

2006

VCE

Mathematical Methods

Trial Examination 1

This Mathematics Methods Trial Examination must be attempted without the use of technology. This is a common examination for Mathematical Methods and Mathematical Methods CAS.



KILBAHA MULTIMEDIA PUBLISHING
PO BOX 2227
KEW VIC 3101
AUSTRALIA

TEL: (03) 9817 5374
FAX: (03) 9817 4334
chemas@chemas.com
www.chemas.com

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ABN 47 065 111 373

PURPOSE OF THIS TRIAL EXAMINATION

This Mathematics Methods Trial Examination is designed to assess

- knowledge of mathematical concepts
- skills in carrying out mathematical algorithms
- ability to apply concepts and skills in standard ways

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VICTORIAN CERTIFICATE OF EDUCATION 2006

	STUDENT NUMBER							Letter
Figures								
Words								

MATHEMATICAL METHODS

Trial Written Examination 1

Reading time: 15 minutes

Writing time: 1 hour

QUESTION AND ANSWER BOOK

Structure of book

<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
11	11	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers.
- Students are NOT permitted to bring into the examination room: notes of any kind, blank sheets of paper and/or white out liquid/tape. Calculators are not permitted.

Materials supplied

- Question and answer book of 11 pages, with a detachable sheet of miscellaneous formulas.
- Working space is provided throughout the book.

Instructions

- Detach the formula sheet from the centre of this book during reading time.
- Write your student number in the space provided above on this page.
- All written responses must be in English

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

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MATHEMATICAL METHODS

Written examinations 1 and 2

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time.

This formula sheet is provided for your reference.

Mathematical Methods and Mathematical Methods CAS

Formulas

Mensuration

area of a trapezium	$\frac{1}{2}(a+b)h$	volume of a pyramid	$\frac{1}{3}Ah$
curved surface of a cylinder	$2\pi rh$	volume of a sphere	$\frac{4}{3}\pi r^3$
volume of a cylinder	$\pi r^2 h$	area of a triangle	$\frac{1}{2}bc \sin A$
volume of a cone	$\frac{1}{3}\pi r^2 h$		

Calculus

$\frac{d}{dx}(x^n) = nx^{n-1}$	$\int x^n dx = \frac{1}{n+1}x^{n+1} + c, n \neq -1$
$\frac{d}{dx}(e^{ax}) = ae^{ax}$	$\int e^{ax} dx = \frac{1}{a}e^{ax} + c$
$\frac{d}{dx}(\log_e(x)) = \frac{1}{x}$	$\int \frac{1}{x} dx = \log_e x + c$
$\frac{d}{dx}(\sin(ax)) = a \cos(ax)$	$\int \sin(ax) dx = -\frac{1}{a} \cos(ax) + c$
$\frac{d}{dx}(\cos(ax)) = -a \sin(ax)$	$\int \cos(ax) dx = \frac{1}{a} \sin(ax) + c$
$\frac{d}{dx}(\tan(ax)) = \frac{a}{\cos^2(ax)} = \sec^2(ax)$	
product rule: $\frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}$	quotient rule: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$
chain rule: $\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}$	approximation: $f(x+h) \approx f(x) + hf'(x)$

Probability

$\Pr(A) = 1 - \Pr(A')$	$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$
$\Pr(A B) = \frac{\Pr(A \cap B)}{\Pr(B)}$	
mean: $\mu = E(X)$	variance: $\text{var}(X) = \sigma^2 = E((X - \mu)^2) = E(x^2) - \mu^2$

probability distribution		mean	variance
discrete	$\Pr(X = x) = p(x)$	$\mu = \sum xp(x)$	$\sigma^2 = \sum (x - \mu)^2 p(x)$
continuous	$\Pr(a < X < b) = \int_a^b f(x) dx$	$\mu = \int_{-\infty}^{\infty} xf(x) dx$	$\sigma^2 = \int_{-\infty}^{\infty} (x - \mu)^2 f(x) dx$

END OF FORMULA SHEET

Instructions

Answer **all** questions in the spaces provided.

A decimal approximation will not be accepted if an exact answer is required to a question.

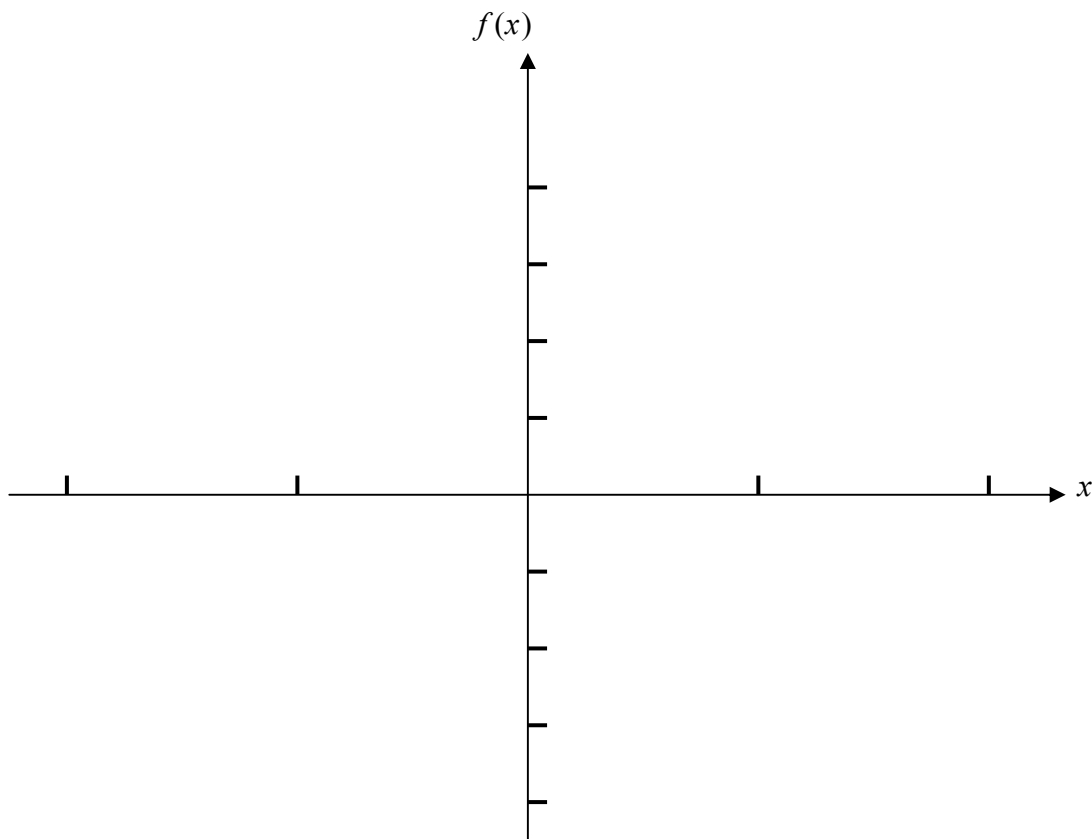
In questions where more than one mark is available, appropriate working must be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1

For the function $f: [0, \infty) \rightarrow \mathbb{R}, f(x) = 1 - e^{-x}$,

- a. Sketch the graph of the function $f(x)$ on the set of axes below. Label any intercepts with the axes and give the equation of any asymptotes.



- b. What is the range of this function?

3 + 1 = 4 marks

Question 2

Find the exact values of the point(s) of intersection of the graphs $y = e^x$ and $y = 6e^{-x} - 1$.

3 marks

Question 3

- a. Find $f'(x)$ if $f(x) = \log_e(x^2 + 5)$.

- b. Find the rule of $f(x)$ if $f'(x) = 1 - 6 \sin 3x$ and $f\left(\frac{\pi}{6}\right) = \frac{\pi}{3}$.

2 + 2 = 4 marks

Question 4

a. $f : \left\{ x : x > \frac{1}{3} \right\} \rightarrow R$, where $f(x) = \frac{1}{3x-1}$.

Find the inverse function $f^{-1}(x)$.

b. Give the domain and range of $f^{-1}(x)$.

2 + 2 = 4 marks

Question 5

Find the solutions of the equation: $2 \sin 3\left(\theta + \frac{\pi}{4}\right) - 1 = 0$, $0 \leq \theta \leq \frac{\pi}{2}$

3 marks

Question 6

A continuous random variable X has a probability density function:

$$f(x) = \begin{cases} 4x^3 & 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

- a.** Find the exact value of a so that $\Pr(X < a) = \frac{1}{4}$.

- b.** Find the probability that $\frac{1}{2} \leq x \leq \frac{3}{4}$.

2 + 2 = 4 marks

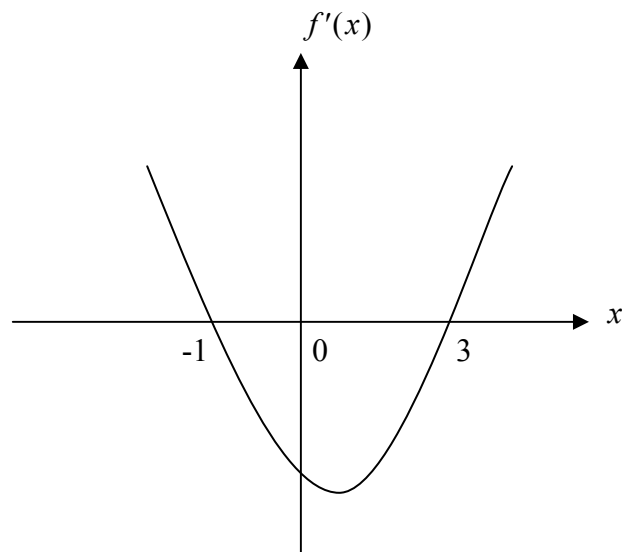
Question 7

- a. If $\Pr(A) = 0.3$, $\Pr(B) = 0.6$ and $\Pr(A \cup B) = 0.8$, find $\Pr(A | B)$.

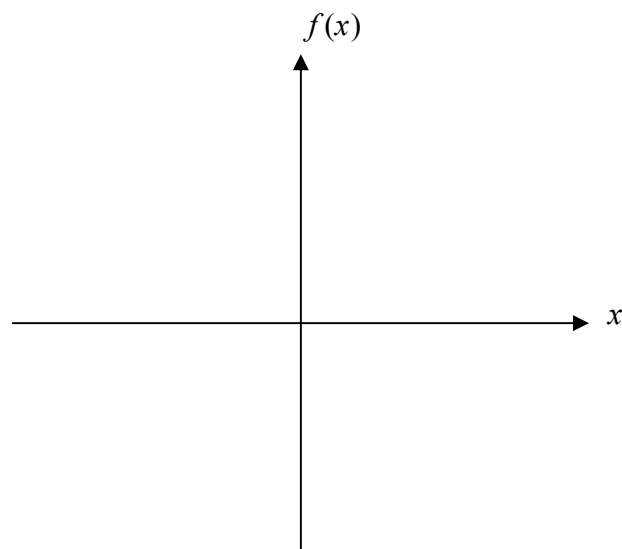
- b. A normal random variable has a mean of 8.0 and a standard deviation of 2.0
Find the probability $X < 12 | X > 8$.

2 + 3 = 5 marks

Question 8



- a. The above is a graph of $f'(x)$. On the axes below, sketch a graph of $f(x)$



- b. If $g(x) = x^3 - 9x^2 + 15x + 13$, find the values of x for which $g(x)$ is an increasing function.

1 + 2 = 3 marks

Question 9

A random variable X has a binomial distribution with $n = 9$ and a standard deviation of 1.5

- a.** Find the probability of a success p .

- b.** Find the probability that X is at least one.

2 + 1 = 3 marks

Question 10

- a. Find the derivative of $e^{3x} \sin 3x + 3$.

- b. Hence, find an antiderivative of $3 \sin 3x(e^{3x} - 1) + 3e^{3x} \cos 3x$.

1 + 2 = 3 marks

Question 11

Water is dripping at a rate of 50 millilitres per second from a cuboidal tank which has a square base of side 100 cm.

- a. Find the rate at which the depth of the water is falling in the tank.

- b. How long will it take for the depth of the water to decrease by 60 cm?
Give your answer in hours.

3 + 1 = 4 marks

End of 2006 Mathematical Methods Trial Examination 1
Question and Answer Book

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