational institutions contact
CAL, Level 15, 233 Castlereagh Street, Sydney, NSW, 2000

Tel: (02) 9394 7600

Fax: (02) 9394 7601

Email: info@copyright.com.au

- Teachers and students are reminded that for the purposes of school requirements and external assessments, students must submit work that is clearly their own.
- Schools which purchase a licence to use this material may distribute this electronic file to the students at the school for their exclusive use. This distribution can be done either on an Intranet Server or on media for the use on stand-alone computers.
- Schools which purchase a licence to use this material may distribute this printed file to the students at the school for their exclusive use.

SECTION 1

Question 1 **Answer C**

$y = x^2$ translate 3 units down $y = x^2 - 3$
 translate 2 units to the right $y = (x - 2)^2 - 3$

Question 2 **Answer B**

$$\tan(2x) = 1$$

$$2x = \tan^{-1}(1) = \frac{\pi}{4}$$

$$x = \frac{\pi}{8} \text{ the smallest positive value}$$

Question 3 **Answer A**

$$f(x) = 5 - x \quad f(-2) = 7 \quad f(7) = -2$$

$$\text{domain } [-2, 7] \quad \text{range } (-2, 7]$$

Question 4 **Answer B**

$$f(x) = 3\sin\left(\frac{\pi}{2} - \pi\right) + 4,$$

$$\text{the range is } 4 \pm 3 \quad [1, 7]$$

Question 5 **Answer D**

3 white, 7 yellow, total 10 balls,
 without replacement

$$\Pr(YYY) = \frac{7}{10} \times \frac{6}{9} \times \frac{5}{8} = \frac{7}{24}$$

Question 6 **Answer D**

$f: [a, \infty) \rightarrow \mathbb{R} \quad f(x) = \log_e(x^4)$
 is not a one-one function, to have
 an inverse function, it must be one-one
 so $a > 0$

Question 7 **Answer B**

$$g(x) = \log_e|x - b| \quad b \in \mathbb{R},$$

$$\text{the maximal domain is}$$

$$x - b \neq 0 \quad \mathbb{R} \setminus \{b\}$$

Question 8 **Answer E**

$$y = \log_3(x)$$

$$\text{reflect in the } x\text{-axis } y = -\log_3(x)$$

$$\text{translate 5 units up } y = -\log_3(x) + 5$$

$$\text{translate 2 units right } y = -\log_3(x - 2) + 5$$

Question 9 **Answer B**

$$y = 3a^{2x} + b \quad y - b = 3a^{2x}$$

$$a^{2x} = \frac{y - b}{3} \quad 2x = \log_a\left(\frac{y - b}{3}\right)$$

$$x = \frac{1}{2} \log_a\left(\frac{y - b}{3}\right)$$

Question 10 **Answer A**

$$\text{Given } \frac{dr}{dt} = 3 \text{ cm/min}$$

$$\frac{dV}{dt} = ? \quad \text{when } r = 6$$

$$V = \frac{4}{3}\pi r^3 \quad \frac{dV}{dr} = 4\pi r^2$$

$$\frac{dV}{dt} = \frac{dV}{dr} \cdot \frac{dr}{dt} = 12\pi r^2$$

$$\left. \frac{dV}{dt} \right|_{r=6} = 432\pi$$

Question 11 **Answer D**

$$|2k + 1| = k + 1 \quad \text{then}$$

$$2k + 1 = k + 1 \quad \text{or} \quad -2k - 1 = k + 1$$

$$k = 0 \quad \quad \quad -3k = 2$$

$$k = 0 \quad \quad \quad \text{or} \quad k = -\frac{2}{3}$$

Question 12 **Answer C**

$$X \sim \text{Bi}\left(n = 10, p = \frac{1}{2}\right)$$

$$\Pr(X \geq 8) = 0.0547$$

Question 13 **Answer E**

$$f \quad y = \frac{2}{3x+6} - 1 \quad f^{-1} \quad x = \frac{2}{3y+6} - 1$$

transposing $x+1 = \frac{2}{3y+6}$

$$3y+6 = \frac{2}{x+1} \quad 3y = \frac{2}{x+1} - 6$$

$$y = f^{-1}(x) = \frac{2}{3x+3} - 2$$

Question 14 **Answer A**

$f'(x)$ or the gradient is negative
for $-3 < x < 3$

Question 15 **Answer D**

Let $A_1 = \int_{-1}^1 f(x) dx$ but $A_1 < 0$

$$A_2 = \int_1^4 f(x) dx \quad \text{and} \quad A_3 = \int_4^6 f(x) dx$$

but $A_3 < 0$ the area is $A_2 - A_1 - A_3$

$$= \int_1^4 f(x) dx - \int_{-1}^1 f(x) dx - \int_1^6 f(x) dx$$

Question 16 **Answer A**

$$f'(x) = g'(x) + 3$$

$$f(x) = g(x) + 3x + c \quad \text{at } x = 0$$

$$f(0) = g(0) + 0 + c$$

$$2 = 1 + c \quad c = 1$$

so $f(x) = g(x) + 3x + 1$

Question 17 **Answer A**

let $y = \frac{\sin(3x)}{2e^x - x}$

Quotient rule

$$u = \sin(3x) \quad v = 2e^x - x$$

$$\frac{du}{dx} = 3\cos(3x) \quad \frac{dv}{dx} = 2e^x - 1$$

$$\frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

$$\frac{dy}{dx} = \frac{(2e^x - x)3\cos(3x) - \sin(3x)(2e^x - 1)}{(2e^x - x)^2}$$

$$\frac{dy}{dx} = \frac{3(2e^x - x)\cos(3x) - (2e^x - 1)\sin(3x)}{4e^{2x} - 4xe^x + x^2}$$

Question 18 **Answer D**

Since $\sum \Pr(X = x) = 1$

$$a + b + 0.4 = 1$$

and $E(X) = \sum x \Pr(X = x)$

$$E(X) = -a + 0.4 = 0.3$$

so $a = 0.1 \quad b = 1 - 0.4 - a = 0.5$

Question 19 **Answer A**

$$\int_0^k (1 + e^{\frac{x}{k}}) dx = \left[x + ke^{\frac{x}{k}} \right]_0^k$$

$$= (k + ke^1) - (0 + k) = 1$$

$$ke = 1$$

$$k = \frac{1}{e} = e^{-1}$$

Question 20 **Answer E**

Let $y = f(\sin(4x)) = f(u)$

$$u = \sin(4x) \quad \frac{dy}{du} = f'(u)$$

$$\frac{du}{dx} = 4\cos(4x)$$

$$\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx} = 4\cos(4x) f'(\sin(4x))$$

Question 21 **Answer D**

$$X \sim N(\mu = 200, \sigma^2 = 10^2)$$

$$\Pr(X < 208) = 0.788$$

Question 22 **Answer C**

$$y = a \log_e(x - b) \quad \text{where } a < 0 \quad b > 0$$

reflect $y = \log_e(x)$ in the x-axis,

translate b units to the right.

SECTION 2

Question 1

$$f : [0, 2\pi] \rightarrow \mathbb{R} \quad f(x) = 2\sin(x)$$

a. i. $f'(x) = 2\cos(x)$

ii. $|f'(x)|_{\max} = 2 \quad |f'(x)|_{\min} = 0$

b. i. $f'(x) = 2\cos(x) = 1 \quad \cos(x) = \frac{1}{2}$

$$x = \frac{\pi}{3}, \quad 2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$$

ii. $f'\left(\frac{\pi}{3}\right) = 2\cos\left(\frac{\pi}{3}\right) = 1 = m_T$

$$f\left(\frac{\pi}{3}\right) = 2\sin\left(\frac{\pi}{3}\right) = \sqrt{3} \text{ at } A\left(\frac{\pi}{3}, \sqrt{3}\right) \quad \text{tangent } y - \sqrt{3} = 1\left(x - \frac{\pi}{3}\right)$$

$$y = x + \sqrt{3} - \frac{\pi}{3}$$

iii. tangent crosses x-axis when $y = 0 \quad x = \frac{\pi}{3} - \sqrt{3} \quad \left(\frac{\pi}{3} - \sqrt{3}, 0\right)$

tangent crosses y-axis when $x = 0 \quad y = \sqrt{3} - \frac{\pi}{3} \quad \left(0, \sqrt{3} - \frac{\pi}{3}\right)$

c. At $B \quad x = \frac{5\pi}{3} \quad f\left(\frac{5\pi}{3}\right) = -\sqrt{3} \quad B\left(\frac{5\pi}{3}, -\sqrt{3}\right)$

tangent at $B \quad y + \sqrt{3} = 1\left(x - \frac{5\pi}{3}\right) \quad y = x - \sqrt{3} - \frac{5\pi}{3}$

so $x_A = \sqrt{3} - \frac{\pi}{3} \quad x_B = -\sqrt{3} - \frac{5\pi}{3}$

$$m = x_A - x_B = 2\sqrt{3} + \frac{4\pi}{3}$$

Question 2

a. i. $\Pr(PPP) = 0.4 \times 0.3^2 = 0.036 = \frac{9}{250}$

ii. $\Pr(GPP) + \Pr(PGP) + \Pr(PPG)$

$$= 0.6 \times 0.4 \times 0.3 + 0.4 \times 0.7 \times 0.4 + 0.4 \times 0.3 \times 0.7 = 0.268 = \frac{67}{250}$$

b. $X \sim N(\mu = 60, \sigma^2 = ?)$

$$\Pr(X < 50) = 0.2$$

$$Z = \frac{X - \mu}{\sigma} \quad \frac{50 - 60}{\sigma} = \frac{-10}{\sigma} = -0.8416$$

$$\sigma = \frac{10}{0.6416} = 11.9 \text{ minutes}$$

c.

$$f(t) = \begin{cases} 4t^3 - 24t^2 + 44t - 24 & \text{for } 1 \leq t \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

d. 75 minutes = 1.25 hours

$$\Pr(T < 1.25) = \int_1^{1.25} (4t^3 - 24t^2 + 44t - 24) dt$$

$$= 0.191$$

e. $Y \sim \text{Bi}(n = 5, p = 0.8086)$

$$\Pr(Y = 4) = \binom{5}{4} 0.8086^4 \times 0.1914$$

$$= 0.41$$

f. $\int_1^m (4t^3 - 24t^2 + 44t - 24) dt = \frac{1}{2}$

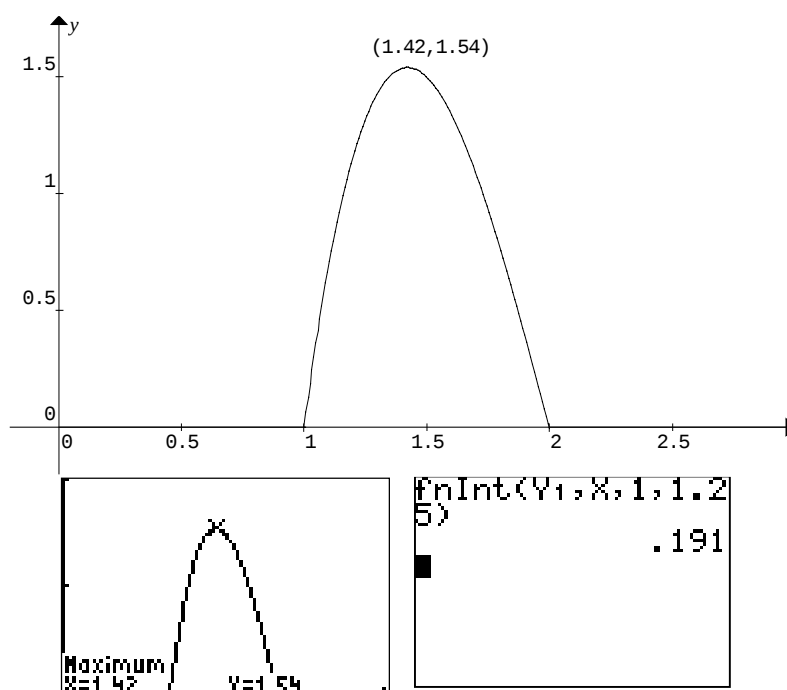
$$\left[t^4 - 8t^3 + 22t^2 - 24t \right]_1^m = \frac{1}{2}$$

$$(m^4 - 8m^3 + 22m^2 - 24m) - (1 - 8 + 22 - 24) = \frac{1}{2}$$

$$m^4 - 8m^3 + 22m^2 - 24m + 8.5 = 0$$

$$m = 1.4588 \text{ hours}$$

$$m = 88 \text{ minutes}$$



```

Plot1 Plot2 Plot3
Y1=X^4-8X^3+22X^2-24X+8.5
Y2=
Y3=
Y4=
Y5=
Y6=

```

```

Zero
X=1.459 Y=0

```

Question 3

a. $f(x) = 3 - e^x - e^{-x}$

$$f(0) = 3 - 1 - 1 = 1 = b$$

b. When $y = 0$ $3 - e^x - e^{-x} = 0$ let $u = e^x$

$$3 - u - \frac{1}{u} = 0 \text{ multiply by } u$$

$$3u - u^2 - 1 = 0$$

$$u^2 - 3u + 1 = 0$$

$$\Delta = 9 - 4 = 5$$

$$u = e^x = \frac{3 \pm \sqrt{5}}{2} \text{ but } u > 0 \text{ take positive}$$

$$a = \log_e \left(\frac{3 + \sqrt{5}}{2} \right)$$

c. i.

x	-0.5	0	0.5
y	0.74	1.00	0.74

ii. Area = $0.5 \times 0.74 + 0.5 \times 1 + 0.5 \times 0.74$
 $= 1.2 \text{ km}^2$

iii. $V = 1.2 \text{ mV}$

d. i. $(0,1)(1,0)(-1,0) y = ax^2 + c$

$$c = 1 \quad a = -1 \quad y = 1 - x^2$$

ii. $A = \int_{-1}^1 (1 - x^2) dx = 2 \int_0^1 (1 - x^2) dx$

$$= 2 \left[x - \frac{1}{3} x^3 \right]_0^1 = 2 \left[\left(1 - \frac{1}{3} \right) - (0) \right]$$

$$= \frac{4}{3} = 1.33 \text{ km}^2$$

Question 4

a. i. $f(x) = px^3 + qx^2 + rx + s$ at $B(0,7)$ $f(0) = 7 = s = 7$

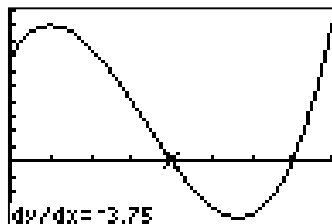
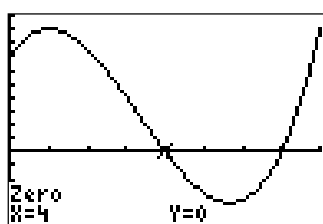
ii. $f'(x) = 3px^2 + 2qx + r$ $f'(0) = 4.25 = r$

b. at $C(1,9)$ $f(1) = 9 \Rightarrow 9 = p + q + 4.25 + 7 \Rightarrow p + q = -2.25$ (1)

$f'(1) = 0 \Rightarrow 3p + 2q + 4.25 = 0 \Rightarrow 3p + 2q = -4.25$ (2)

so $f(x) = 0.25x^3 - 2.5x^2 + 4.25x + 7$

c. $D(4,0)$ $F(7,0)$ from CALC



d. $f'(4) = -3.75$

e. $E(5.66, -3.704)$ so the distance is 3.70

f. inflexion point is $(3.33, 2.648)$ max value $f'(3.33) = -4.08$

absolute value of $f'(3.33) = 4.1$ from CALC

g. $g(x) = \frac{a}{1-bx}$ at $B(0,7)$ $g(0) = 7 = a$ $a = 7$

$g'(x) = ba(1-bx)^{-2} = \frac{ba}{(1-bx)^2}$ $g'(0) = ba = 4.25$ $b = \frac{4.25}{7} = \frac{17}{28}$

h. $A_1 = \int_{-2}^0 \frac{a}{1-bx} dx = -\frac{a}{b} \log_e((1-bx)) \Big|_{-2}^0 = \frac{-196}{17} [\log_e(1) - \log_e(1+2b)]$
 $= \frac{196}{17} \log_e\left(\frac{31}{14}\right) \approx 9.1651$

$A_2 = \int_0^4 (0.25x^3 - 2.5x^2 + 4.25x + 7) dx = \left[\frac{1}{16}x^4 - \frac{5}{6}x^3 + \frac{17}{8}x^2 + 7x \right]_0^4$
 $= \left[\frac{1}{16} \times 256 - \frac{5}{6} \times 64 + \frac{17}{8} \times 16 + 7 \times 4 - 0 \right] = 24 \frac{2}{3}$

total area $A_1 + A_2 = 9.1651 + 24.667 = 33.83$ units²



Mathematics 2007

Kilbaha Multimedia Publishing (Est 1978)
PO Box 2227
Kew Vic 3101 Australia

Tel: (03) 9817 5374
Fax: (03) 9817 4334
chemas@chemas.com www.chemas.com

SCHOOL ORDER NUMBER _____ DATE _____

NAME _____

SCHOOL _____

ADDRESS _____

TEL _____ FAX _____

EMAIL _____

Please ✓ those required. All prices are for a CD-ROM with an unlimited site licence. GST and freight included.

✓	Title	Price	✓	Title	Price
	SumsOnline Numbers: Year 1	\$59.95		Flying with Fractions	\$495
	SumsOnline Numbers: Year 2	\$59.95		Flying with Decimals	\$495
	SumsOnline Numbers: Year 3	\$59.95		Flying with Factorisation	\$495
	SumsOnline Numbers: Year 4	\$59.95		Flying with Shapes	\$495
	SumsOnline Numbers: Year 5	\$59.95		Flying with Mathematics	\$495
	SumsOnline Numbers: Year 6	\$59.95		Mathematics Intranet	\$495
	SumsOnline Calculations: Year 1	\$59.95		2006 Further Maths Trial Exam 1	\$55.00
	SumsOnline Calculations: Year 2	\$59.95		2006 Further Maths Trial Exam 2	\$55.00
	SumsOnline Calculations: Year 3	\$59.95		2006 Maths Methods Trial Exam 1	\$55.00
	SumsOnline Calculations: Year 4	\$59.95		2006 Maths Methods Trial Exam 2	\$55.00
	SumsOnline Calculations: Year 5	\$59.95		2006 Maths Methods Trial Exam 2(CAS)	\$55.00
	SumsOnline Calculations: Year 6	\$59.95		2006 Specialist Maths Trial Exam 1	\$55.00
	SumsOnline Shape & Space: Year 3	\$59.95		2006 Specialist Maths Trial Exam 2	\$55.00
	SumsOnline Shape & Space: Year 4	\$59.95		2007 Further Maths Trial Exam 1	\$55.00
	SumsOnline Shape & Space: Year 5	\$59.95		2007 Further Maths Trial Exam 2	\$55.00
	SumsOnline Shape & Space: Year 6	\$59.95		2007 Maths Methods Trial Exam 1	\$55.00
	SumsOnline Handling Data: Year 3	\$59.95		2007 Maths Methods Trial Exam 2	\$55.00
	SumsOnline Handling Data: Year 4	\$59.95		2007 Maths Methods Trial Exam 2(CAS)	\$55.00
	SumsOnline Handling Data: Year 5	\$59.95		2007 Specialist Maths Trial Exam 1	\$55.00
	SumsOnline Handling Data: Year 6	\$59.95		2007 Specialist Maths Trial Exam 2	\$55.00
	School resources Year 7 Mathematics	\$99.00		"In Training for the Olympics" (book) Please indicate the number required.	\$5.00
	"In Search of the Holy Grail" (book) Please indicate the number required.	\$5.00		"In the Running for the Olympics"(book) Please indicate the number required.	\$5.00
	Detailed worked solutions to all 2006 VCAA VCE Mathematics Examinations				FREE
	CD-ROM of eLearning in Mathematics containing samples of all of the above				FREE

FREE downloads are also available from our Web Site <http://www.chemas.com>

Total for order = \$ _____

Fax to 03 9817 4334