

Year 2006

VCE

Mathematical Methods

Examination 1

Suggested Solutions



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Solutions Mathematical Methods Examination 1 2006**Question 1**

- a. $f(x) = x^2 + 1$ and $g(x) = 2x + 1$
 $f(g(x)) = f(2x + 1) = (2x + 1)^2 + 1 = 4x^2 + 4x + 2$

Question 2

- a. $f: R \rightarrow R$, $f(x) = 3e^{2x} - 1$ so the function is $f: y = 3e^{2x} - 1$
the inverse $f^{-1}: x = 3e^{2y} - 1$ transposing $x + 1 = 3e^{2y}$
 $\frac{x+1}{3} = e^{2y}$ $2y = \frac{1}{2} \log_e \left(\frac{x+1}{3} \right)$
 $f^{-1}(x) = \frac{1}{2} \log_e \left(\frac{x+1}{3} \right)$
- b. $\text{dom } f^{-1} = \text{ran } f = (-1, \infty)$

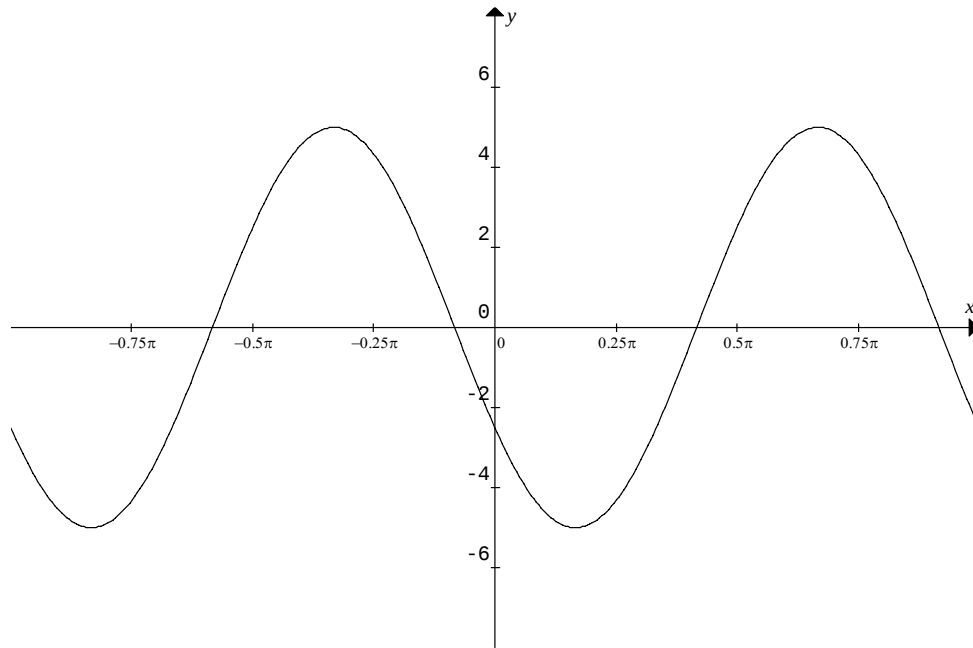
Question 3

- a. $f(x) = e^{\cos(x)}$ using the chain rule
 $y = e^u$ where $u = \cos(x)$
 $\frac{dy}{du} = e^u$ $\frac{du}{dx} = -\sin(x)$
 $f'(x) = \frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx} = -\sin(x) e^{\cos(x)}$
- b. $y = x \tan(x)$ product rule
 $\frac{dy}{dx} = \tan(x) + x \sec^2(x) = \tan(x) + \frac{x}{\cos^2(x)}$
 $\left. \frac{dy}{dx} \right|_{x=\frac{\pi}{6}} = \tan\left(\frac{\pi}{6}\right) + \frac{\frac{\pi}{6}}{\cos^2\left(\frac{\pi}{6}\right)} = \frac{1}{\sqrt{3}} + \frac{\frac{\pi}{6}}{\left(\frac{\sqrt{3}}{2}\right)^2}$
 $\left. \frac{dy}{dx} \right|_{x=\frac{\pi}{6}} = \frac{\sqrt{3}}{3} + \frac{2\pi}{9} = \frac{3\sqrt{3} + 2\pi}{9}$

Question 4

- a. $f: [-\pi, \pi] \rightarrow R$, $f(x) = 5 \cos\left(2\left(x + \frac{\pi}{3}\right)\right)$
amplitude is 5 period is $\frac{2\pi}{2} = \pi$

b.



end-points $(-\pi, -2.5)$ $(\pi, -2.5)$ y-intercept $(0, -2.5)$

x-intercepts $\left(-\frac{7\pi}{12}, 0\right), \left(-\frac{\pi}{12}, 0\right), \left(\frac{5\pi}{12}, 0\right), \left(\frac{11\pi}{12}, 0\right)$ since the period is π

Question 5

$$X \stackrel{d}{=} N(\mu = 150, \sigma^2 = 15^2)$$

a. $\Pr(X > 80)$

$$= \Pr\left(Z > \frac{80 - 72}{8}\right) = \Pr(Z > 1)$$

$$= 1 - \Pr(Z < 1) = 1 - 0.84$$

$$= 0.16$$

b. $\Pr(64 < X < 72)$

$$= \Pr\left(\frac{64 - 72}{8} < Z < \frac{72 - 72}{8}\right) = \Pr(-1 < Z < 0)$$

$$= \Pr(Z < 0) - \Pr(Z < -1.0) = 0.5 - (1 - 0.84)$$

$$= 0.34$$

c. $\Pr(X < 64 / X < 72)$

$$= \frac{\Pr(X < 64)}{\Pr(X < 72)} = \frac{\Pr(Z < -1)}{\Pr(Z < 0)}$$

$$= \frac{0.16}{0.5}$$

$$= 0.32$$

Question 6

$$f(x) = \begin{cases} \frac{x}{12} & 1 \leq x \leq 5 \\ 0 & \text{otherwise} \end{cases}$$

a. $\Pr(X < 3) = \int_1^3 \frac{x}{12} dx$

$$\Pr(X < 3) = \left[\frac{x^2}{24} \right]_1^3 = \frac{1}{24}(9-1)$$

$$\Pr(X < 3) = \frac{1}{3}$$

b. $\Pr(X \geq a) = \int_a^5 \frac{x}{12} dx$

$$\Pr(X \geq a) = \left[\frac{x^2}{24} \right]_a^5 = \frac{1}{24}(25 - a^2) = \frac{5}{8}$$

$$25 - a^2 = 15 \quad a^2 = 10$$

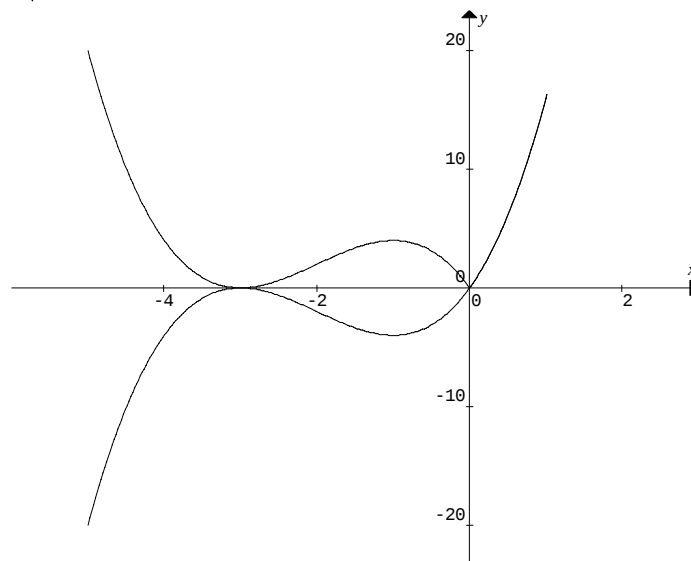
$$a = \pm\sqrt{10} \quad \text{but } 1 < a < 5 \text{ take positive}$$

$$a = \sqrt{10}$$

Question 7

$$f: [-5, 1] \rightarrow \mathbb{R} \quad \text{where} \quad f(x) = x^3 + 6x^2 + 9x$$

a. graph of $y = |f(x)|$



b. $f(x) = x(x^2 + 6x + 9) = x(x+3)^2$

$$f(1) = 16 \quad f(-5) = -5(3-5)^2 = -20$$

$$\text{the range of } y = |f(x)| = [0, 20]$$

Question 8

$$y = \sqrt{x} = x^{\frac{1}{2}} \quad \frac{dy}{dx} = \frac{1}{2} x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}} = m_T$$

$$m_N = -2\sqrt{x} = -4 \quad \text{so} \quad \sqrt{x} = 2 \quad x = 4 \quad y = \sqrt{4} = 2 \quad \text{point is } (4, 2) \text{ on the normal}$$

$$2 = -16 + a$$

$$a = 18$$

Question 9

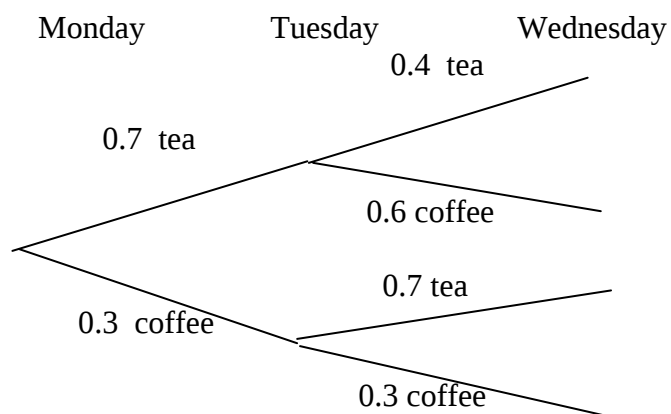
a. $A = 2ab$ but $b = 9 - 3a^2$

$$A = 2a(9 - 3a^2)$$

b. $\frac{dA}{da} = 18 - 18a^2 = 0$ for a maximum value of A

$$18a^2 = 18 \quad a^2 = 1 \quad a = \pm 1$$

$$\text{when } a = 1 \quad A_{\max} = 12$$

Question 10

$$\Pr(\text{tea Wednesday}) = 0.7 \times 0.4 + 0.3 \times 0.7 = 0.28 + 0.21 = 0.49$$

Question 11

$$A = \int_0^3 (-x^2 + ax + 12) dx = 45$$

$$A = \left[-\frac{1}{3}x^3 + \frac{1}{2}ax^2 + 12x \right]_0^3 = \left(-9 + \frac{9a}{2} + 36 \right) = 45$$

$$\frac{9a}{2} = 18 \quad a = 4$$

$$f(x) = -x^2 + 4x + 12 = -(x^2 - 4x - 12) = -(x + 2)(x - 6)$$

$$a = 4 \quad m = 6 \quad n = -2$$



Mathematics 2007

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