

SECTION 1

1	2	3	4	5	6	7	8	9	10	11
E	E	D	D	E	A	D	C	A	A	C

12	13	14	15	16	17	18	19	20	21	22
A	B	D	A	D	C	D	C	B	D	D

Q1 $e^{2x+2} = e^x$, $e^{2x+2} - e^x = 0$, $e^x(e^{x+2} - 1) = 0$.

Since $e^x \neq 0$, $\therefore e^{x+2} - 1 = 0$, $e^{x+2} = 1$, $x+2 = 0$, $x = -2$.

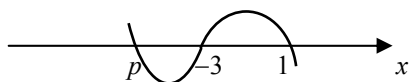
Q2 $3\cos(7x)+1$ is an even function. If $x = a$ and $x = b$ are the first two positive solutions to $3\cos(7x)+1 = 0$, then $x = -a$ and $x = -b$ are the first two negative solutions. Hence the sum = 0.

Q3 $\log_2(4a^p) = \log_2 4 + \log_2 a^p = 2 + \frac{\log_a a^p}{\log_a 2} = 2 + \frac{p}{\log_a 2}$.

Q4 For $f(x)$ to be defined, $(x+1)^2 > 0$, $\therefore x \neq -1$.

Q5 $|2x-1| < 1$ is equivalent to $(2x-1)^2 < 1$, $(2x-1)^2 - 1 < 0$, $[(2x-1)-1][(2x-1)+1] < 0$, $\therefore 4x(x-1) < 0$, $\therefore 0 < x < 1$.

Q6



Q7 From graph, $b = -\frac{5}{2}$, $c = 2$, $\therefore y = a\left(x - \frac{5}{2}\right)^2 + 2$.

The graph passes through $(0,0)$, $\therefore 0 = a\left(0 - \frac{5}{2}\right)^2 + 2$,

$\therefore a = -\frac{8}{25}$.

Q8 Transformation of $y = |x|$: From graph, $y = a|x-p|+3$.

The graph passes through $(0,0)$, $\therefore 0 = a|-p|+3$, $\therefore ap+3 = 0$,

$a = -\frac{3}{p}$.

Hence $y = -\frac{3|x-p|}{p} + 3 = 3\left(1 - \frac{|x-p|}{p}\right) = 3\left(1 - \frac{|p-x|}{p}\right)$.

Q9 Any relation has an inverse.

Q10 $f(x) \rightarrow f\left(x + \frac{1}{2}\right) \rightarrow f\left(x + \frac{1}{2}\right) - \frac{1}{4} \rightarrow -\left[f\left(x + \frac{1}{2}\right) - \frac{1}{4}\right]$,
 $\therefore g(x) = -f\left(x + \frac{1}{2}\right) + \frac{1}{4} = -\left[-\left(x + \frac{1}{2}\right)^2 + \left(x + \frac{1}{2}\right)\right] + \frac{1}{4} = x^2$.

Q11 Use graphics calculator to display $y = x + \sin\left(\frac{\pi x}{2}\right)$. In the interval $[0,4]$, the local minimum value is 1.7895 and the local maximum value is 2.2105. $\therefore x + \sin\left(\frac{\pi x}{2}\right) - c = 0$ will have more than one solution if $1.8 < c < 2.2$.

Q12 Use graphics calculator to display $N = 5 \times 2^{0.1t}$, determine $\frac{dN}{dt}$ at $t = 10$. $\frac{dN}{dt} \approx 0.7$.

Q13 At 6.00 am, $t = 6$, $h = 1.5 + 0.6\cos\pi = 0.9$.

At 8.00 am, $t = 8$, $h = 1.5 + 0.6\cos\frac{8\pi}{6} = 1.2$.

Average rate = $\frac{1.2 - 0.9}{8 - 6} = 0.150$.

Q14 Total area = $-\int_a^b (f(x) - g(x))dx + \int_b^c (f(x) - g(x))dx$
 $= \int_b^a (f(x) - g(x))dx + \int_b^c (f(x) - g(x))dx$.

Q15 $\int_0^1 2(x - f(x))dx = 2\int_0^1 (x - f(x))dx = 2\left(\int_0^1 xdx - \int_0^1 f(x)dx\right)$
 $= 2\left[\frac{x^2}{2}\right]_0^1 - 2[F(x)]_0^1 = 1 - 2(F(1) - F(0)) = 1 - 2F(1) + 2F(0)$.

Q16 For $\pi < x < 3\pi$, $f(x) = \left|\cos\left(\frac{x}{2}\right)\right| = -\cos\left(\frac{x}{2}\right)$,

$f'(x) = \frac{1}{2}\sin\left(\frac{x}{2}\right)$, $\therefore f'(a) = \frac{1}{2}\sin\left(\frac{a}{2}\right)$.

Q17 Let $f(x) = \sqrt{x}$, $f'(x) = \frac{1}{2\sqrt{x}}$, $a = 16$, $h = -1$.

$\sqrt{15} = \sqrt{16 + (-1)} \approx \sqrt{16} + (-1) \times \frac{1}{2\sqrt{16}} = 4 - 0.125 = 3.875$.

Q18 Check the gradient of the curve. As $x \rightarrow -\infty$, $f'(x) \rightarrow 0$. As $x \rightarrow \infty$, $f'(x) \rightarrow 0$. Gradient is always negative. Slope is steepest (most negative) at $x = 0$.

Q19 Graph becomes more symmetrical as n increases. Graph becomes more asymmetrical if p increases or decreases past 0.5.

$$\begin{aligned} \text{Q20 } \Pr(X \leq 2 | X \geq 1) &= \frac{\Pr(X \leq 2 \cap X \geq 1)}{\Pr(X \geq 1)} \\ &= \frac{\Pr(X=1) + \Pr(X=2)}{\Pr(X \geq 1)} = \frac{0.7}{0.9} = \frac{7}{9}. \end{aligned}$$

$$\begin{aligned} \text{Q21 } \Pr(X > \mu + 8) &= \Pr(X > \mu + 2\sigma) = \Pr(Z > 2) = 1 - \Pr(Z < 2). \end{aligned}$$

$$\begin{aligned} \text{Q22 } \int_1^2 k \sin(\pi x) dx &= 1, \left[\frac{-k \cos(\pi x)}{\pi} \right]_1^2 = 1, \\ \frac{-k \cos(2\pi)}{\pi} - \frac{-k \cos(\pi)}{\pi} &= 1, \frac{-2k}{\pi} = 1, \therefore k = -\frac{\pi}{2}. \end{aligned}$$

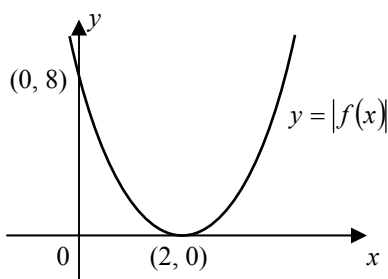
SECTION 2

$$\begin{aligned} \text{Q1a. } f(x) &= (x+b)^3 + c = x^3 + 3bx^2 + 3b^2x + b^3 + c \\ &= x^3 - 6x^2 + 12x + p, \therefore 3b = -6 \text{ and } b^3 + c = p, \\ \therefore b &= -2 \text{ and } c = p + 8. \end{aligned}$$

$$\begin{aligned} \text{Q1b. } x^3 - 6x^2 + 12x + p &= 0, \therefore (x-2)^3 + p + 8 = 0, \\ (x-2)^3 &= -(p+8), \therefore x-2 = \sqrt[3]{-(p+8)} = -\sqrt[3]{p+8}, \\ x &= 2 - \sqrt[3]{p+8}, \text{ which is defined for all real } p. \end{aligned}$$

$$\text{Q1ci. For } f(x) = (x-2)^3 + p + 8 \text{ to have a stationary point on the } x\text{-axis, } p + 8 = 0, p = -8.$$

Q1cii.



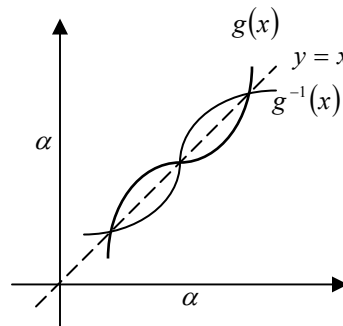
$$\begin{aligned} \text{Q1d. Since } f(x) &= (x+b)^3 + c, \\ \therefore f(x-b) &= ((x-b)+b)^3 + c = x^3 + c, \\ \therefore f(x-b) - c &= x^3. \\ \text{Compare with } f(x+u) + v &= x^3, u = -b = 2 \text{ and } \\ v = -c &= -p - 8. \end{aligned}$$

$$\begin{aligned} \text{Q1ei. For } p &= -7, f(x) = x^3 - 6x^2 + 12x + p = (x-2)^3 + 1. \\ \text{Equation of function } f &: y = (x-2)^3 + 1. \end{aligned}$$

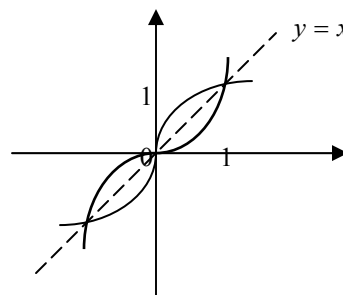
$$\begin{aligned} \text{Equation of function } f^{-1} &: x = (y-2)^3 + 1. \text{ Express } y \text{ as the} \\ \text{subject of the equation, } x-1 &= (y-2)^3, y-2 = \sqrt[3]{x-1}, \\ y &= \sqrt[3]{x-1} + 2. \therefore f^{-1}(x) = \sqrt[3]{x-1} + 2. \end{aligned}$$

$$\begin{aligned} \text{Q1eii. } y &= \sqrt[3]{x-1} + 2 = (x-1)^{\frac{1}{3}} + 2, \\ \frac{dy}{dx} &= \frac{1}{3}(x-1)^{-\frac{2}{3}} = \frac{1}{3(x-1)^{\frac{2}{3}}}. \text{ Maximal domain is } R \setminus \{1\}. \end{aligned}$$

Q1f. The graphs of $g(x) = (x-\alpha)^3 + \alpha$ and $g^{-1}(x)$ are shown below.



The total area of the enclosed regions is the same as the total area enclosed after vertical and horizontal translations by α .



$$\text{Total area} = 4 \times \int_0^1 (x - x^3) dx = 4 \left[\frac{x^2}{2} - \frac{x^4}{4} \right]_0^1 = 1.$$

$$\text{Q2a. } P(t) = Ae^{-at}. \text{ At } t = 0, P(0) = Ae^0 = A.$$

$$\text{Q2bi. } P(t) = Ae^{-at}, \frac{dP}{dt} = -aAe^{-at} = -aP, \therefore \frac{dP}{dt} \propto P.$$

$$\begin{aligned} \text{Q2bii. Since } \frac{dP}{dt} &\propto P, \therefore \frac{dP}{dt} \text{ is halved when } P \text{ is halved, i.e.} \\ P &= \frac{1}{2}A. \therefore \frac{1}{2}A = Ae^{-at}, e^{-at} = \frac{1}{2}, e^{at} = 2, at = \log_e 2, \\ t &= \frac{\log_e 2}{a}. \end{aligned}$$

$$\text{Q2ci. } D(t) = P(0) - P(t) = A - P(t).$$

$$\begin{aligned} \text{Q2cii. } A &= D(t) + P(t), \frac{A}{P(t)} = \frac{D(t) + P(t)}{P(t)}, \frac{A}{P(t)} = \frac{D(t)}{P(t)} + 1, \\ e^{at} &= \frac{D(t)}{P(t)} + 1, \therefore at = \log_e \left(\frac{D(t)}{P(t)} + 1 \right), t = \frac{1}{a} \log_e \left(\frac{D(t)}{P(t)} + 1 \right). \end{aligned}$$

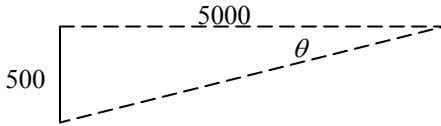
Q2di.

$$t = \frac{1}{a} \log_e \left(\frac{D(t)}{P(t)} + 1 \right) = \frac{1}{1.39 \times 10^{-11}} \log_e (0.0196 + 1) = 1.40 \times 10^9$$

Q2dii. Let $r = \frac{D(t)}{P(t)}$, $t = \frac{1}{a} \log_e (r + 1)$, $\frac{dt}{dr} = \frac{1}{a(r+1)}$.

$$\Delta t \approx \frac{dt}{dr} \Delta r = \frac{\Delta r}{a(r+1)} = \frac{0.00130}{1.39 \times 10^{-11} (0.0196 + 1)} = 9.17 \times 10^7.$$

Q3ai.



$$\tan \theta = \frac{500}{5000}, \theta = 0.100. \text{ Lower bound for } \theta \text{ is } -0.100.$$

Q3aii. $\frac{h-500}{5000} = \tan \theta, \therefore h = 5000 \tan \theta + 500.$

Q3b. $h = 5000 \tan \theta + 500, \frac{dh}{d\theta} = 5000 \sec^2 \theta = \frac{5000}{\cos^2 \theta}$

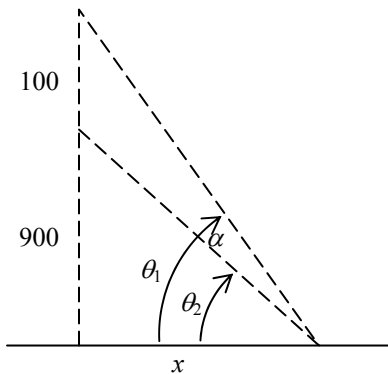
Related rates: $\frac{dh}{dt} = \frac{dh}{d\theta} \times \frac{d\theta}{dt}, \therefore \frac{dh}{dt} = \frac{5000}{\cos^2 \theta} \times \frac{d\theta}{dt}.$

Hence $\frac{d\theta}{dt} = \frac{\cos^2 \theta}{5000} \times \frac{dh}{dt}.$

Q3c. If $\frac{dh}{dt}$ is constant, then $\frac{d\theta}{dt} \propto \cos^2 \theta.$

Since $-0.100 \leq \theta < \frac{\pi}{2}$, $\therefore \cos^2 \theta$ and hence $\frac{d\theta}{dt}$ is maximum when $\theta = 0.$

Q3di.



$$\tan \theta_1 = \frac{1000}{x}, \theta_1 = \tan^{-1} \left(\frac{1000}{x} \right).$$

$$\tan \theta_2 = \frac{900}{x}, \theta_2 = \tan^{-1} \left(\frac{900}{x} \right).$$

$$\therefore \alpha = \theta_1 - \theta_2 = \tan^{-1} \left(\frac{1000}{x} \right) - \tan^{-1} \left(\frac{900}{x} \right).$$

Q3dii. Use graphics calculator to sketch

$\alpha = \tan^{-1} \left(\frac{1000}{x} \right) - \tan^{-1} \left(\frac{900}{x} \right).$ Find x where maximum α occurs, $x = 949$ m.

Q3diii. $\alpha = 0.052656^\circ = 0.052656 \times \frac{180^\circ}{\pi} = 3.02^\circ.$

Q3div. For $x \geq 2000$, α is maximum at $x = 2000$,

$$\alpha = 0.040794 \times \frac{180^\circ}{\pi} = 2.34^\circ.$$

Q4a.

$$\Pr(4.95 \leq L \leq 5.05) = \text{normalcdf}(4.95, 5.05, 5.00, 0.02) = 0.988$$

Q4b. $\Pr(3.92 \leq d \leq 4.08) = \int_{3.92}^{4.08} 750(d - 3.9)(4.1 - d) dd$
 $= 0.944$ (by graphics calculator)

Q4c. Proportion acceptable $= 0.988 \times 0.944 = 0.933$,
 $\therefore$ proportion unacceptable $= 1 - 0.933 = 0.067.$

Q4d.

	L	L'	
d	0.933	0.011	0.944
d'	0.055	0.001	0.056
	0.988	0.012	1

$$\text{Required proportion} = \frac{0.055}{0.067} = 0.821.$$

Q4e. $95\% \times 20 = 19.$

Binomial distribution: $n = 20$, $p = 0.933$, $x \geq 19$,

$$\Pr(X \geq 19) = \Pr(X = 19) + \Pr(X = 20) = 0.3588 + 0.2498 = 0.609$$

Q4f. $\Pr(\text{second inspection}) = 0.609 \times 0.3 + 0.391 \times 0.9 = 0.535$

$$\Pr(\text{no second inspection}) = 1 - 0.535 = 0.465.$$

Please inform mathline@itute.com re conceptual, mathematical and/or typing errors